Concepts and Examples
Right Triangle Trigonometry

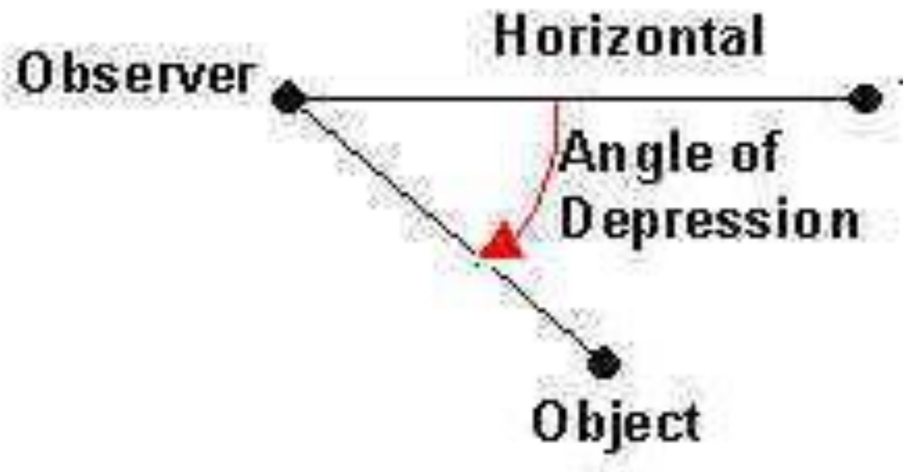
Based on power point presentations by Pearson Education, Inc.
Revised by Ingrid Stewart, Ph.D.

Learning Objectives

1. Find the angle of depression.
2. Find the angle of elevation.
3. Solve right triangles.

1. The Angle of Depression (1 of 3)

When we look at something **below our location**, the angle between the line of sight and the horizontal is called the **Angle of Depression**. In this case, the line of sight is depressed below the horizontal.

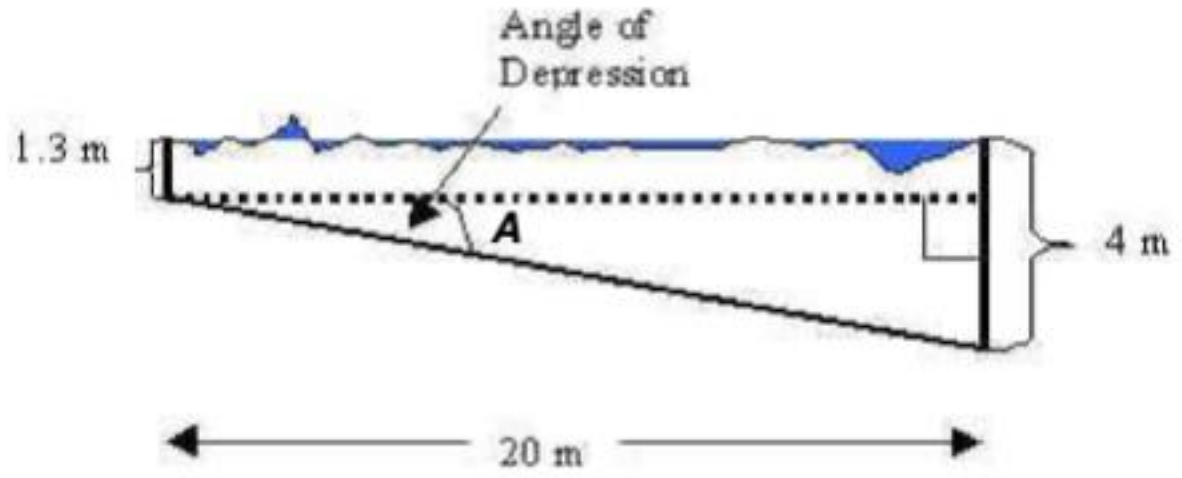


The Angle of Depression (2 of 3)

Example 1:

A swimming pool is 20 meters long. The bottom of the pool has a constant slant so that the water depth is 1.3 meters at the shallow end and 4 meters at the deep end. Find the angle of depression of the bottom of the pool rounded to two decimal places.

Let’s first draw a picture.



