Concepts
Introduction to Polar Coordinates

Based on power point presentations by Pearson Education, Inc.

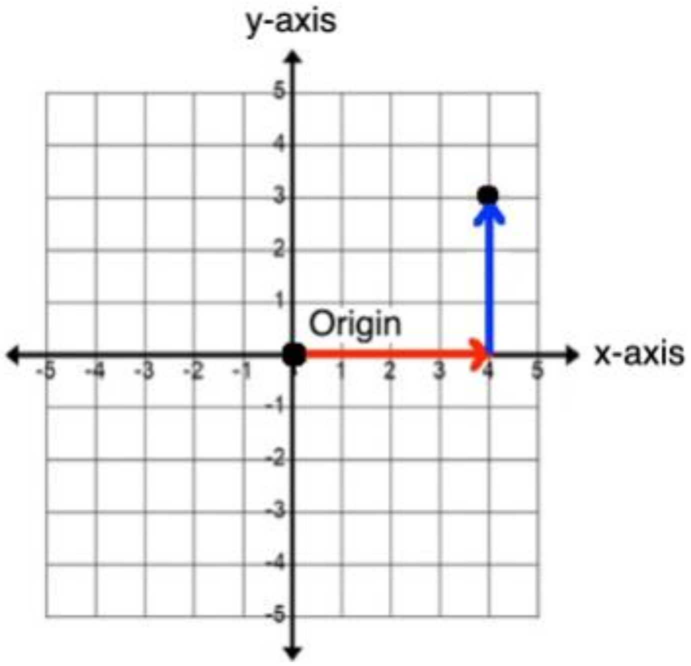
Revised by Ingrid Stewart, Ph.D.

Learning Objectives

1. Graph points in the polar coordinate system.
2. Change polar coordinates into rectangular coordinates.
3. Change rectangular coordinates into polar coordinates.

NOTE: This lesson contains some examples. You can find more examples in the "Examples" document also located in the appropriate MOM Learning Materials folder.

So far, we have been representing graphs of equations as a collection of points (x, y) in the **Rectangular Coordinate System**. For example, in the picture below, we see that the point (x, y) is (4, 3).



Now, we will study a second system called the **Polar Coordinate System**. The initial motivation for the introduction of this system was the study of circular and orbital motion. Polar coordinates are now used in navigation and for many phenomena in the physical world.

1. The Polar Coordinate System (1 of 7)

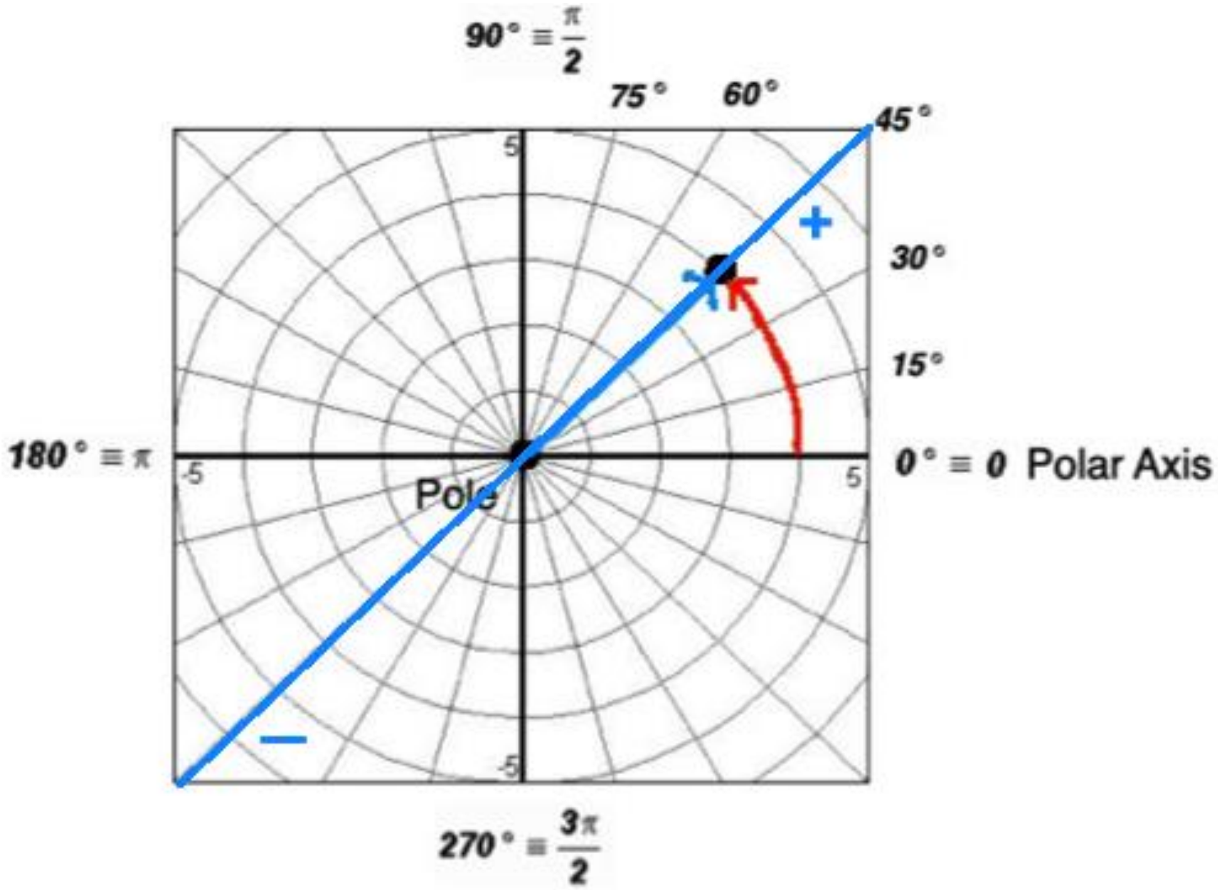
To create a **Polar Coordinate System**, we first draw a horizontal and a vertical line that intersect at a right angle. The point where the lines intersect is no longer called the Origin, but the **Pole**.

