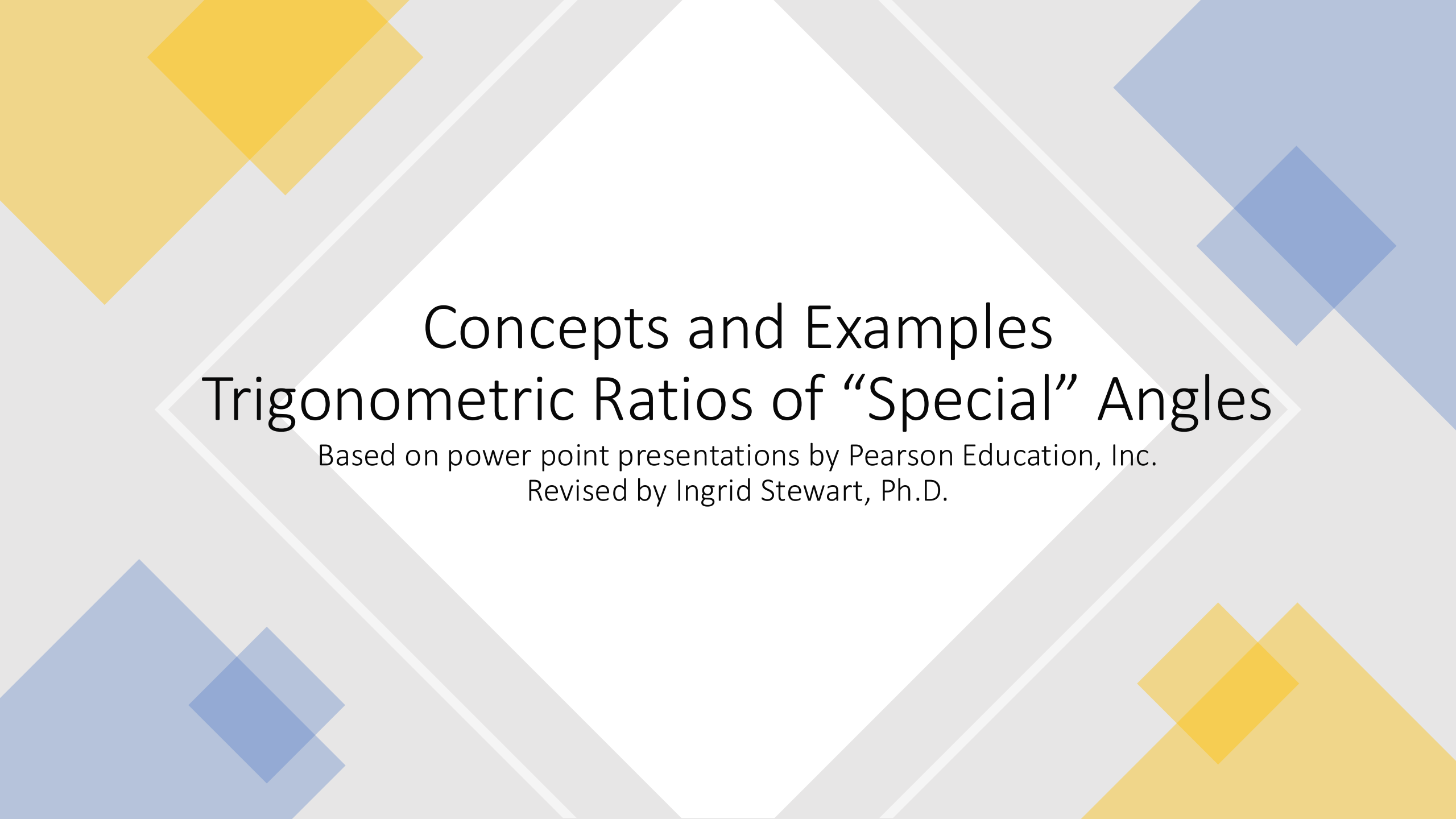




Concepts and Examples

Trigonometric Ratios of “Special” Angles

Based on power point presentations by Pearson Education, Inc.

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Learning Objective

Find and **memorize** the EXACT values of trigonometric ratios of the "special" angles 30° , 45° , and 60° . We will NOT use a calculator!