The Angle of Depression (3 of 3)

Example 1 continued:

For our calculations we are going to use the right triangle shown in the picture. The side opposite the angle ***A*** is 4 – 1.3 = 2.7 m and the side adjacent to the angle ***A*** is 20 m.

Using the tangent function, we find the following:

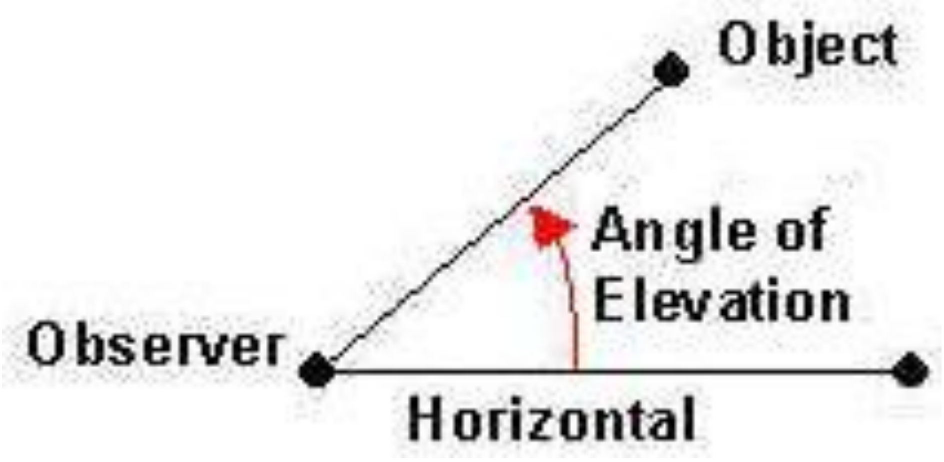
$\tan A = \frac{opp}{adj} = \frac{2.7}{20}$ and then $A = \tan^{-1} \frac{2.7}{20} \approx 7.69^{o}$

The angle of depression is approximately 7.69^{o}.

NOTE: In mathematics, we usually try to work with exact numbers as long as possible. In this problem, the fraction is considered to be more exact than its decimal equivalent because it may end up being a rounded value.

2. The Angle of Elevation (1 of 5)

When we look at something **above our location**, the angle between the line of sight and the horizontal is called the **Angle of Elevation**. In this case, the line of sight is elevated above the horizontal.

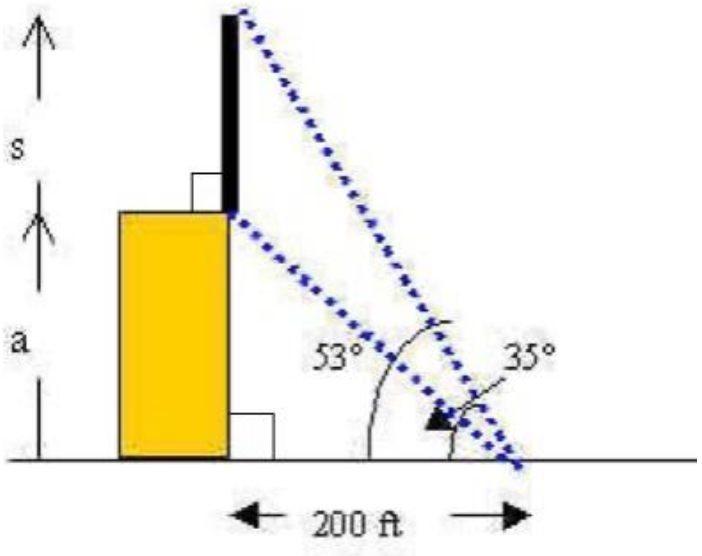


The Angle of Elevation (2 of 5)

Example 2:

A building of height ***a*** has an antenna attached to its flat roof at a right angle. It is considered an extension of the side of the building! At a point 200 feet from the base of the building, the angle of elevation to the bottom of the antenna is 35^o, and the angle of elevation to the top of the antenna is 53^o. Find the height ***s*** of the antenna rounded to one decimal place.

Let's first draw a picture.



Incidentally, we are assuming the building makes a right angle with the ground!

The Angle of Elevation (3 of 5)

Example 2 continued:

Note from the picture that this problem involves two right triangles. Using the tangent function, we find the height ***s*** of the antenna as follows:

$$\tan 53^\circ = \frac{a+s}{200}$$

However, we need to find the height ***a*** of the building first. For this, we will use the following:

$$\tan 35^\circ = \frac{a}{200}$$

We conclude that the height ***a*** of the building is $a = 200 \tan 35^o$.