We then draw angles with their vertices at the *Pole* and their initial side at the **polar axis**, which is equivalent to the positive *x*-axis in a rectangular coordinate system. The terminal sides of the angles are called *r*-axes.

A point in the *Polar Coordinate System* is represented by the polar coordinates (r, θ). The number r lies on an r-axis which is the terminal side of angle θ.

The Polar Coordinate System (2 of 7)

In the picture below, we see a Polar Coordinate System containing the point (4, 45°).



NOTES:

1. The *r*-axis is **positive** along the terminal side of the angle and **negative** along the EXTENSION of the terminal side through the pole.

2. The number ***r*** and can be positive or negative.

3. The angle **θ** can be expressed in radians or degrees.

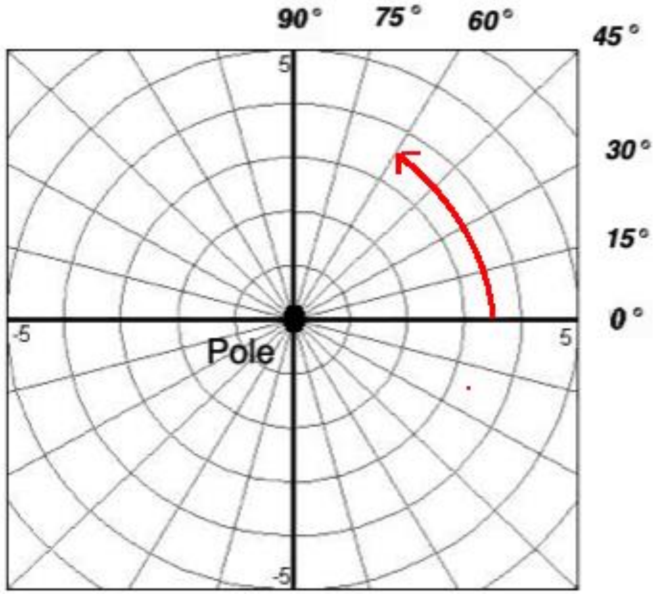
The Polar Coordinate System (3 of 7)

Strategy for Graphing Points (r, θ) in the Polar Coordinate System

Step 1 - Starting at the polar axis, draw the angle θ to find its terminal side. This is the positive r-axis.

