Concepts and Examples
Introduction to Vectors – Part 1

Based on power point presentations by Pearson Education, Inc.

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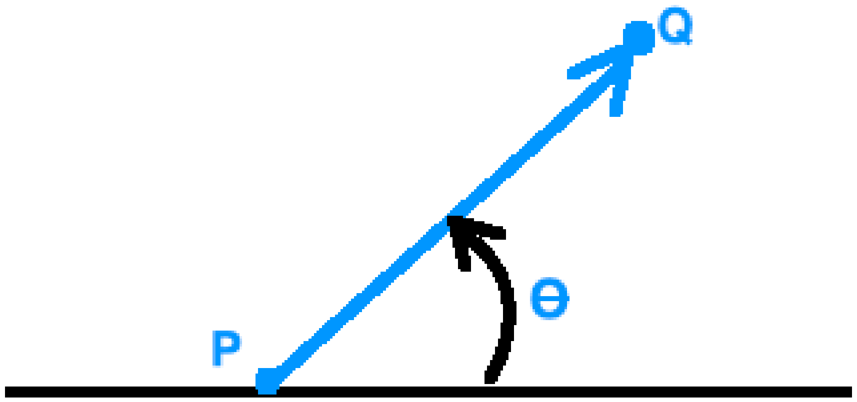
Learning Objectives

1. Memorize the definition of a vector.
2. Use vector notation.
3. Write a vector in component form.
4. Find the magnitude of a vector.
5. Find the direction of a vector.
6. Perform arithmetic operations on vectors.

1. Definition of a Vector (1 of 2)

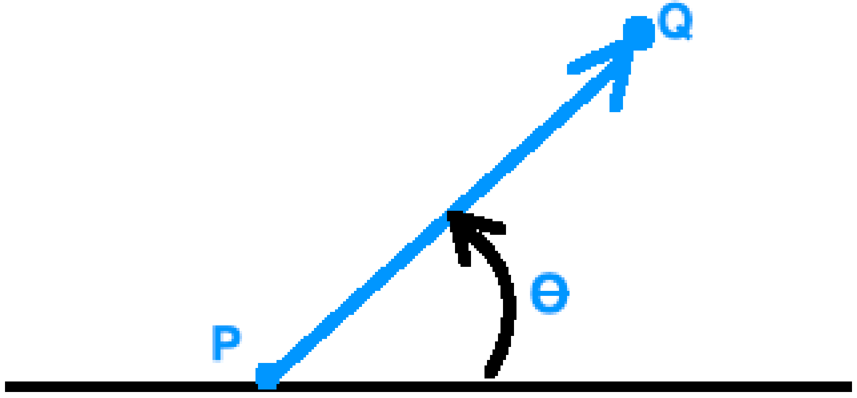
Many quantities in physics, such as area, time, and temperature, can be represented by a single number. Other quantities, such as force and velocity, must be represented by **vectors**. They require both magnitude and direction.

Let's draw a line segment (arrow on one end) making a positive angle θ with a horizontal line. Let's give it a beginning point ***P*** and an ending point ***Q*** at the tip of its arrow.



Definition of a Vector (2 of 2)

Let's copy the picture from the previous slide. When we describe the line segment in terms of the angle **θ** and the distance between the points ***P*** and ***Q***, we call it a vector.



The angle **θ** is the **direction** of the vector and is always positive. The distance between point ***P*** and point ***Q*** is called the **magnitude** of the vector.

Please note, from now on we will call point ***P*** the **initial point** and point ***Q*** the **terminal point** of a vector

2. Vector Notation

Vectors are denoted by lower-case letters, such as **a**, **b**, or **c**, etc. or by their initial and terminal points. In mathematics, we often use **u**, **v**, and **w**.

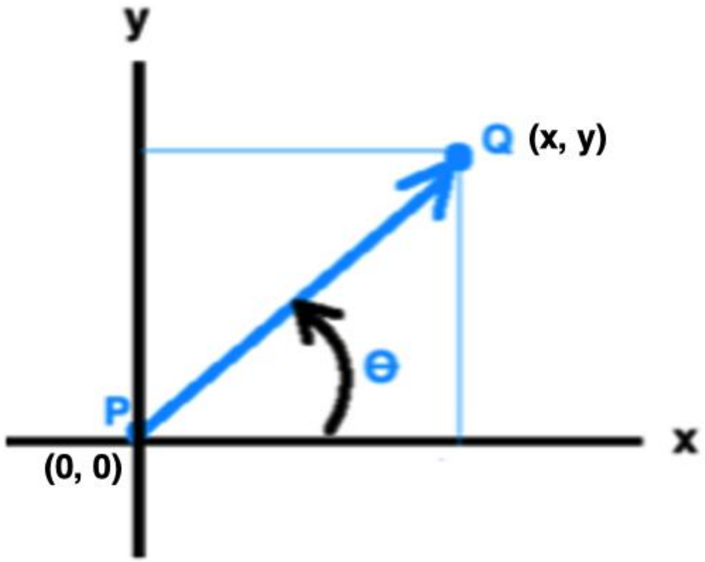
In handwritten documents, a half arrow is placed over either notation. For example, given a vector with initial point ***P*** and terminal point ***Q***, we could name the vector $\overrightarrow{PQ}$. Or we could simply call it, say, $\overrightarrow{v}$.