The Angle of Elevation (4 of 5)

Example 2 continued:

At this point, we will not show the decimal approximation for the height of the building. In mathematics, we usually try to work with exact numbers as long as possible. Therefore, we try not to round until the final calculation.

Now, given $\tan 53^\circ = \frac{a+s}{200}$, we will use ***a = 200 tan 35º*** to find the height of the antenna ***s***.

$$\tan 53^\circ = \frac{200\tan 35^\circ + s}{200}$$

Then ***200 tan 53º = 200 tan 35º + s***

and ***s = 200 tan 53º – 200 tan 35º***.

The Angle of Elevation (5 of 5)

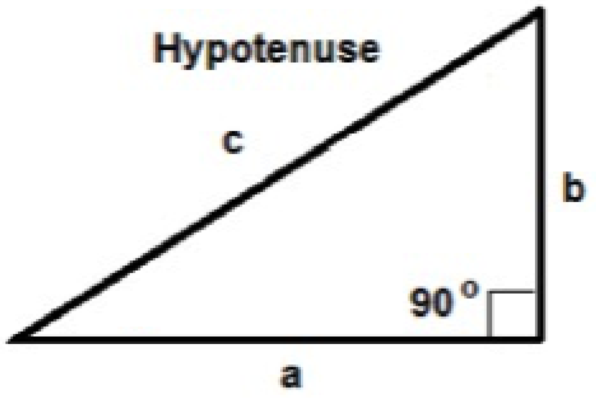
We will input ***200 tan 53º – 200 tan 35º*** into a calculator in its entirety to find that ***s*** = ***125.3674567*** … .

Thus, the height ***s*** of the antenna is approximately ***125.4*** feet. Here we can finally round!

3. Solve Right Triangles (1 of 8)

The process of finding the missing measurements in a triangle knowing some measurements is known as **solving the triangle**. We are going to solve right triangles by using the *Pythagorean Theorem* and the definitions of the trigonometric ratios.

We learned that the sides of a right triangle are related via the **Pythagorean Theorem** which states $a^2 + b^2 = c^2$, where c is the length of the *hypotenuse* and a and b are the length of the legs. Remember, the *hypotenuse* is opposite the right angle!



Solve Right Triangles (2 of 8)

Previously, we memorized the following trigonometric ratios for some angle θ in a right triangle:

$$\sin\theta = \frac{opp}{hyp} \qquad \cos\theta = \frac{adj}{hyp} \qquad \tan\theta = \frac{opp}{adj}$$

In the formulas, we used the following abbreviation:

adj - the side of a right triangle adjacent to the angle θ

opp - the side of a right triangle opposite the angle θ

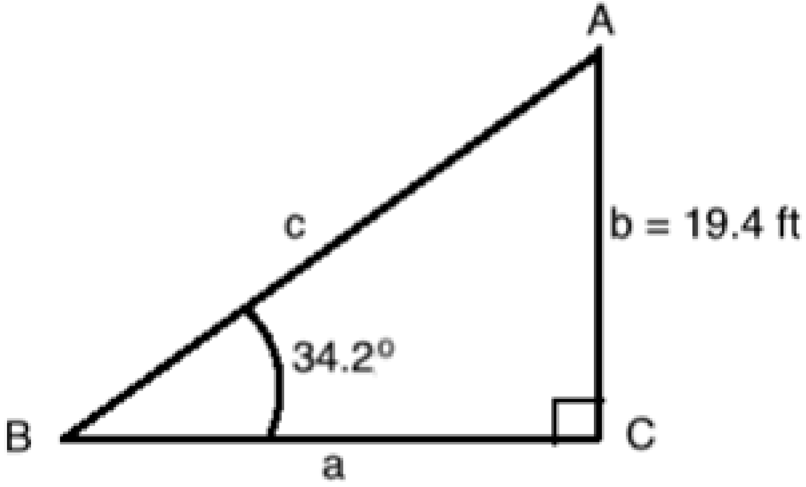
hyp - the hypotenuse of a right triangle

A handy Memorization Aid for the Sine, Cosine, and Tangent Ratios (there may be others): SOH CAH TOA

Solve Right Triangles (3 of 8)

Example 3:

Solve the right triangle pictured below. That is, find all unknown sides and angles. Round the final solutions to one decimal place!



