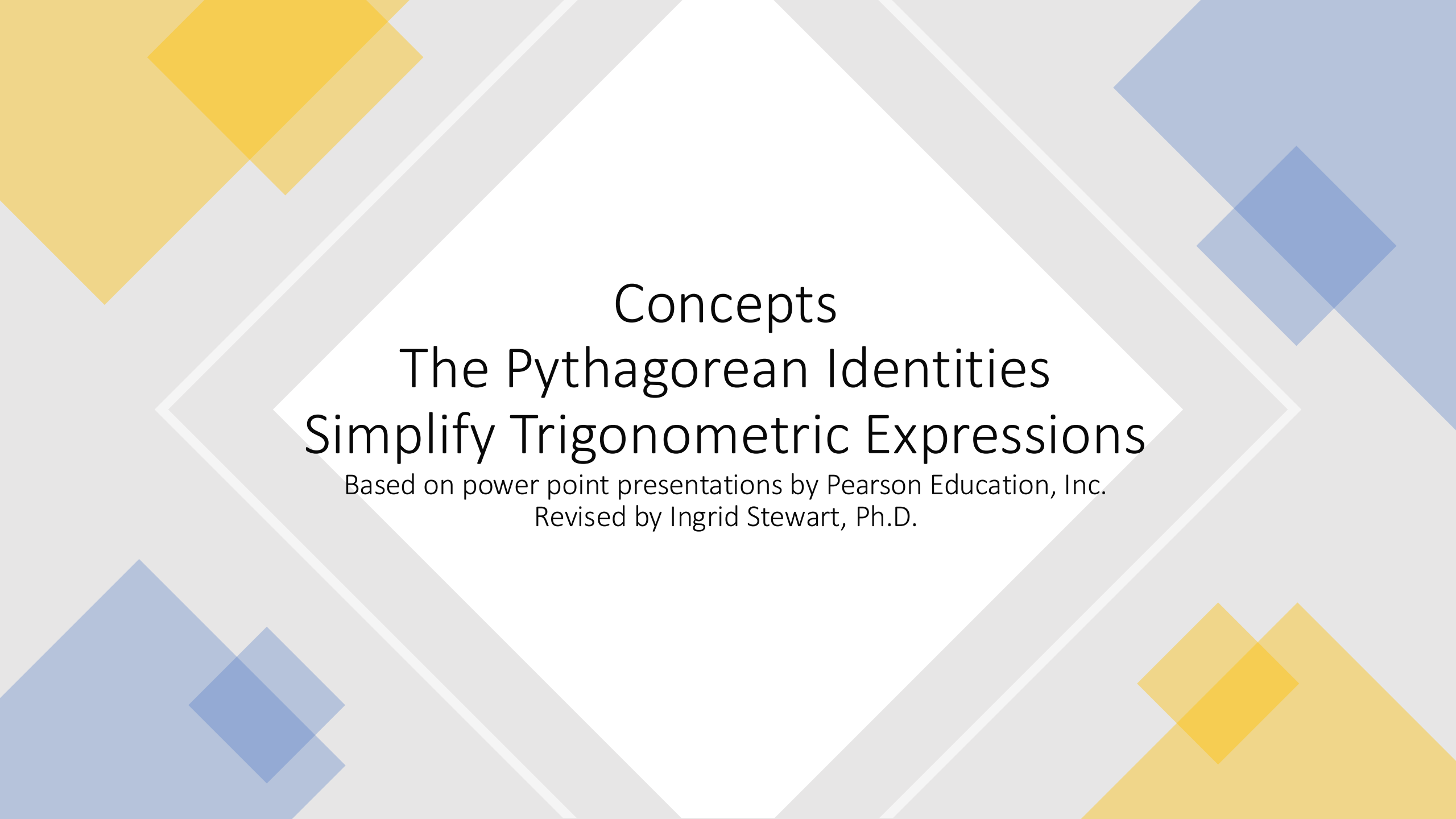




Concepts

The Pythagorean Identities

Simplify Trigonometric Expressions

Based on power point presentations by Pearson Education, Inc.
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Learning Objectives

1. Memorize and use the Pythagorean Identities.
2. Use fundamental identities to rewrite trigonometric expressions.
3. Use addition and subtraction to simplify trigonometric expressions.
4. Use multiplication to simplify trigonometric expressions.
5. Use factoring to simplify trigonometric expressions.
6. Separate fractions containing trigonometric expressions into two or more fractions.
7. Change complex fractions containing trigonometric expressions to simple fractions.