Find the EXACT Values of Trigonometric Ratios of “Special” Angles (1 of 7)

In the previous lesson, we found values of the six trigonometric ratios of an angle using the sides of a right triangle. You may have noticed that we did not need actual angle measures to do this.

In this lesson, we will find values of trigonometric ratios when the angle measures in a right triangle are known. Specifically, we will find the EXACT** values of their trigonometric ratios of the special angles 30° , 45° , and 60° within a right triangle. **NOTE: We will NOT use a calculator for homework and on the Module 1 Quiz!**

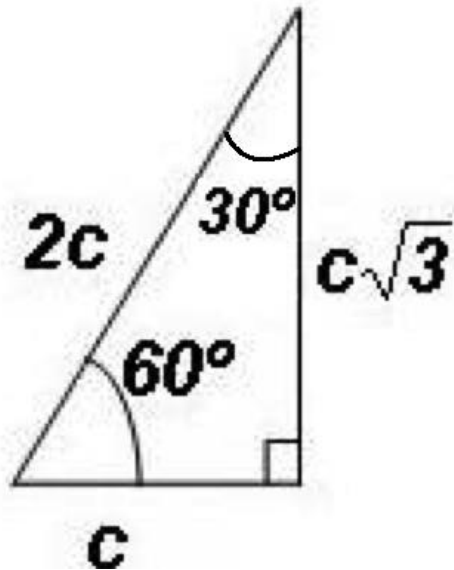
** What is an EXACT value? For example, $\sqrt{3}$ is considered an EXACT value, while its decimal approximation 1.732 is not considered to be exact.

Likewise, $\sqrt{2}$ is an EXACT value, while its decimal approximation is 1.414 is not considered exact.

Find the Values of Trigonometric Ratios of “Special” Angles

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First, let's derive the values of the six trigonometric ratios of the 30° and 60° angles. They can both exist in a right triangle because $30^\circ + 60^\circ + 90^\circ = 180^\circ$ which is always the sum of any triangle.



Hopefully, from your high school geometry class you remember that the sides of a 30-60-90 triangle are in the proportions $c, c\sqrt{3}, 2c$, where $2c$ is the length of the hypotenuse of the triangle. Note that the 90° angle is indicated by a square in the right corner!

Find the Values of Trigonometric Ratios of “Special” Angles

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We will let $c = \frac{1}{2}$, although c could take on any value and we would still get the same answers!!! Then the length of the hypotenuse is $2c = 2(\frac{1}{2}) = 1$ and the length of the other leg is $c\sqrt{3} = \frac{1}{2}(\sqrt{3}) = \frac{\sqrt{3}}{2}$.

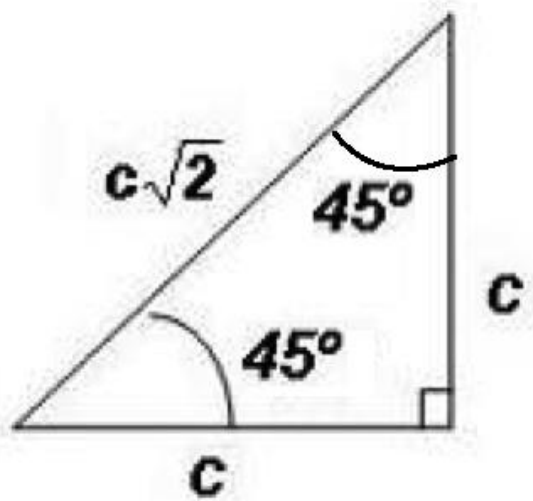
Using the definition of the six trigonometric ratios, please convince yourself that the EXACT values of the six trigonometric ratios of the 30° and 60° angles are as follows. Please note that certain denominators were rationalized!

x	$\sin x$	$\cos x$	$\tan x$	$\csc x$	$\sec x$	$\cot x$
$30^\circ \equiv \frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
$60^\circ \equiv \frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$

Find the Values of Trigonometric Ratios of “Special” Angles

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Next, let's derive the values of the six trigonometric ratios of the 45° angle. Two exist in a right triangle because $45^\circ + 45^\circ + 90^\circ = 180^\circ$.



