Concepts
Sine and Cosine Functions

Based on power point presentations by Pearson Education, Inc.

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Learning Objectives

1. Memorize the shape and characteristics of the graph of the basic sine function.
2. Memorize the shape and characteristics of the graph of the basic cosine function.
3. Know what transformations do to the basic sine and cosine functions.
4. Graph some sine and cosine functions by hand.

NOTE: This lesson contains some examples. You can find more examples in the "Examples" document also located in the appropriate MOM Learning Materials folder.

We will now create functions using the trigonometric ratios. In this lesson, we will look at the basic sine and cosine functions and their graphs. We will also examine some transformations.

Please note that you must memorize the graphs and the characteristics of the sine and cosine functions and those of their transformations!

1. The Basic Sine Function and its Graph

The definition of the basic sine function is

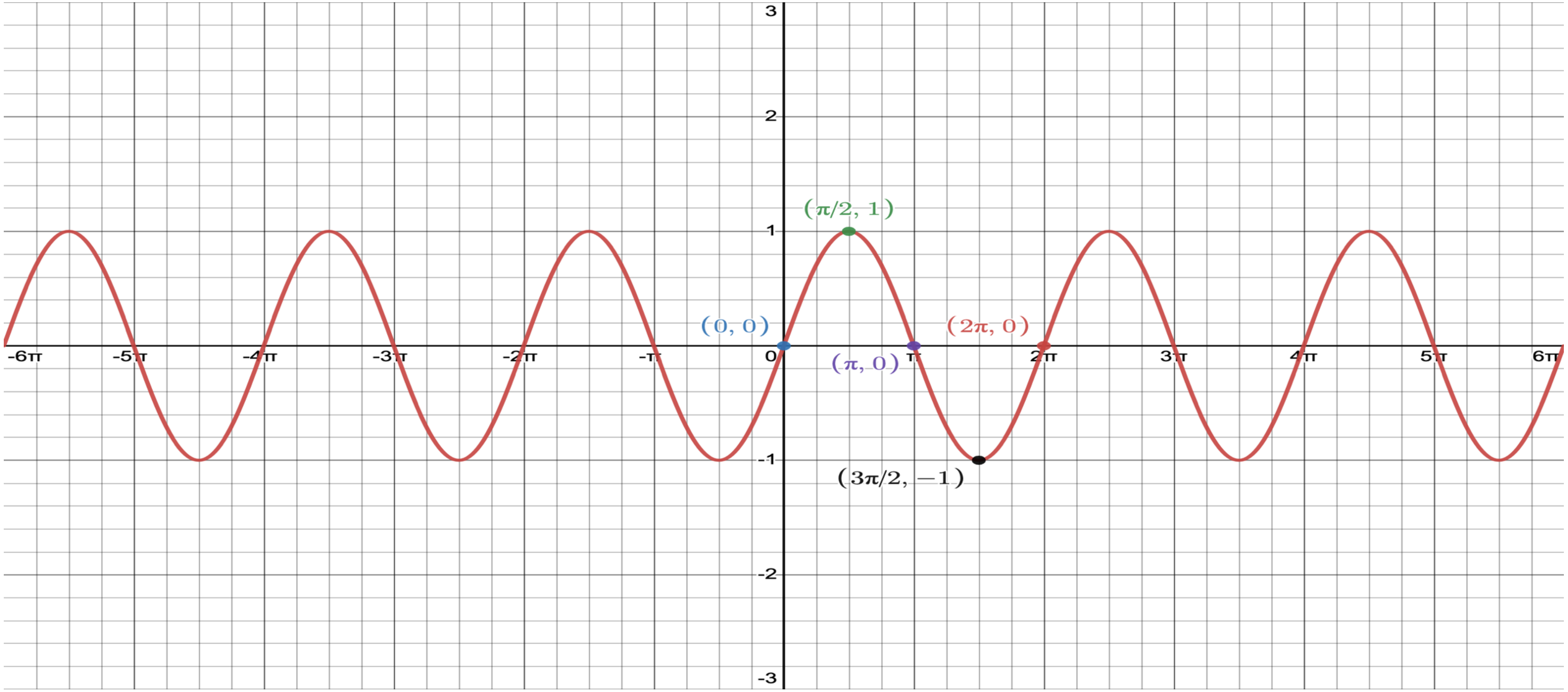
g(x) = sin x

Domain: *All real numbers* or $(-\infty, \infty)$ in Interval Notation. Please note that the numbers in the domain of ALL trigonometric functions are radians. NO degree measures can be used in their domain.

Range: *All real numbers between – 1 and 1 including – 1 and 1* or [– *1*, *1*] in Interval Notation!

The Basic Sine Function and its Graph (2 of 5)

We will now use the Desmos online graphing calculator to create a picture of ***g(x) = sin x***. Please note that the graph has peaks and valleys which are somewhat parabolic in shape! You must memorize the graph!



The Basic Sine Function and its Graph (3 of 5)

NOTE: You can find instructions on how to use Desmos at http://profstewartmath.com/Math127/A_CONTENTS/desmos.htm .