Example 1:

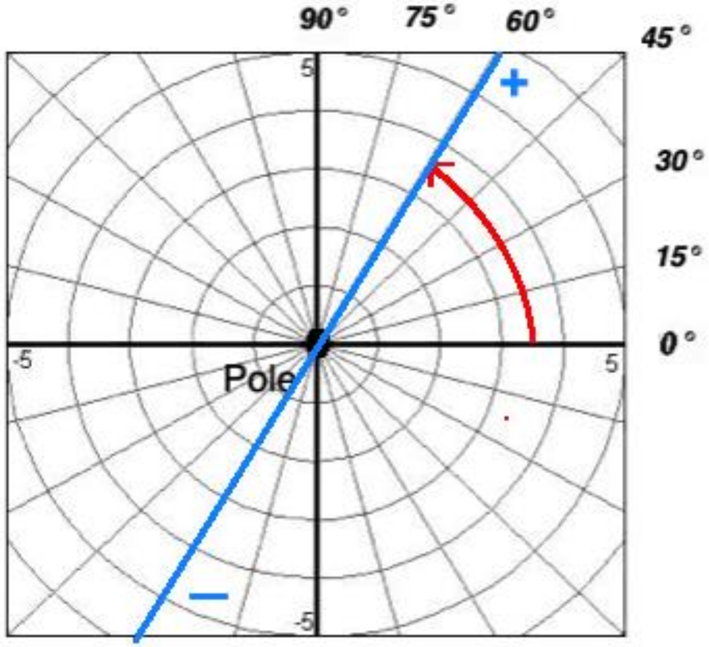
Plot the point ***A*** with coordinates (-3, 60^{o}) in the Polar Coordinate System.



The Polar Coordinate System (4 of 7)

Step 2 - Extend the terminal side of the angle through the pole. This is the negative *r*-axis.

Example 1 continued

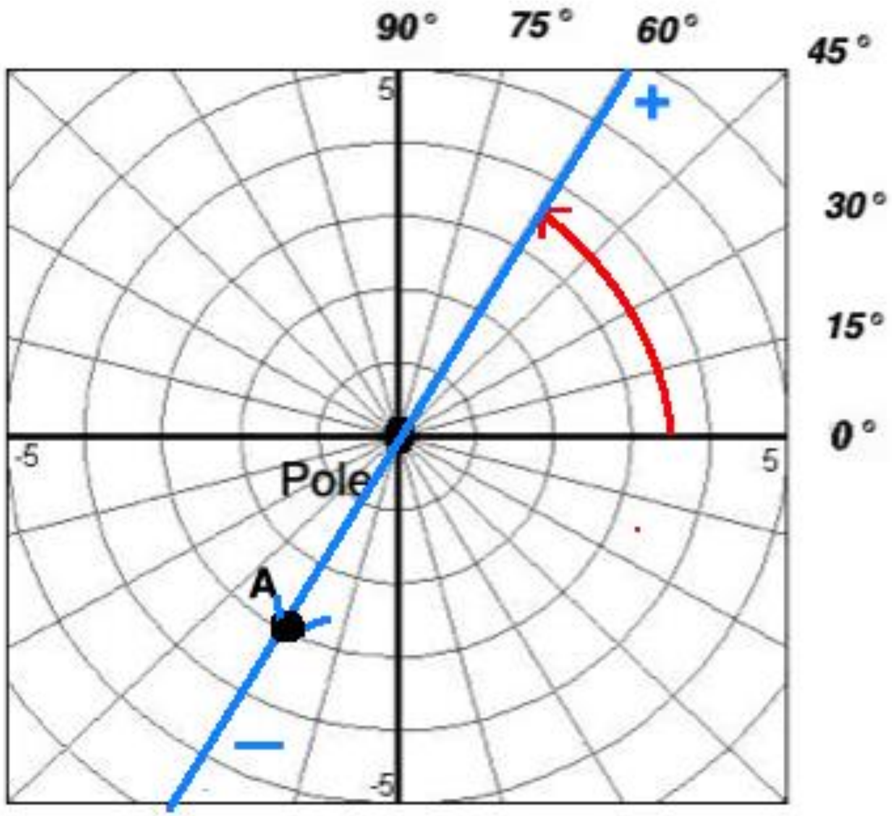


- If ***r*** is positive, plot the point along the positive *r*-axis.
- If ***r*** is negative, plot the point along the negative *r*-axis.

The Polar Coordinate System (5 of 7)

Example 1 continued:

Since ***r*** is negative, we plot the point ***A*** along the negative *r*-axis.



The Polar Coordinate System

Example 2:

Plot the following points **(r, θ)** with $\boldsymbol{\theta}$ in radians in a *Polar Coordinate System*. Its angles are spaced in increments of $\frac{\pi}{6} \equiv 30^{o}$.