In printed documents, vector names are shown in bold print without the half arrow. For example, **PQ** or **v**.

3. The Component Form of a Vector (1 of 5)

Vectors are often written in what is called **component form**. To accomplish this, we must first place the vector into **standard position**. That is, we must place its initial point at the origin of a rectangular coordinate system. Let's name its horizontal axis ***x*** and its vertical axis ***y***.

Let's use the vector $\overrightarrow{PQ}$ from our earlier discussion and place it into standard position.



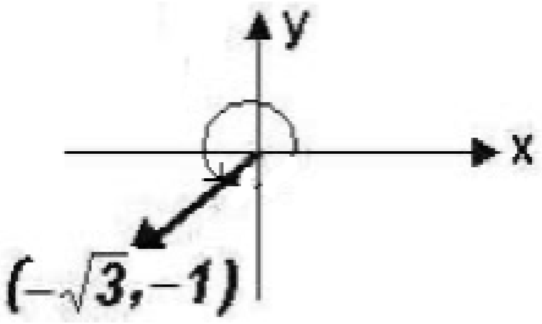
We will let (*x*, *y*) be the coordinates of the point ***Q***.

These coordinates are then used to write the **component form** of the vector $\overrightarrow{PQ}$ as $\langle x, y\rangle$ using angle brackets $\langle\ \rangle$. In vector algebra, we call *x* and *y* the **components** of a vector.

The Component Form of a Vector (2 of 5)

Example 1:

Write the following vector in component form.

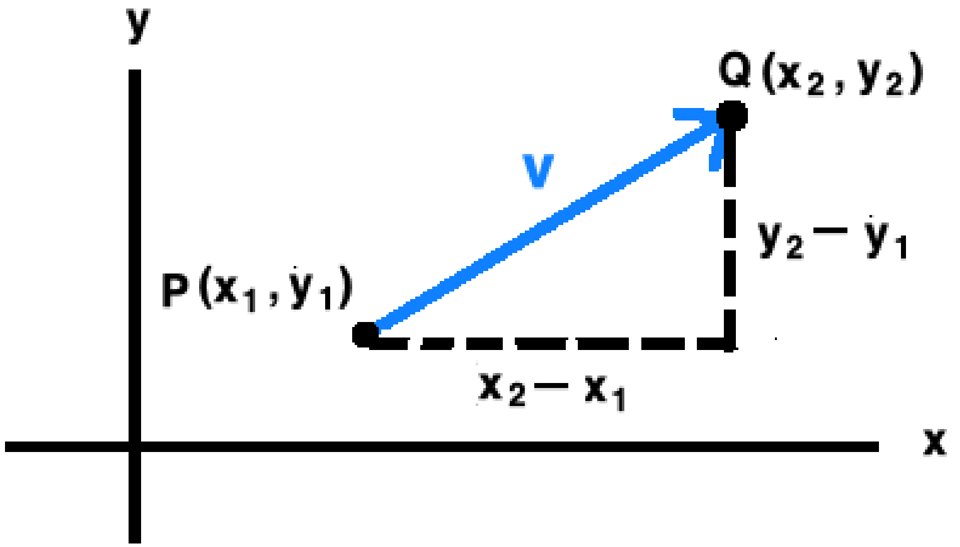


We see that the terminal point has coordinates $(-\sqrt{3},-1)$. Therefore, the component form of the vector is $\langle -\sqrt{3},-1 \rangle$.

The Component Form of a Vector (3 of 5)

Sometimes, vectors are not in standard position in a rectangular coordinate system. Then we must perform some calculations if we want to find their component form.

Let's assume we are given a vector **v** with initial point ***P*** and terminal point ***Q***. It lies in a rectangular coordinate system, but it is not in standard position. See picture below.



Then, when we move vector **v** to standard position, its component form is the formula

$$\langle x_2 - x_1, y_2 - y_1 \rangle$$

Please note it is "terminal minus initial" for both the *x* and the *y* components.

The Component Form of a Vector (4 of 5)

Example 2:

Let **v** be a vector in a rectangular coordinate system with initial point ***P*** at (3, – 1) and terminal point ***Q*** at (– 2, 1). Write the vector **v** in component form.