Some of the points on the graph are labeled. Specifically, (0, 0), $\left(\frac{\pi}{2}, 1\right)$, $(\pi, 0)$, $\left(\frac{3\pi}{2}, -1\right)$, and $(2\pi, 0)$. How were these points found?

Given ***g(x) = sin x***, use the calculator in radian mode and find the following:

$g(0) = \boldsymbol{\sin} 0 = 0$

$g\left(\frac{\pi}{2}\right) = \boldsymbol{\sin} \frac{\pi}{2} = 1$

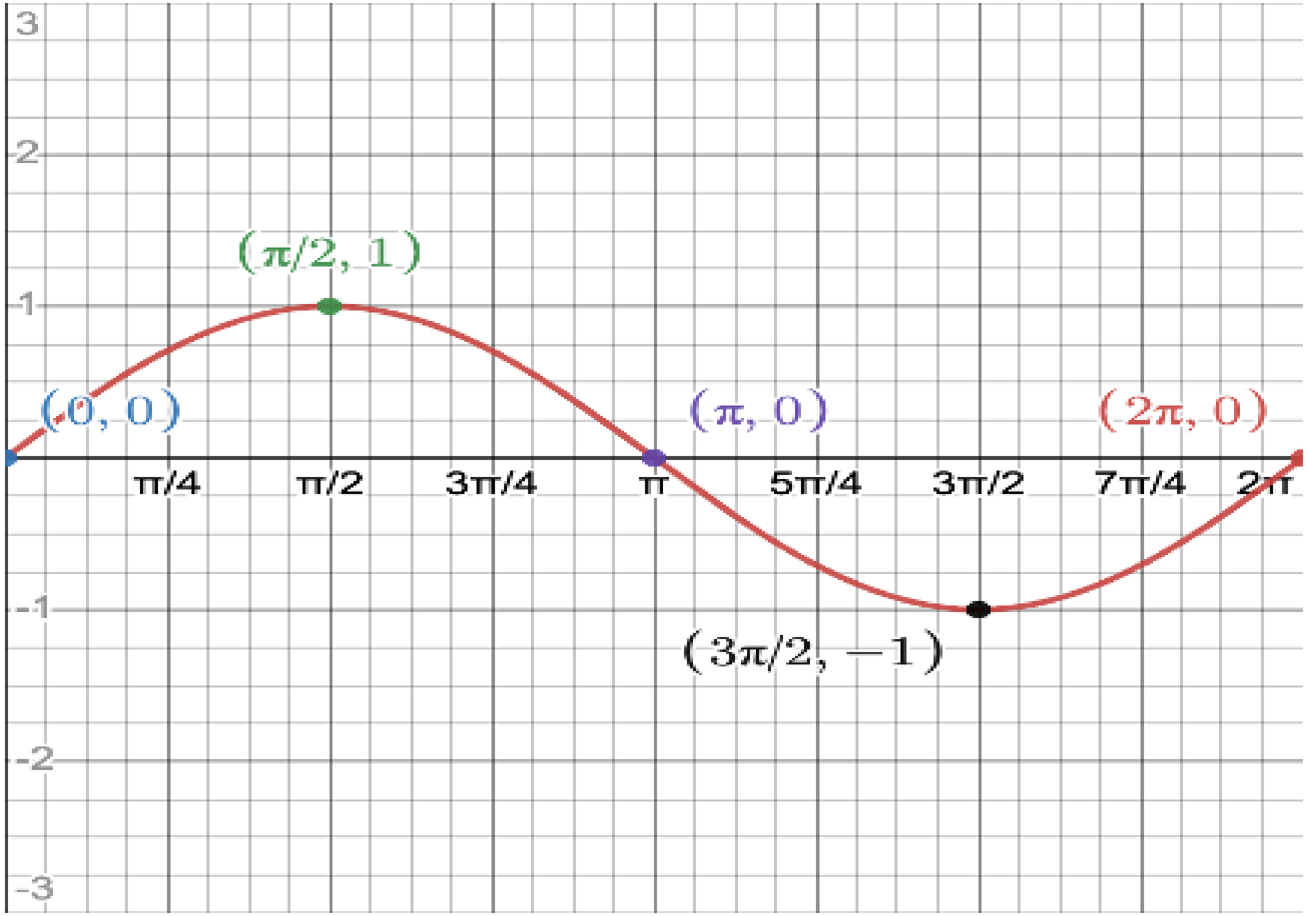
$g(\pi) = \boldsymbol{\sin} \pi = 0$

$g\left(\frac{3\pi}{2}\right) = \boldsymbol{\sin} \frac{3\pi}{2} = -1$

$g(2\pi) = \boldsymbol{\sin} 2\pi = 0$

The Basic Sine Function and its Graph (4 of 5)

We will now only plot the 5 points from the previous slide into the coordinate system and then connect them keeping in mind the shape of the graph of the sine function. You must memorize this portion of the graph!



We call this portion of the graph of the sine function the **representative picture**. It lies on the interval $[\mathbf{0}, \mathbf{2\pi}]$which is called the **period** of the sine graph. Clearly, the period is **2π** in length.

Note that the graph is no higher than 1 and lower than – 1.