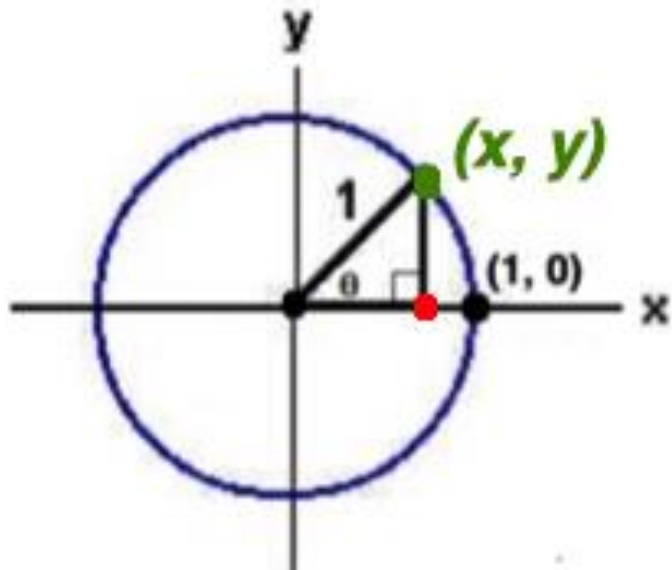
NOTE: This lesson contains some examples. You can find more **complex examples** in the “Examples” document also located in the appropriate MOM Learning Materials folder.

NOTE: For this lesson it is absolutely necessary that you study and rework the problems in the “Examples” document.

1. The Pythagorean Identities (1 of 3)

We have already discussed the *Quotient* and *Reciprocal Identities*. Now we will discuss other important identities called the Pythagorean Identities.

The *Pythagorean Identities* are derived from the unit circle, which is a circle of radius 1.



Please note that $\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{x}{1} = x$ and

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{y}{1} = y$$

This indicates that the ordered pair (x, y) can be called $(\cos \theta, \sin \theta)$.

The Pythagorean Identities (2 of 3)

From precalculus we know that the *unit circle* in a rectangular coordinate system with center at the origin has the equation $x^2 + y^2 = 1$.

Given the discussion on the previous slide, we will now rewrite this equation as $(\cos \theta)^2 + (\sin \theta)^2 = 1$.

It is common practice to then alter the equation somewhat, namely to $\sin^2 \theta + \cos^2 \theta = 1$. Please note that the exponents are usually written immediately next to the sin, cos, tan, csc, sec, and cot.

We call $\sin^2 \theta + \cos^2 \theta = 1$ the **primary Pythagorean Identity**. It must be memorized!

The Pythagorean Identities (3 of 3)

From the primary *Pythagorean Identity* $\sin^2 \theta + \cos^2 \theta = 1$, we will now derive four (4) more identities **which must be memorized**.

(1) If we subtract $\cos^2 \theta$ from both sides, we get $\sin^2 \theta = 1 - \cos^2 \theta$

(2) If we subtract $\sin^2 \theta$ from both sides, we get $\cos^2 \theta = 1 - \sin^2 \theta$

(3) If we divide every term by $\cos^2 \theta$, we get $1 + \tan^2 \theta = \sec^2 \theta$

(4) If we divide every term by $\sin^2 \theta$, we get $1 + \cot^2 \theta = \csc^2 \theta$

In trigonometry we are often asked to simplify trigonometric expressions.

For example, we can simplify $\frac{1 + \tan x}{1 + \cot x}$ to $\tan x$. See Example 2 in the "Examples" document.

We simplify trigonometric expressions primarily to make later work easier. For example, simplifying trigonometric expression is often necessary when solving complex equations or when we "differentiate" and "integrate" in a calculus course.

We will now discuss six (6) most common methods used to simplify trigonometric expressions. Know them well!

Note that sometimes you must use more than one method or use one method more than once to most efficiently rewrite a trigonometric expression into its simplest form.

2. Use Fundamental Trigonometric Identities

We can use fundamental trigonometric identities to simplify trigonometric expressions. This is often used as a first step in the simplification process. But sometimes one of the other five methods (discussed below) might work better.

Reciprocal Identities (must be memorized):

$$\sin \theta = \frac{1}{\csc \theta} \quad \cos \theta = \frac{1}{\sec \theta} \quad \tan \theta = \frac{1}{\cot \theta} \quad \csc \theta = \frac{1}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta} \quad \cot \theta = \frac{1}{\tan \theta}$$

Quotient Identities (must be memorized):

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

Pythagorean Identities (see discussion above)

3. Add and Subtract Trigonometric Ratios (1 of 2)

When can simplify trigonometric expressions by adding and subtracting them. We simply add/subtract the coefficients of “like” ratios.

