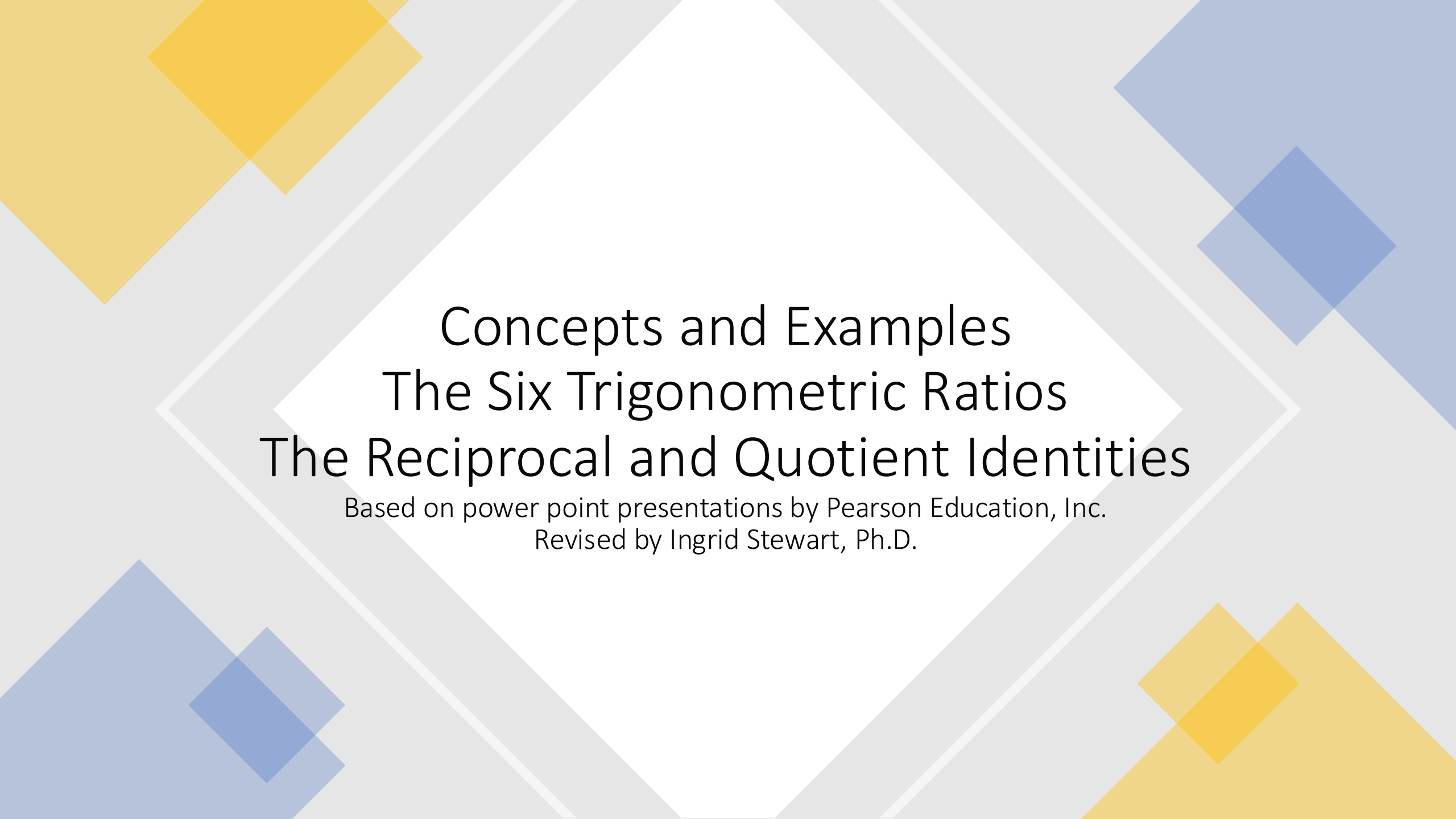




Concepts and Examples The Six Trigonometric Ratios The Reciprocal and Quotient Identities

Based on power point presentations by Pearson Education, Inc.
Revised by Ingrid Stewart, Ph.D.