Hopefully, from your high school geometry class you remember that the sides of a 45-45-90 triangle are in the proportions $c, c, c\sqrt{2}$, where $c\sqrt{2}$ is the length of the hypotenuse of the triangle. Note that the 90° angle is indicated by a square in the right corner!

Find the Values of Trigonometric Ratios of “Special” Angles

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We will let $c = \frac{1}{2}$, although c could take on any value and we would still get the same answers!!! Then the length of the hypotenuse is $c\sqrt{2} = \frac{1}{2}(\sqrt{2}) = \frac{\sqrt{2}}{2}$.

Using the definition of the six trigonometric ratios, please convince yourself that the EXACT values of the six trigonometric ratios of the 45° angle are as follows. Please note that certain denominators were rationalized!

x	$\sin x$	$\cos x$	$\tan x$	$\csc x$	$\sec x$	$\cot x$
$45^\circ \equiv \frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1

Find the Values of Trigonometric Ratios of “Special” Angles

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The following table is a summary of what we just found in the previous slides. The EXACT values of trigonometric ratios of the "special" angles 30° , 45° , and 60° MUST be memorized because they are used frequently in mathematics, physics, and engineering. We will NOT use a calculator to find these values for homework and the Module 1 Quiz!

x	$\sin x$	$\cos x$	$\tan x$	$\csc x$	$\sec x$	$\cot x$
$30^\circ \equiv \frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
$45^\circ \equiv \frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$60^\circ \equiv \frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$

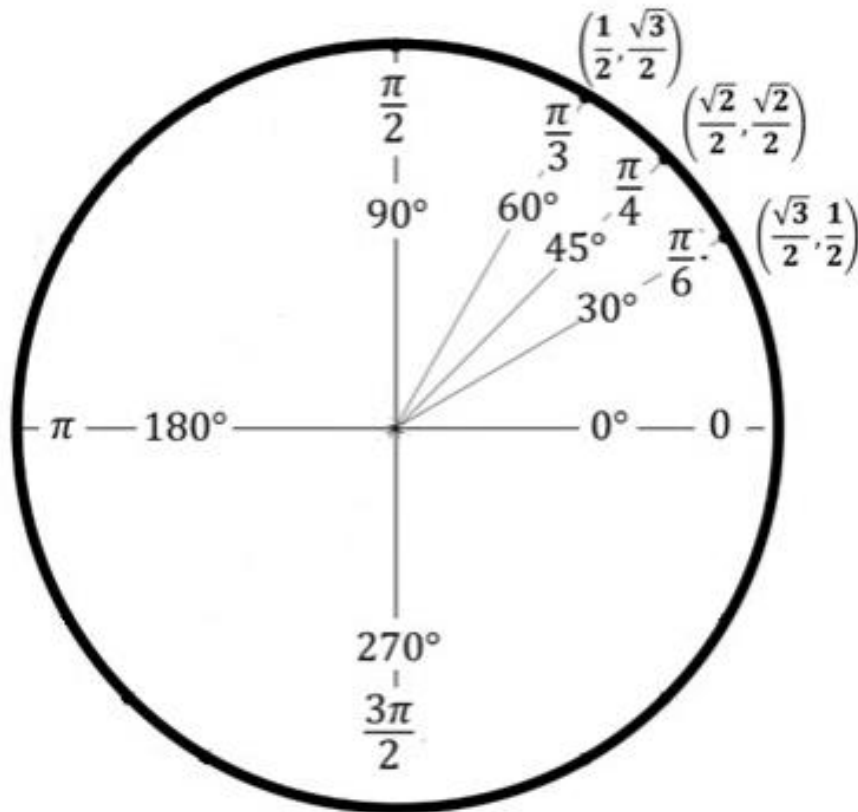
Memorization Hint: Only memorize the values for sine and cosine. Then use the *Reciprocal* and/or *Quotient Identities* to find the remaining values.

Find the Values of Trigonometric Ratios of “Special” Angles

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NOTE:

Some students who have had trigonometry in the past might have memorized the EXACT values of the trigonometric ratios of the "special" angles using the *unit circle*.



The x-coordinates of the points on the circle are the values of cosine of certain special angles and the y-coordinates are the values of sine. The values of the tangent can be calculated using the *Quotient Identity*.