Basically, we are asked to place the vector into standard position.

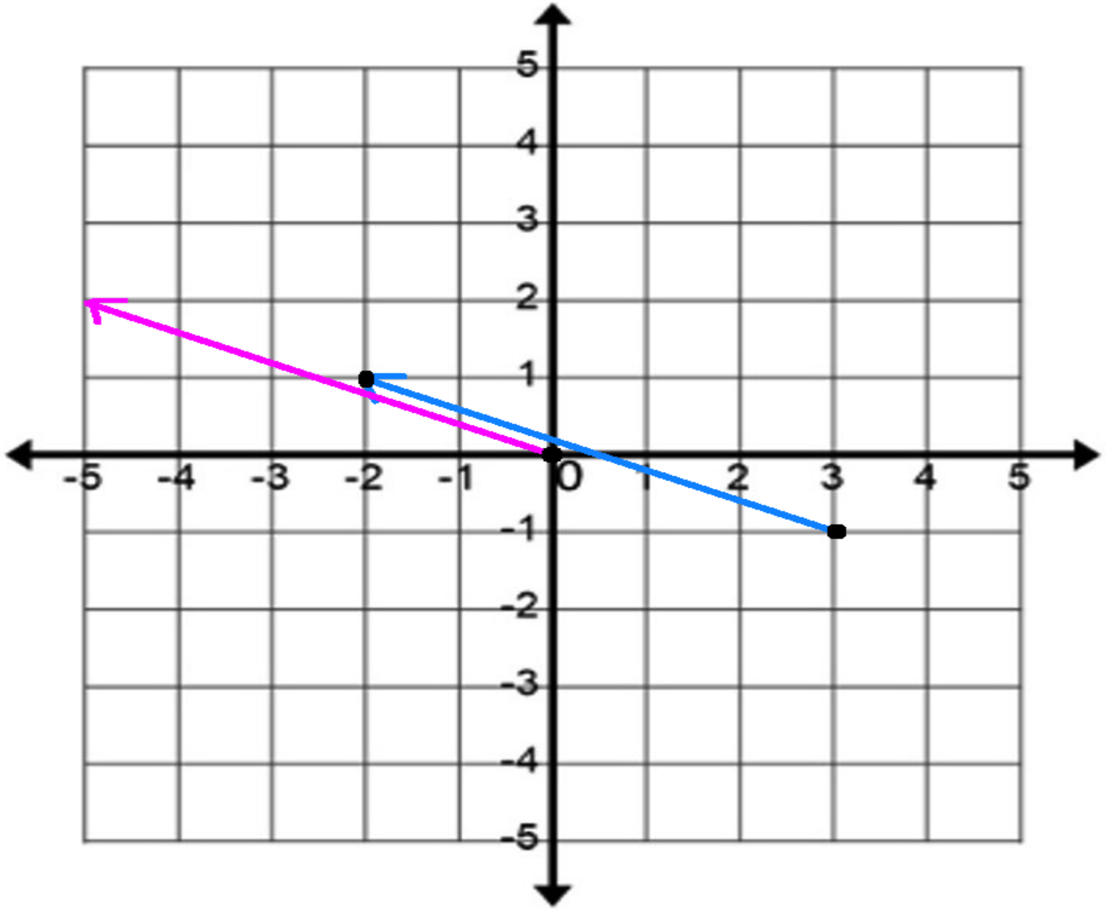
We just found the formula $\langle x_2 - x_1, y_2 - y_1 \rangle$ which is the component form of a vector that has been moved from somewhere in the coordinate system to standard position. While developing the formula, we assigned the coordinates (x_1, y_1) to the initial point ***P*** and (x_2, y_2) to the terminal point **Q**.

Therefore, the component form of vector **v** must be $\langle -2 - 3, 1 - (-1) \rangle$.
Simplifying, we find the component form of vector **v** to be $\langle -5, 2 \rangle$.

The Component Form of a Vector (5 of 5)

Example 2 continued:

Following is a graph of vector **v** in non-standard form (blue vector) and vector **v** in standard form (pink vector). Note that they are parallel!



4. The Magnitude of a Vector

The "length" of a vector is called **magnitude**. In physics it often represent speed or weight among other things.

The magnitude of some vector **v** in standard position in a rectangular coordinate system whose component form is, say $\langle x, y \rangle$, is denoted by $||\mathbf{v}||$ where $||\mathbf{v}||= \sqrt{x^2 + y^2}$.

Example 3:

Calculate the EXACT magnitude of vector **w** = $\langle 3{,}2 \rangle$.