A. $(-1, \frac{\pi}{6})$

B. $(2, \frac{5\pi}{6})$

C. $(4, \pi)$

D. $(-4.5, \frac{7\pi}{6})$

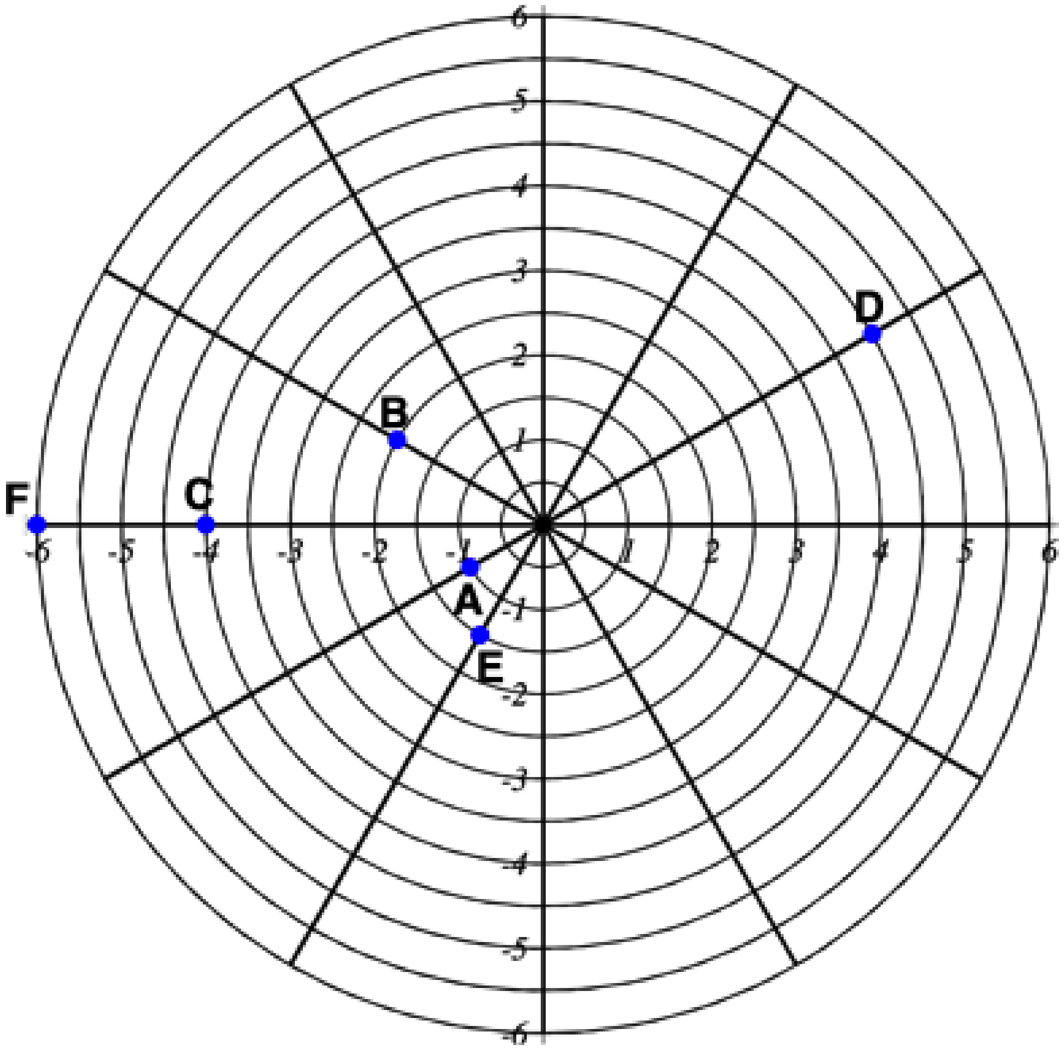
E. $(1.5, \frac{4\pi}{3})$

F. $(-6, 2\pi)$

Hint: It might be easier to convert the angles to degrees and then plot the points.

The Polar Coordinate System (7 of 7)

Example 2 continued:



Please examine the coordinates and points they produce carefully! Can you reproduce the plots?

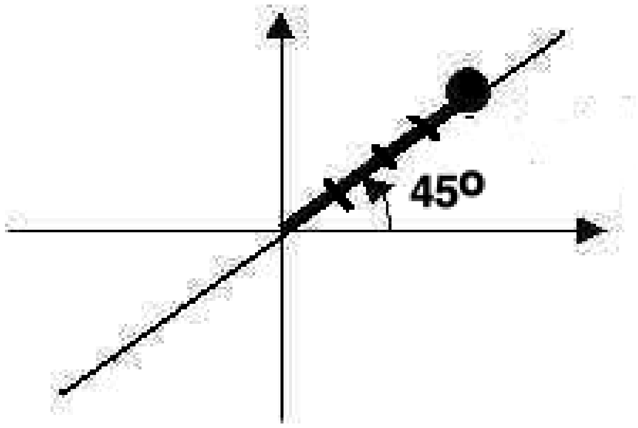
2. Change Polar Coordinates to Rectangular Coordinates (1 of 2)

Step 1 - Sketch the point **(r, θ)** in the *Polar Coordinate System* to give you an idea of its location in the *Rectangular Coordinate System*.