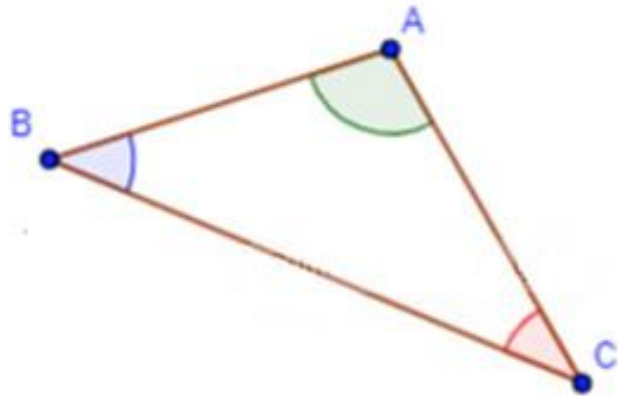
Learning Objectives

1. Define the right triangle and the unit circle.
2. Memorize and use the six trigonometric ratios in a right triangle.
3. Memorize and use the reciprocal and quotient identities.

1. The Right Triangle and the Unit Circle (1 of 3)

Before we begin our study of trigonometry, we will review the **right triangle** and the **unit circle** because we will be using them frequently throughout the semester.

A triangle is a closed shape in the plane formed by three (3) line segments that intersect only at their endpoints.



Interior Angles of a Triangle

The **interior angles** of a triangle are the angles inside the triangle. **The sum of the measures of the three interior angles is always 180° .**

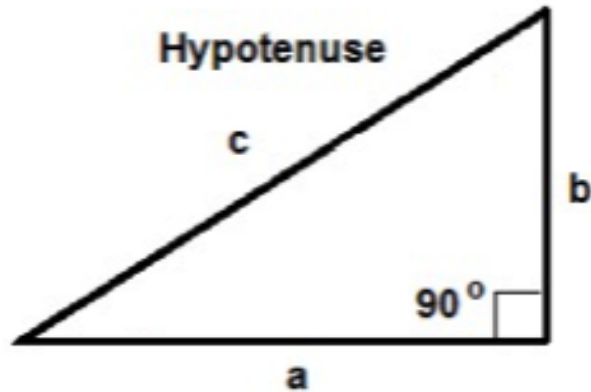
Sides of a Triangle

The longest side of a triangle is opposite the largest angle and vice versa. The shortest side of a triangle is opposite the smallest angle and vice versa.

The Right Triangle and the Unit Circle (2 of 3)

A triangle in which one angle is 90° is called a **right triangle**. The side opposite the right angle is called the **hypotenuse** and the remaining two sides are called **legs**.

In the picture below, the length of the hypotenuse is called ***c*** and the lengths of the two legs are called ***a*** and ***b***. Often the 90° is indicated by a square in one corner.