The Basic Sine Function and its Graph (5 of 5)

The representative picture repeats "forever" along the positive and negative *x*-axis and creates the graph of the basic sine function. Therefore, the sine function is called "periodic”.

Note that the representative picture is divided into four (4) equal intervals.

- The graph starts at (0, 0) where there is an *x*-intercept and $x = 0$.
- At the end of the first interval the graph has a peak..
- At the end of the second interval the graph has an *x*-intercept.
- At the end of the third interval the graph has a valley.
- At the end of the fourth interval the graph has an *x*-intercept.

You must memorize these characteristics!

2. The Basic Cosine Function and its Graph

The definition of the basic cosine function is

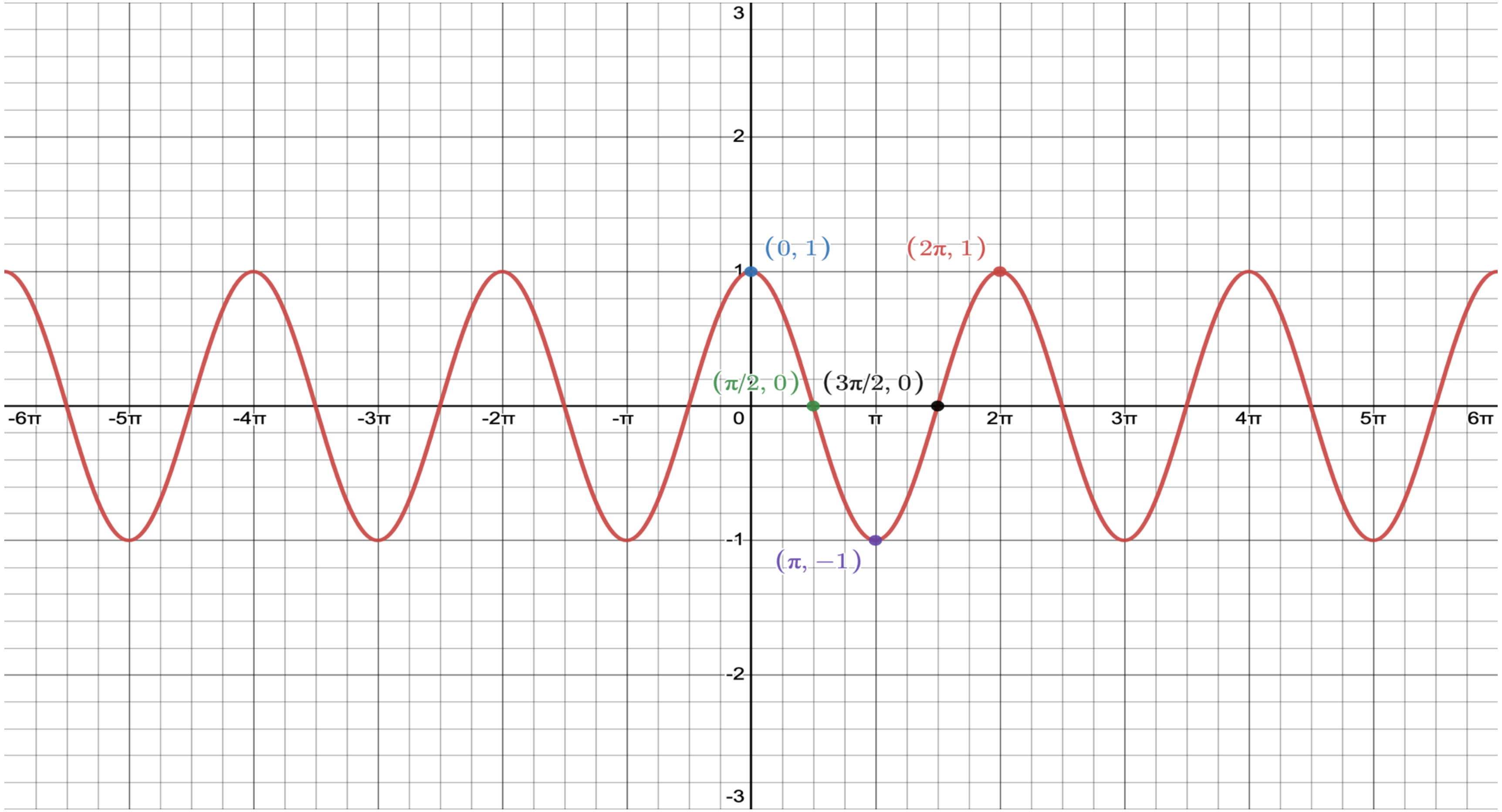
h(x) = cos x

Domain: *All real numbers* or $(-\infty, \infty)$ in Interval Notation. Please note that the numbers in the domain of ALL trigonometric functions are radians. NO degree measures can be used in their domain.

Range: *All real numbers between – 1 and 1 including – 1 and 1* or [*– 1, 1*] in Interval Notation.

The Basic Cosine Function and its Graph

We will now use the Desmos online graphing calculator to create a picture of ***h(x) = cos x***. Please note that the graph has peaks and valleys which are somewhat parabolic in shape! You must memorize the graph!