Solve for Angle ***A***:

Since the sum of the interior angles of any triangle always equals 180°, we find that $\boldsymbol{A = 180^{o} - 90^{o} - 34.2^{o} = 55.8^{o}}$.

Solve Right Triangles (4 of 8)

Example 3 continued:

Solve for side ***a***:

Let's use the fact that $\tan A = \dfrac{opp}{adj} = \dfrac{a}{b}$.

Given the information in the problem, we can then write $\tan 55.8° = \dfrac{a}{19.4}$.

Solving for side ***a***, we get $a = 19.4 \tan 55.8° \approx 28.5$.

Solve for side ***c***:

Let's use the fact that $\sin B = \dfrac{opp}{hyp}$ because then we do not have to use a previously rounded value (e.g., side ***a***), something we should avoid if at all possible.

Solve Right Triangles (5 of 8)

Example 3 continued:

Given the information in the problem, we can then write $\sin 34.2^\circ = \dfrac{19.4}{c}$.

Solving for side ***c***, we first get $c \sin 34.2^\circ = 19.4$.

Now we divide both sides of the equal sign by **sin 34.2º**, to get to following:

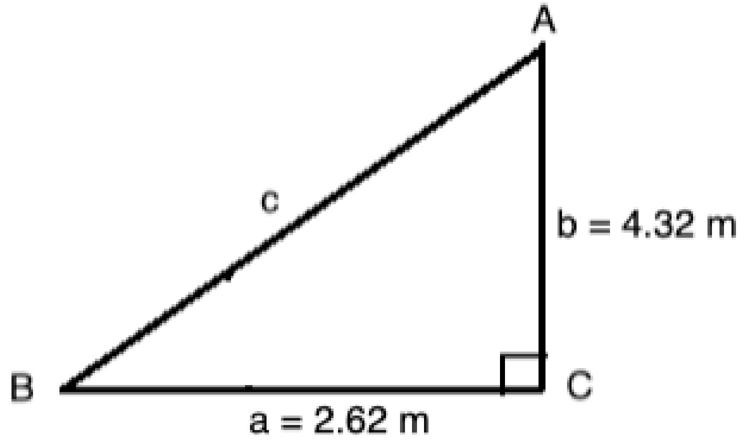
$$c = \frac{19.4}{\sin 34.2^\circ} \approx 34.5$$

In summary, we found that ***A* = 55.8º**, ***a*** ≈ **28.5 ft**, and ***c*** ≈ **34.5 ft**.

Solve Right Triangles (6 of 8)

Example 4:

Solve the right triangle pictured below. That is, find all unknown sides and angles. Round the final solutions to two decimal place!



Solve Right Triangles (7 of 8)

Example 4 continued:

Solve for Angle ***A***:

Let's use the fact that $\tan A = \frac{opp}{adj} = \frac{a}{b}$.

Given the information in the problem, we can then write $\tan A = \frac{2.62}{4.32}$.

Solving for angle ***A***, we get $A = \tan^{-1}\left(\frac{2.62}{4.32}\right) \approx 31.24^{\circ}$.

Solve for Angle ***B***:

To be sure that the the sum of the interior angles of the triangle equals 180^{o}, here we will use the previously rounded angle ***A***. That is,

$\boldsymbol{B = 180^{o} - 90^{o} - 31.24^{o} \approx 58.76^{o}}$.

Solve Right Triangles (8 of 8)

Example 4 continued:

Solve for side ***c***:

Here we can use the Pythagorean Theorem, given a right triangle and the measure of its two legs.

$$c^2 = (2.62)^2 + (4.32)^2$$

Solving for ***c***, we will use the *Square Root Property* to get to following

$$c = \pm\sqrt{(2.62)^2 + (4.32)^2} \quad \text{and} \quad c \approx \pm 5.05$$

Since a side of a triangle is never negative, we will only use ***c*** ≈ ***5.05***.

In summary, we found that ***A*** ≈ **31.24°**, ***B*** ≈ **58.76°**, and ***c*** ≈ **5.05 m**.