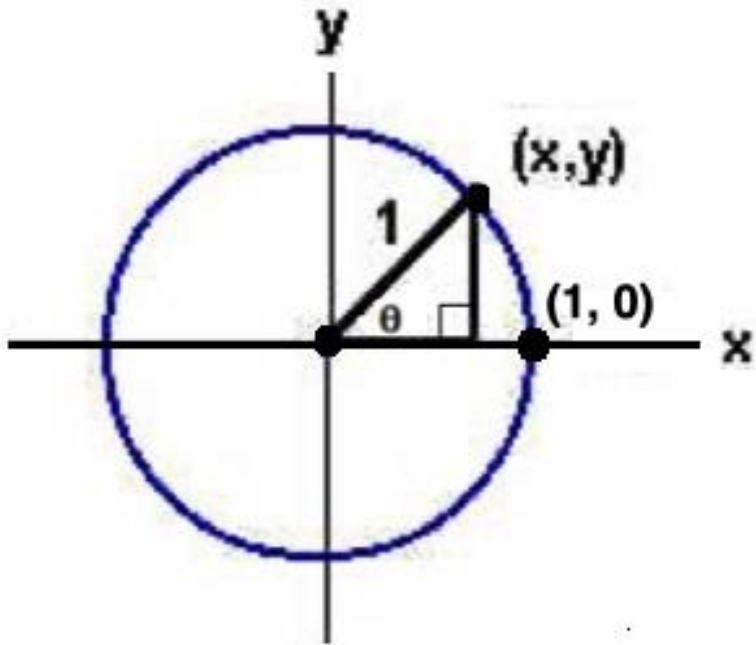


The sides of a right triangle are related via the *Pythagorean Theorem* as follows:

$$a^2 + b^2 = c^2$$

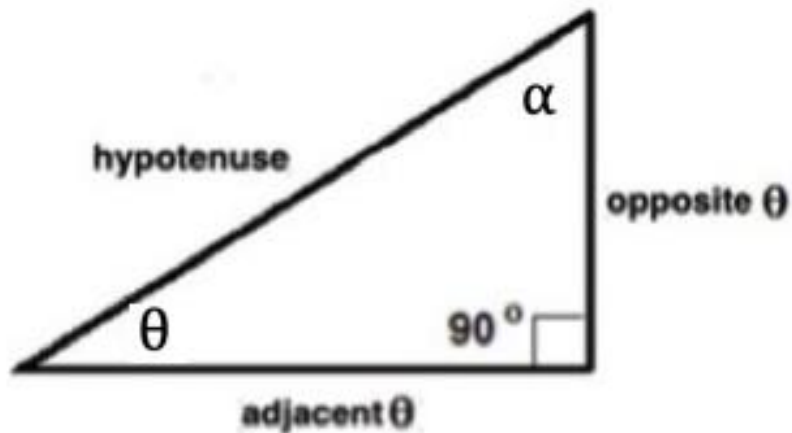
The Right Triangle and the Unit Circle (3 of 3)

A circle with **radius 1** is called a **unit circle**. In the picture below, the unit circle is placed into a rectangular coordinate system with an x - and y -axis.



2. Definition of the Six Trigonometric Ratios (1 of 8)

Trigonometric ratios are the “backbone” of trigonometry. **They are always based on some angle.** In this course, we will use the angle θ in a right triangle (see picture below) to define the six trigonometric ratios.



In the definitions of the trigonometric ratios, we will use the following abbreviations:

adj - the side adjacent to angle θ

opp - the side opposite angle θ

hyp - the hypotenuse

NOTE: In some mathematics classes the six trigonometric ratios are defined using the unit circle. But we will not do this in our course.

Definition of the Six Trigonometric Ratios (2 of 8)

Sine Ratio: $\sin \theta = \frac{\text{opp}}{\text{hyp}}$

Pronounce "sin" as "sine". Do NOT ever pronounce it as **sin**!

Cosine Ratio: $\cos \theta = \frac{\text{adj}}{\text{hyp}}$

Pronounce "cos" as "cosine". Here you can simply say **cos**.

Tangent Ratio: $\tan \theta = \frac{\text{opp}}{\text{adj}}$

Pronounce "tan" as "tangent". Here you can simply say **tan**.

You **MUST** memorize these three ratios! A handy Memorization Aid for the Sine, Cosine, and Tangent Ratios (there may be others):

SOH CAH TOA

Definition of the Six Trigonometric Ratios (3 of 8)

Cosecant Ratio: $\csc \theta = \frac{\text{hyp}}{\text{opp}}$

Pronounce "csc" as "cosecant". Do NOT ever pronounce it as **csc**!

NOTE: The Cosecant Ratio is the reciprocal of the Sine Ratio!

Secant Ratio: $\sec \theta = \frac{\text{hyp}}{\text{adj}}$

Pronounce "sec" as "secant". Do NOT ever pronounce it as **sed**!

NOTE: The Secant Ratio is the reciprocal of the Cosine Ratio!