The Basic Cosine Function and its Graph (3 of 5)

Some of the points on the graph are labeled. Specifically, (0, 1), $\left(\frac{\pi}{2}, 0\right)$, $(\pi, -1)$, $\left(\frac{3\pi}{2}, 0\right)$, and $(2\pi, 1)$. How were these points found?

Given ***h(x) = cos x***, use the calculator in radian mode and find the following:

$h(0) = \boldsymbol{\cos} 0 = 1$

$h\left(\frac{\pi}{2}\right) = \boldsymbol{\cos} \frac{\pi}{2} = 0$

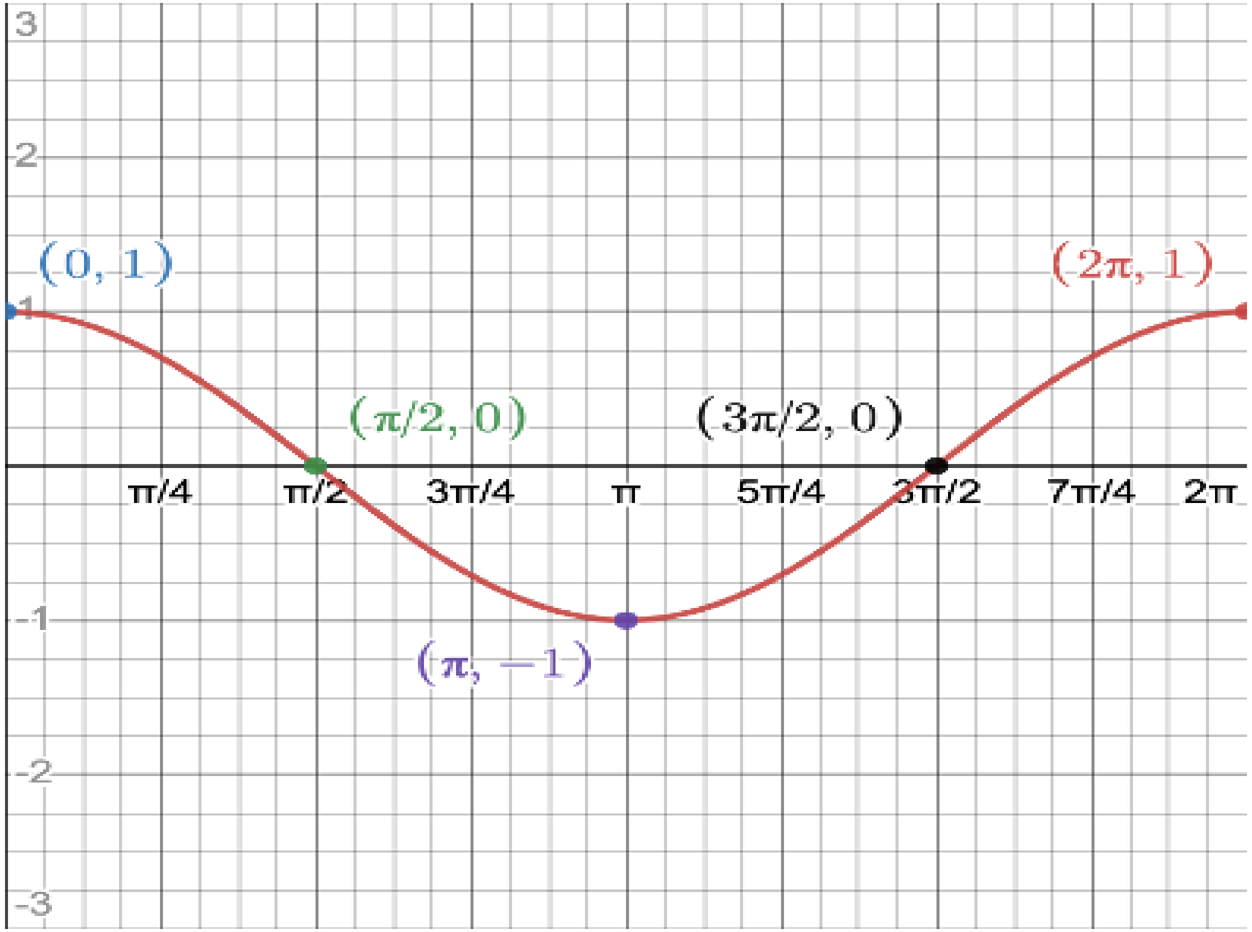
$h(\pi) = \boldsymbol{\cos} \pi = -1$

$h\left(\frac{3\pi}{2}\right) = \boldsymbol{\cos} \frac{3\pi}{2} = 0$

$h(2\pi) = \boldsymbol{\cos} 2\pi = 1$

The Basic Cosine Function and its Graph (4 of 5)

We will now only plot the 5 points from the previous slide into the coordinate system and then connect them keeping in mind the shape of the graph of the cosine function. You must memorize this portion of the graph!



We call this portion of the graph of the cosine function the **representative picture**. It lies on the interval $[\mathbf{0}, \mathbf{2\pi}]$ which is called the **period** of the cosine graph. Clearly, the period is $\mathbf{2\pi}$ in length.

Note that the graph is no higher than 1 and lower than – 1.