Example 1:

Add **$5 \sin x + 3 \sin x + 6 \cos x$** .

We add the coefficients of like ratios, that is **$(5 + 3) \sin x + 6 \cos x$** which equals **$8 \sin x + 6 \cos x$** .

Example 2:

Subtract **$2 \sec x - 7 \sec x - 2 \csc x$** .

We subtract the coefficients of like ratios, that is **$(2 - 7) \sec x - 2 \csc x$** which equals **$-5 \sec x - 2 \csc x$** .

Add and Subtract Trigonometric Ratios (2 of 2)

Example 3:

Add $\frac{1}{\sin x} + \frac{\tan x}{\cos x}$.

Here we are dealing with a fraction. To add and subtract coefficients of “like” ratios, we must first find a common denominator.

That is, we preferably find the smallest number that is divisible by both given denominators. In our case, it is the product ***sin x cos x***.

Next, we will change all denominators to ***sin x cos x*** as follows:

$$\frac{1}{\sin x} \left(\frac{\cos x}{\cos x} \right) + \frac{\tan x}{\cos x} \left(\frac{\sin x}{\sin x} \right) = \frac{\cos x + \tan x \sin x}{\sin x \cos x}$$

We notice that there are no like ratios to combine.

4. Multiply Trigonometric Ratios (1 of 2)

We can simplify trigonometric expressions by multiplying them.

Example 4:

Multiply $\sin x (\sin x + \cos x)$.

Here we use the **Distributive Property** and then the **Laws of Exponents** to find the following:

$$\sin x (\sin x) + \sin x (\cos x) = \sin^2 x + \sin x \cos x$$

Multiply Trigonometric Ratios (2 of 2)

Example 5:

Multiply $(\sin x + \cos x)^2$.

We know that $(\sin x + \cos x)^2 = (\sin x + \cos x)(\sin x + \cos x)$.

Using **FOIL**, we get $\sin^2 x + 2 \sin x \cos x + \cos^2 x$.

5. Factor Trigonometric Ratios (1 of 2)

We can simplify trigonometric expressions by factoring them.

Example 6:

Factor **$\sin x \cos x - \sin x$** .

We see that both terms have a common factor, namely **$\sin x$** . We will factor it out as follows:

$$\sin x (\cos x - 1)$$

Factor Trigonometric Ratios (2 of 2)

Example 7:

Factor **$\tan^2 \theta + 5 \tan \theta + 6$** .

The trigonometric expression is "like" the quadratic expression **$ax^2 + bx + c$** . However, in this case, the **x** is **$\tan \theta$** .

How do you factor **$x^2 + 5x + 6$** ?

Therefore, we can factor **$\tan^2 \theta + 5 \tan \theta + 6$** as **$(\tan \theta + 3)(\tan \theta + 2)$** .

6. Separate Fractions Containing Trigonometric Ratios

We can simplify trigonometric expressions by separating fractions.

Example 8:

Separate $\frac{\sec x - \csc x}{\sec x \csc x}$ into two fractions.

Any time we have terms in the numerator separated by a minus or plus sign, we can assign to each term the entire denominator as follows:

$$\frac{\sec x}{\sec x \csc x} - \frac{\csc x}{\sec x \csc x}$$

Now, we can cancel the secants in the first term and the cosecants in the second term to get the following:

$$\frac{1}{\csc x} - \frac{1}{\sec x}$$

7. Change Complex Fractions Containing Trigonometric Ratios into Simple Fractions (1 of 2)

We can simplify trigonometric expressions by changing complex fractions to simple fractions.

Example 9:

Change the following complex fraction into a simple fraction:

$$\frac{\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}}{\frac{1}{\sin x \cos x}}$$

Let's take the middle fraction bar and write it as $\div$.

$$\frac{\sin^2 x + \cos^2 x}{\sin x \cos x} \div \frac{1}{\sin x \cos x}$$

Change Complex Fractions Containing Trigonometric Ratios into Simple Fractions (2 of 2)

Example 9 continued:

From arithmetic we know that dividing by a fraction is the same as multiplying by its reciprocal. We will do this next.

$$\frac{\sin^2 x + \cos^2 x}{\sin x \cos x} \cdot \frac{\sin x \cos x}{1}$$

Now, we can cancel ***sin x cos x*** to get the following:

$$\frac{\sin^2 x + \cos^2 x}{\cancel{\sin x \cos x}} \cdot \frac{\cancel{\sin x \cos x}}{1} = \frac{\sin^2 x + \cos^2 x}{1}$$

This can simply be written as ***sin² x + cos² x***.