Given $||v||= \sqrt{x^2 + y^2}$, we find the magnitude of **w** to be

$$\sqrt{3^2 + 2^2} = \sqrt{13}\ .$$

5. The Direction of a Vector (1 of 4)

The **direction** of some vector **v** in standard position whose component form is, say $\langle x, y \rangle$, is given by a positive angle $\boldsymbol{\theta}$ between the positive horizontal axis and the vector **v**.

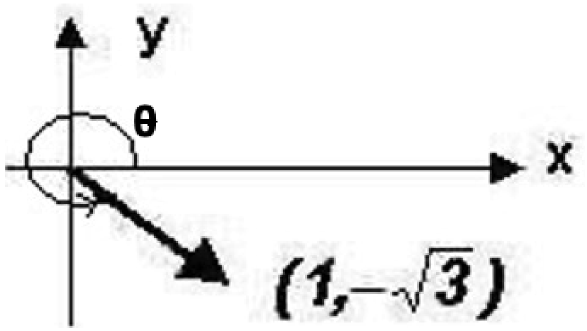
Let's assume the vector **v** lies in an *xy*-coordinate system. Then we calculate $\boldsymbol{\theta}$ using $\tan\theta = \frac{y}{x}$ which was derived in the lesson on polar coordinates.
Note that *x* and *y* refer to the terminal point of the vector **v**.

The Direction of a Vector (2 of 4)

Example 4:

Find the direction of vector $\mathbf{v} = \langle 1, -\sqrt{3} \rangle$.

Let's look at a picture of the vector and the angle $\boldsymbol{\theta}$ it makes with the positive *x*-axis. This is the angle that will tell us the direction of the vector.



We will now use $\tan\theta = \dfrac{y}{x}$ and the coordinates of the terminal point to find $\tan\theta = \dfrac{-\sqrt{3}}{1} = -\sqrt{3}$.

The Direction of a Vector (3 of 4)

Example 4 continued:

When solving $\tan\theta = \frac{-\sqrt{3}}{1} = -\sqrt{3}$ for the angle $\boldsymbol{\theta}$ using a calculator, we find $\theta = \tan^{-1}(-\sqrt{3}) = -60^{o}$. Be sure the calculator is in degree mode!

Remember that the range of the inverse tangent consists of angles between -90^{o} and 90^{o} !!! The calculator will give us no other angles.

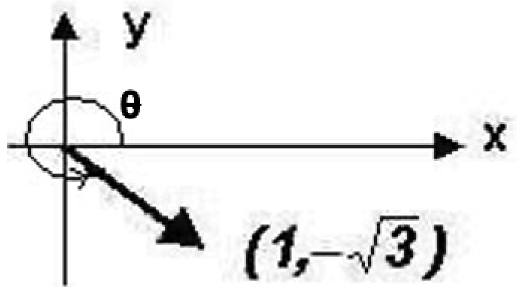
Since all vector directions are given as positive angles, we need to find an angle that is coterminal to -60^{o} and positive. We do this by using the reference angle of -60^{o} which is 60^{o}.

You might want to review Lesson 4 on reference angles!

The Direction of a Vector (4 of 4)

Example 4 continued:

From the picture we drew previously, that is



we know that the vector lies in QIV. Therefore, we can easily conclude that the angle $\boldsymbol{\theta} = 360^o - 60^o = 300^o$ is the angle that tells us the direction of the vector.

6. Arithmetic Operations on Vectors (1 of 2)

Addition

Given a vector in component form, we add corresponding components.

For example, given vectors $\mathbf{v} = < 2, 5 >$ and $\mathbf{w} = < 3, -4 >$, then

$\mathbf{v} + \mathbf{w} = < 2 + 3, 5 + (-4) > = < 5, 1 >$

Subtraction

Given a vector in component form, we subtract corresponding components.

For example, given vectors $\mathbf{v} = < 2, 5 >$ and $\mathbf{w} = < 3, -4 >$, then

$\mathbf{v} - \mathbf{w} = < 2 - 3, 5 - (-4) > = < -1 , 9 >$ and

$\mathbf{w} - \mathbf{v} = < 3 - 2, -4 - 5 > = < 1 , -9 >$

Arithmetic Operations on Vectors (2 of 2)

Multiplication

There are several different types of multiplications when working with vectors. Some result in vectors and some in a **scalars****. In this lesson, we will discuss **scalar** multiplication**. It results in a vector.

******In vector algebra, all "non-vectors" are called scalars.

In scalar multiplication, every vector component is multiplied by a given scalar.

For example, multiply vector $\mathbf{v} = < 2, 5 >$ by the scalar 3, which is just a number. We write $3\mathbf{v} = 3 < 2, 5 > = < 3(2), 3(5) > = < 6, 15 >$.

Note that there is implied multiplication between the scalar and the vector notation **v** as well as the scalar and the component form of the vector **v**.