The Basic Cosine Function and its Graph (5 of 5)

The representative picture repeats "forever" along the positive and negative *x*-axis and creates the graph of the basic cosine function. Therefore, the cosine function is called "periodic”.

Note that the representative picture is divided into four (4) equal intervals.

- The graph starts at (0, 1) where there is a peak.
- At the end of the first interval the graph has an *x*-intercept.
- At the end of the second interval the graph has a valley.
- At the end of the third interval the graph has an *x*-intercept.
- At the end of the fourth interval the graph has a peak.

You must memorize these characteristics!

3. Transformations of the Basic Sine and Cosine Functions

We will now investigate transformations of the basic sine and cosine functions.

NOTE: From algebra, we remember that transformations allow us to move and resize basic functions by shifting them vertically and horizontally; by reflecting them in the *x*- and *y*-axis; and by vertically and horizontally stretching and compressing them.

The transformations of the basic sine and cosine functions are described as follows:

$y = a \sin(bx \pm c) \pm d$ and $y = a \cos(bx \pm c) \pm d$

We will now discuss how the numbers ***a***, ***b***, ***c***, and ***d*** transform the graphs.

Transformations of the Basic Sine and Cosine Functions

The number ***a***:

It indicates a vertical stretch or compression of the graph of the basic sine and cosine functions. It changes the height of the peaks and valleys. The absolute value of ***a*** or $|a|$ is called the **amplitude**.

Example 1:

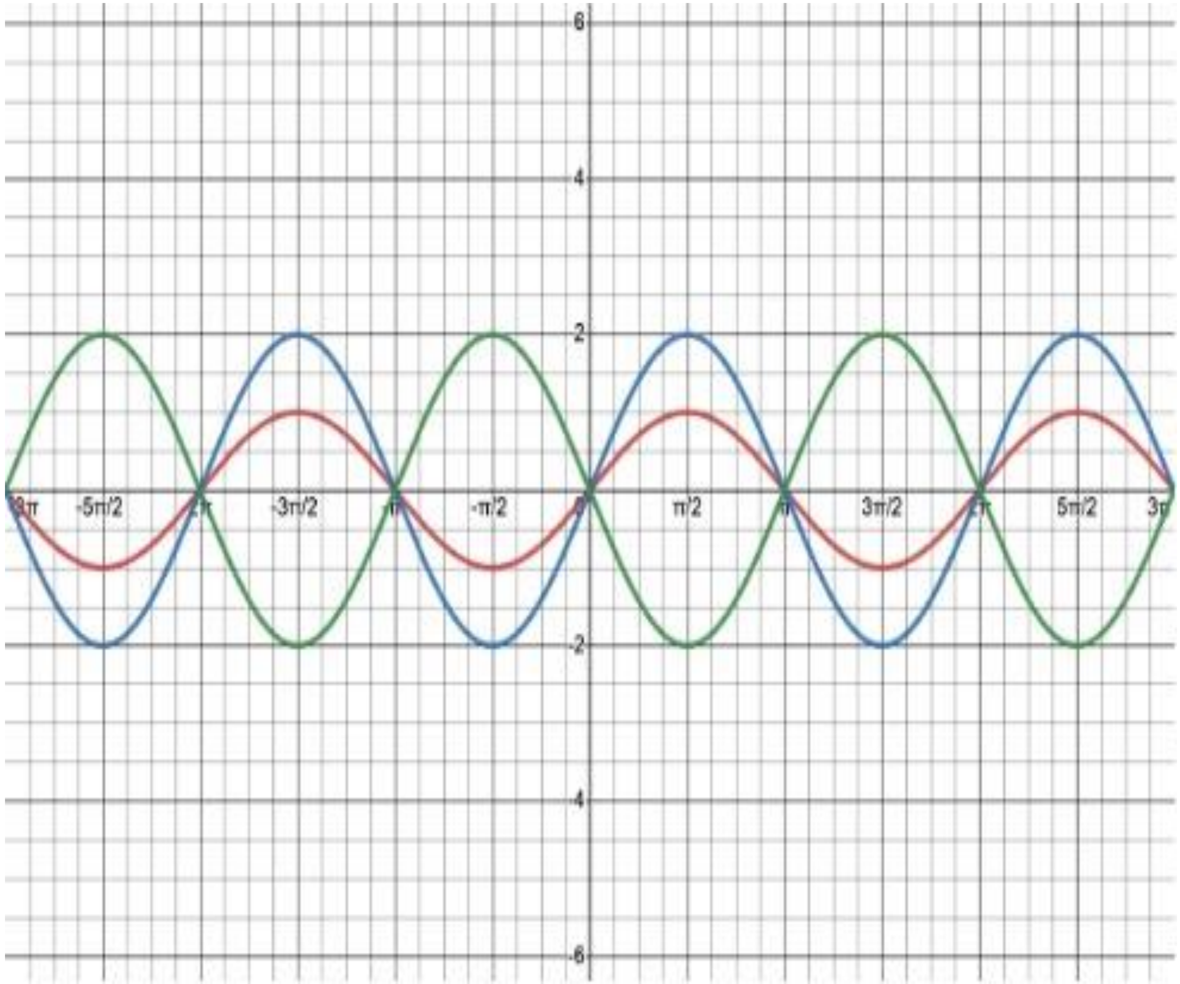
Graph $y = 2\sin x$ and $y = -2\sin x$ using Desmos.