Cotangent Ratio: $\cot \theta = \frac{\text{adj}}{\text{opp}}$

Pronounce "cot" as "cotangent". Do NOT ever pronounce it as **cot**!

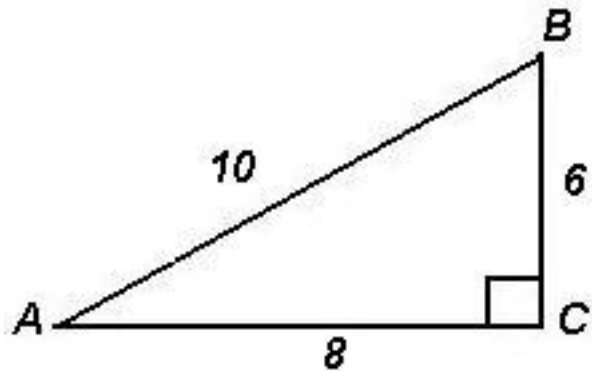
NOTE: The Cotangent Ratio is the reciprocal of the Tangent Ratio!

You MUST memorize these three ratios!

Definition of the Six Trigonometric Ratios (4 of 8)

Example 1:

Given the following triangle, find the numeric values of the sine, cosine, and tangent ratios of angles **A** and **B**. Express your answers both as a fraction and a decimal.



Ratios for Angle A:

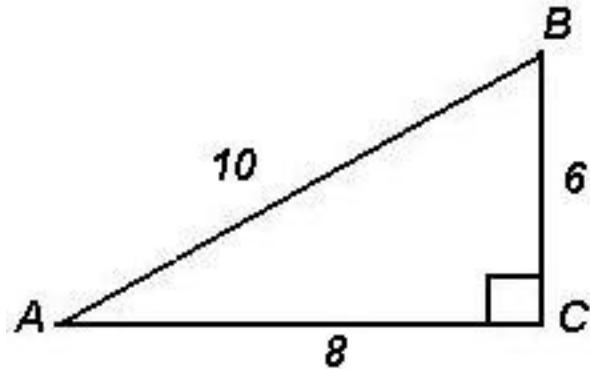
$$\sin A = \frac{\text{opp}}{\text{hyp}} = \frac{6}{10} = \frac{3}{5} = 0.6$$

$$\cos A = \frac{\text{adj}}{\text{hyp}} = \frac{8}{10} = \frac{4}{5} = 0.8$$

$$\tan A = \frac{\text{opp}}{\text{adj}} = \frac{6}{8} = \frac{3}{4} = 0.75$$

Definition of the Six Trigonometric Ratios (5 of 8)

Example 1 continued:



Ratios for Angle B:

$$\sin B = \frac{\text{opp}}{\text{hyp}} = \frac{8}{10} = \frac{4}{5} = 0.8$$

$$\cos B = \frac{\text{adj}}{\text{hyp}} = \frac{6}{10} = \frac{3}{5} = 0.6$$

$$\tan B = \frac{\text{opp}}{\text{adj}} = \frac{8}{6} = \frac{4}{3} \approx 1.33$$

Please note that the location of the "side opposite" and the "side adjacent" changes with the location of the angle in the triangle. The values for the trigonometric ratios of angle **A** are different from the values of the trigonometric ratios of angle **B**.

Definition of the Six Trigonometric Ratios (6 of 8)

Example 2:

Given $\cos \theta = \frac{\sqrt{2}}{2}$, use appropriate trigonometric ratios to find the exact value of the *sine* and *tangent* ratios of angle θ .

Knowing that $\cos \theta = \frac{\text{adj}}{\text{hyp}}$, we also know the following:

adj (side adjacent to the angle θ) is $\sqrt{2}$ units in length

hyp (hypotenuse) is **2** units in length

We can now find **opp** (side opposite the angle θ) by using the *Pythagorean Theorem* $a^2 + b^2 = c^2$.

NOTE: Unless you are told otherwise, **c** is always the hypotenuse of a right triangle and **a** and **b** are the legs.