Example 3:

Change the polar coordinates $\left(4, \frac{\pi}{4}\right)$ to EXACT rectangular coordinates.

Let's sketch the point in a simple *Polar Coordinate System*.



Change Polar Coordinates to Rectangular Coordinates (2 of 2)

Step 2 - Use the conversions ***x = r cos*** $\boldsymbol{\theta}$ and ***y = r sin*** $\boldsymbol{\theta}$ (review Lecture #7) to find the rectangular coordinates.

Example 3 continued:

Given $\left(4, \frac{\pi}{4}\right)$, we can now write the following:

$$x = 4\cos\left(\frac{\pi}{4}\right) = 4\left(\frac{\sqrt{2}}{2}\right) = 2\sqrt{2}$$

$$y = 4\sin\left(\frac{\pi}{4}\right) = 4\left(\frac{\sqrt{2}}{2}\right) = 2\sqrt{2}$$

The polar coordinates $\left(4, \frac{\pi}{4}\right)$ are equivalent to the rectangular coordinates $\left(2\sqrt{2}, 2\sqrt{2}\right)$.

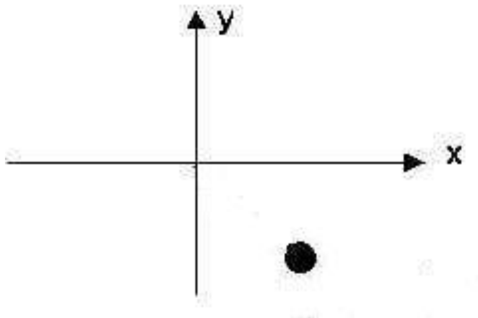
3. Change Rectangular Coordinates to Polar Coordinates (1 of 4)

Step 1 - Sketch the point **(*x, y*)** in the *Rectangular Coordinate System* to give you an idea of its location in the *Polar Coordinate System*.

Example 4:

Change the rectangular coordinates $\left(1, -\sqrt{3}\right)$ to EXACT polar coordinates. Let ***r*** **> 0** and $\boldsymbol{\theta}$ between **0º** and **360º**. Express $\boldsymbol{\theta}$ in degrees.

Let's sketch the point in a simple *Rectangular Coordinate System*.



Change Rectangular Coordinates to Polar Coordinates (2 of 4)

Step 2 - Use the conversion $\boldsymbol{r^2 = x^2 + y^2}$ (review Lecture #7) to find the *r*-value of the polar coordinates.

Example 4 continued:

Given $(1, -\sqrt{3})$, we can now write the following:

$$r^2 = (1)^2 + (-\sqrt{3})^2 = 4$$

and $\boldsymbol{r = \pm 2}$

Given the restriction $\boldsymbol{r > 0}$, we will only keep $\boldsymbol{r = 2}$.

Change Rectangular Coordinates to Polar Coordinates (3 of 4)

Step 3 - Use the conversion $tan\,\theta = \frac{y}{x}$ (review Lecture #7) to find the value of $\boldsymbol{\theta}$ of the polar coordinates.

NOTE: Depending on the instructions and/or the location of the point, you may have to work with a *reference angle* to find the appropriate value of $\boldsymbol{\theta}$. Don't forget that quadrantal angles do not have reference angles!

Example 3 continued:

Given $(1, -\sqrt{3})$, we can now write the following:

$$tan\,\theta = \frac{-\sqrt{3}}{1} = -\sqrt{3}$$

and $\theta = tan^{-1}(-\sqrt{3})$

Change Polar Coordinates to Rectangular Coordinates (4 of 4)

Example 3 continued:

Given the restriction θ between **0^o** and **360^o**, we must find a positive angle.

Knowing the location of the point in the *Rectangular Coordinate System*, we realize that we must find a QIV angle.

We will use the reference angle of – 60^o, which equals 60^o, to find the measure of angle θ. Remember, negative angles and their positive counterparts have the same reference angle!

Namely, $\boldsymbol{\theta = 360^o - 60^o = 300^o}$.

The rectangular coordinates $\left(1, -\sqrt{3}\right)$ are equivalent to the polar coordinates **$(2, 300^o)$**.