Please note that if the number ***a*** is negative, the graph of $y = \sin x$ is reflected in the *x*-axis.

Transformations of the Basic Sine and Cosine Functions

Example 1 continued:

Following are the graphs together with ***y* = sin *x*** (red graph):



blue graph: ***y* = 2 sin *x***

Indicates a change of the amplitude of the graph of ***y* = sin *x*** to ***a*** = | 2 | = 2.

green graph: ***y* = – 2 sin *x***

Indicates a change of the amplitude of the graph of ***y* = sin *x*** to ***a*** = | – 2 | = 2. Notice the reflection in the *x*-axis!

Transformations of the Basic Sine and Cosine Functions (4 of 9)

The number ***b***:

It indicates a horizontal stretch or compression of the graph of the basic sine and cosine functions. This will affect the length of the **period *P***.

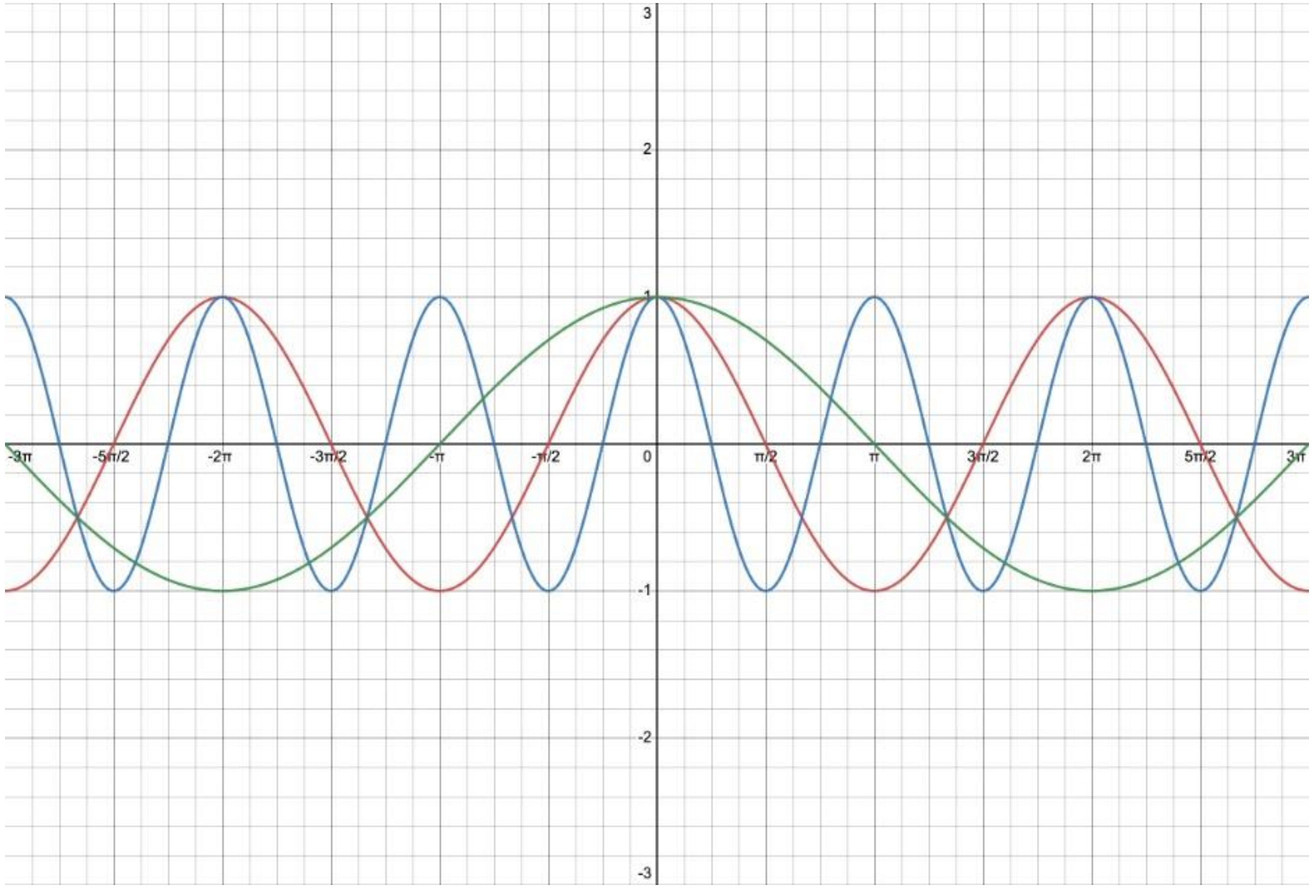
The formula $P = \frac{2\pi}{b}$ calculates the period of the graph of the sine and cosine functions.

- If ***b*** is between 0 and 1, the period of the basic sine and cosine function gets larger. This is a horizontal stretch.
- If ***b*** is greater than 1, the period of the basic sine and cosine function gets smaller. This is a horizontal compression.