Definition of the Six Trigonometric Ratios (7 of 8)

Example 2 continued:

Let's now find **opp** (side opposite the angle). Let's assume that it is ***b*** in the *Pythagorean Theorem*!

$$2^2 = (\sqrt{2})^2 + b^2$$

$$4 = 2 + b^2$$

$$2 = b^2$$

Now we'll use the *Square Root Property* to find the length of ***b = opp***:

$$\pm\sqrt{2} = b$$

Since we are working with a right triangle whose sides are never negative, we find ***b = opp*** = $\sqrt{2}$. DO NOT CHANGE TO A DECIMAL EQUIVALENT!

Definition of the Six Trigonometric Ratios (8 of 8)

Example 2 continued:

Now we are ready to find the sine and cosine ratios given ***opp*** = $\sqrt{2}$,
adj = $\sqrt{2}$, and ***hyp*** = 2.

Reminder: $\sin \theta = \frac{\text{opp}}{\text{hyp}}$ and $\tan \theta = \frac{\text{opp}}{\text{adj}}$

Finally, we find what was requested:

$$\sin \theta = \frac{\sqrt{2}}{2}$$

$$\tan \theta = \frac{\sqrt{2}}{\sqrt{2}} = 1$$

3. The Reciprocal and Quotient Identities (1 of 5)

In trigonometry, a great deal of time is spent studying relationships between trigonometric ratios. We call these relationships "identities." In this lesson we will discuss the Reciprocal and the Quotient Identities. **You MUST memorize these identities!**

Reciprocal Identities:

$$\csc \theta = \frac{1}{\sin \theta} \qquad \sec \theta = \frac{1}{\cos \theta} \qquad \cot \theta = \frac{1}{\tan \theta}$$

OR

$$\sin \theta = \frac{1}{\csc \theta} \qquad \cos \theta = \frac{1}{\sec \theta} \qquad \tan \theta = \frac{1}{\cot \theta}$$

The Reciprocal and Quotient Identities (2 of 5)

Quotient Identities:

$$\mathbf{\tan \theta = \frac{\sin \theta}{\cos \theta}} \quad \text{and} \quad \mathbf{\cot \theta = \frac{\cos \theta}{\sin \theta}}$$

Definition of the Six Trigonometric Ratios (3 of 5)

Example 3:

Given a right triangle for which $\sin \theta = \frac{\sqrt{3}}{2}$ and $\cos \theta = \frac{1}{2}$, use the *Reciprocal* and *Quotient Identities* to find the remaining four trigonometric ratios.

Let's use the *Reciprocal Identities* $\csc \theta = \frac{1}{\sin \theta}$ and $\sec \theta = \frac{1}{\cos \theta}$ to find the *cosecant* and *secant ratios*.

$$\csc \theta = \frac{1}{\frac{\sqrt{3}}{2}} = 1 \cdot \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \quad \text{and} \quad \sec \theta = \frac{1}{\frac{1}{2}} = 1 \cdot \frac{2}{1} = 2$$

Please note that there is a radical in the denominator of the cosecant. While this is usually a "no-no" in algebra, in trigonometry it is often okay to leave the radical in the denominator.

Definition of the Six Trigonometric Ratios (4 of 5)

Example 3 continued:

If you do want to rationalize the denominator, you must multiply the numerator and denominator by $\sqrt{3}$ as follows:

$$\csc \theta = \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

Now let's use the *Quotient Identities* $\tan \theta = \frac{\sin \theta}{\cos \theta}$ and $\cot \theta = \frac{\cos \theta}{\sin \theta}$ to find the *tangent* and *cosecant* ratios.

$$\tan \theta = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{2} \cdot \frac{2}{1} = \sqrt{3} \quad \text{and} \quad \cot \theta = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

Definition of the Six Trigonometric Ratios (5 of 5)

Example 3 continued:

If you do want to rationalize the denominator of the *cotangent* ratio, you must multiply the numerator and denominator by $\sqrt{3}$ as follows:

$$\cot \theta = \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$