Transformations of the Basic Sine and Cosine Functions

Example 2:

Graph $y = \cos 2x$ and $y = \cos \frac{1}{2} x$ using Desmos together with $y = \cos x$.



red graph: $y = \cos x$

blue graph: $y = \cos 2x$

Indicates a change of the period of the graph of $y = \cos x$ to $\frac{2\pi}{2} = \pi$.

green graph: $y = \cos \frac{1}{2} x$

Indicates a change of the period of the graph of $y = \cos x$ to $\frac{2\pi}{\frac{1}{2}} = 2\pi \cdot \frac{2}{1} = 4\pi$.

Transformations of the Basic Sine and Cosine Functions

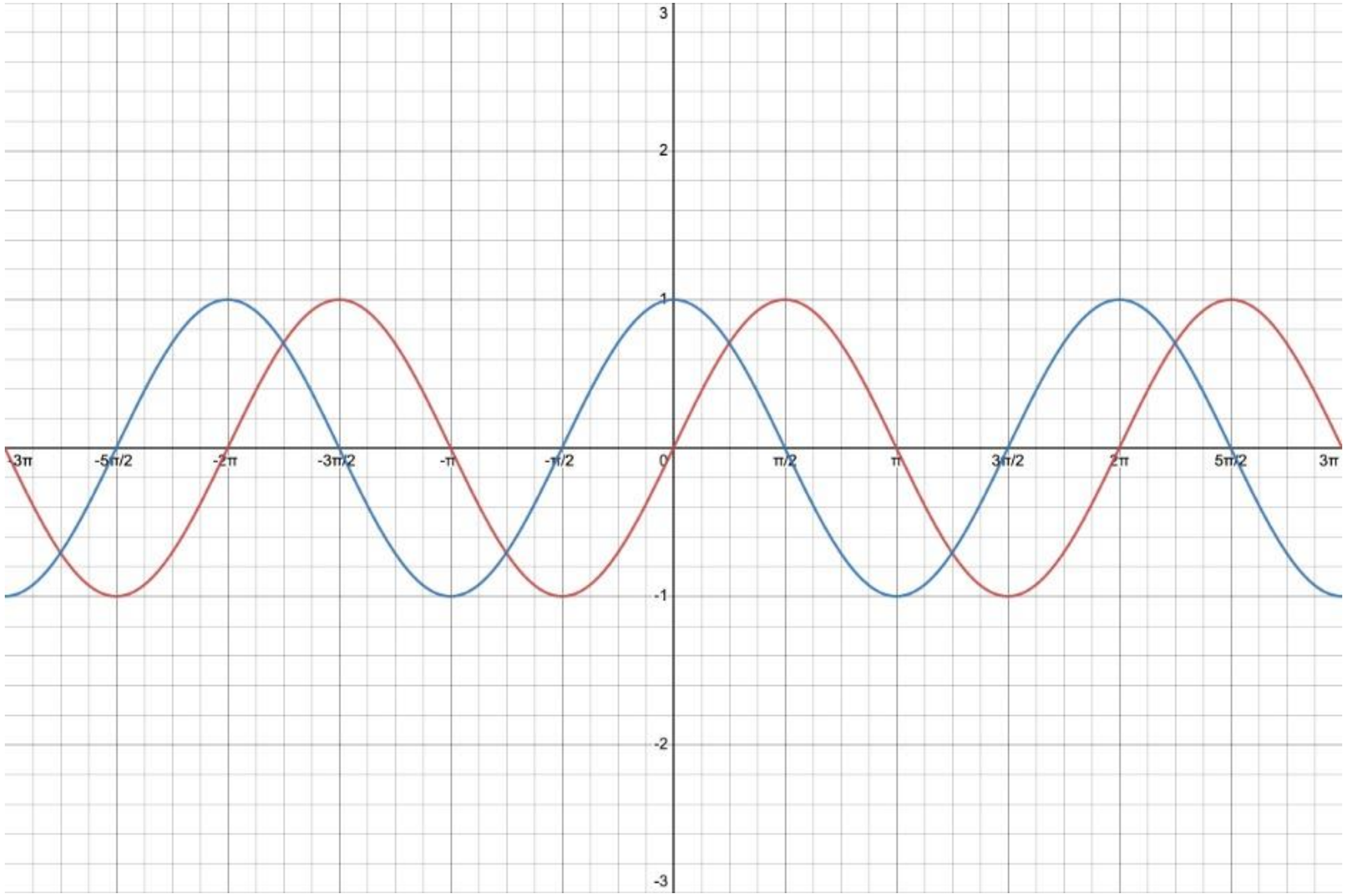
The number ***c***:

It indicates a horizontal shift to the right or left of the graph of the basic sine and cosine functions. This is often called a **phase shift**.

Transformations of the Basic Sine and Cosine Functions

Example 3:

Graph $\boldsymbol{y = \sin\left(x + \frac{\pi}{2}\right)}$ using Desmos together with $\boldsymbol{y = \sin x}$.



red graph: $\boldsymbol{y = \sin x}$

blue graph: $\boldsymbol{y = \sin\left(x + \frac{\pi}{2}\right)}$

Indicates a shift to the left of the graph of $\boldsymbol{y = \sin x}$ by $\frac{\pi}{2}$ units.

Transformations of the Basic Sine and Cosine Functions

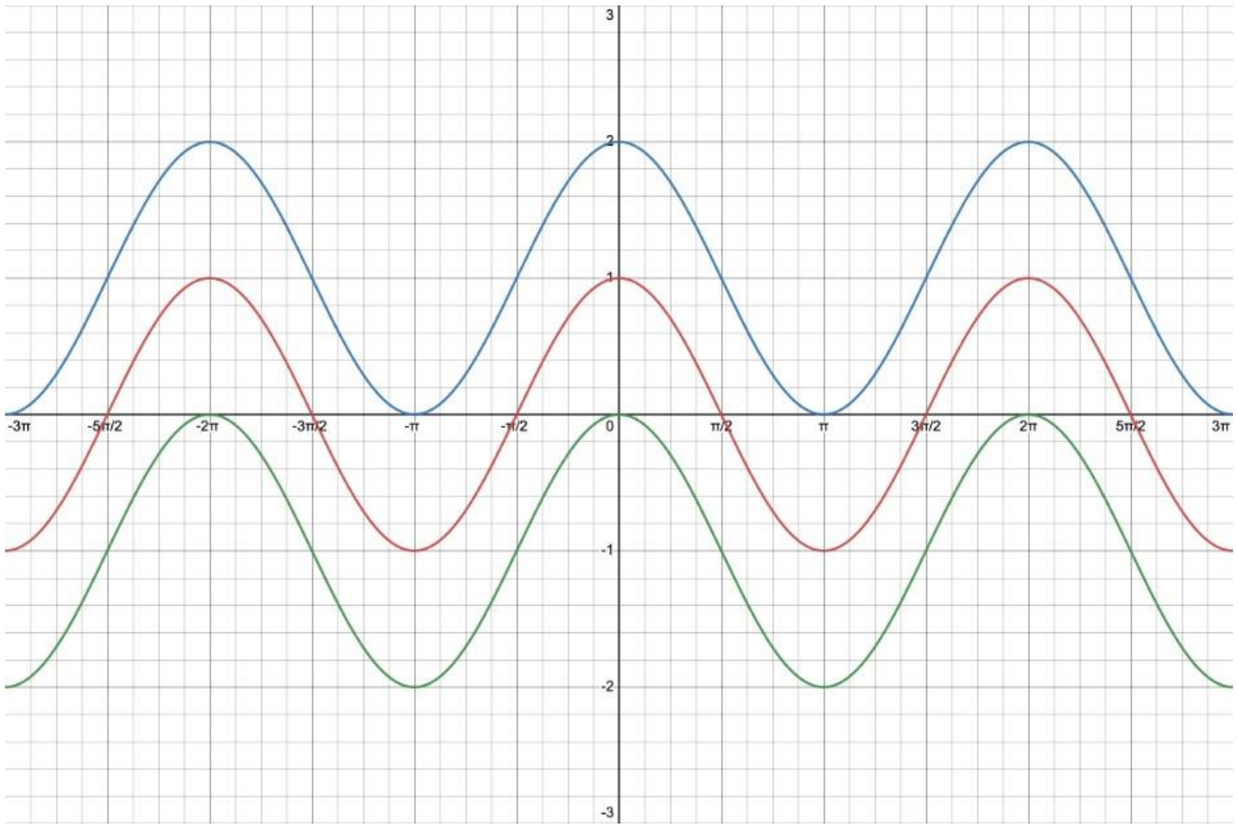
The number ***d***:

It indicates a vertical shift up or down of the graph of the basic sine and cosine functions.

Transformations of the Basic Sine and Cosine Functions (9 of 9)

Example 2:

Graph $y = \cos x + 1$ and $y = \cos x - 1$ using Desmos together with $y = \cos x$.



red graph: $y = \cos x$

blue graph: $y = \cos x + 1$

Indicates a shift up of the graph of $y = \cos x$ by 1 unit.

green graph: $y = \cos x - 1$

Indicates a shift down of the graph of $y = \cos x$ by 1 unit.

4. Graph Some Sine and Cosine Functions by Hand (1 of 2)

We will only graph functions of the form ***y = a sin bx*** and ***y = a cos bx*** by hand. Following are the eight (8) steps necessary to properly graph these functions by hand. Please study and then rework the examples in the corresponding "Examples" document.

1. Determine the amplitude using $|a|$.
2. Determine the period using $P = \frac{2\pi}{b}$.
3. Keep in mind the graph and characteristics of the basic function.
4. Mark off a distance along the *x*-axis to represent the period of the representative picture.
5. Divide the period into four equal intervals.

Steps continued on next slide!

Graph Some Sine and Cosine Functions by Hand (2 of 2)

Graphing strategy continued:

6. Create the appropriate representative picture by using the beginning/ending point of each interval. Keep in mind the amplitude. Be mindful of ***a*** being positive or negative!
7. Connect the points found in (6) to form the representative picture.
8. Copy the representative picture several more times along the negative and positive *x*-axis.