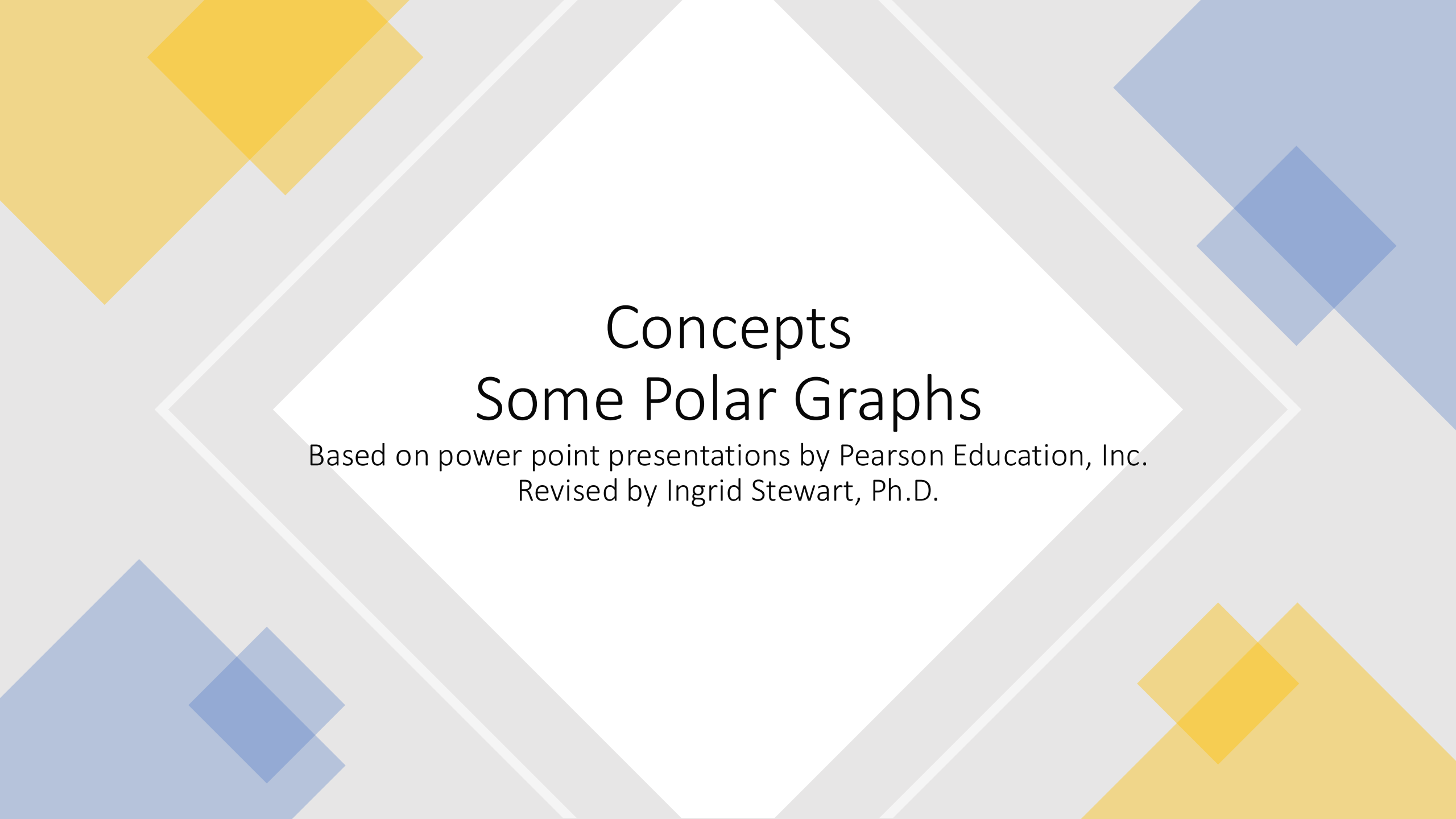




Concepts Some Polar Graphs

Based on power point presentations by Pearson Education, Inc.
Revised by Ingrid Stewart, Ph.D.

Learning Objectives

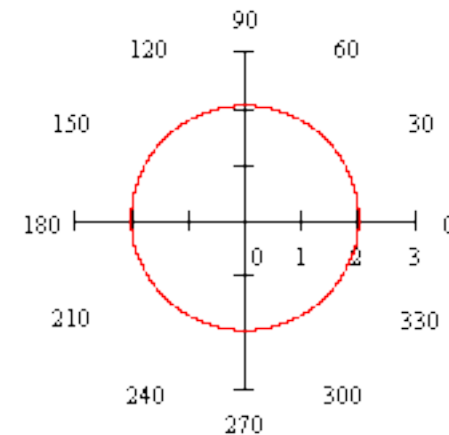
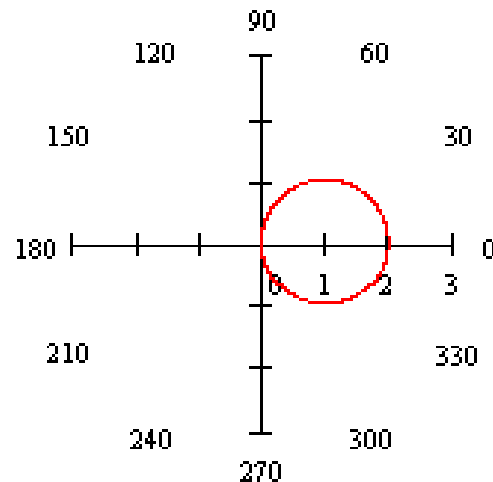
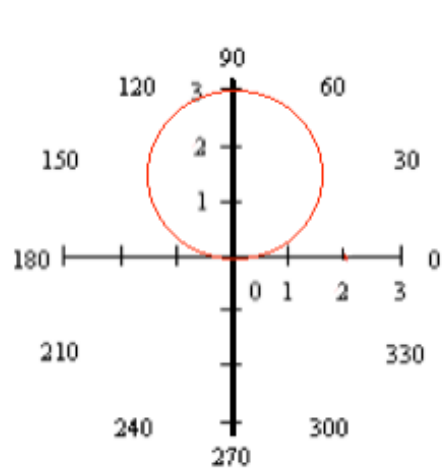
1. Memorize the general polar forms of circles and associate them with their graphs.
2. Find the radius of a circle equation in polar form.
3. Associate the names of limaçons with their graphs.
4. Recognize the graphs of rose curves given their polar form and find the number of petals.

NOTE: This lesson contains some examples. You can find more examples in the “Examples” document also located in the appropriate MOM Learning Materials folder.

Many polar equations have graphs we already studied in algebra (e.g., lines, parabolas, circles). In this lesson we will discuss a few polar equations whose graphs we did not encounter in algebra. However, we will also revisit circles which we already studied in algebra, but this time their equations will be in polar form.

1. Circle Equations in Polar Form (1 of 4)

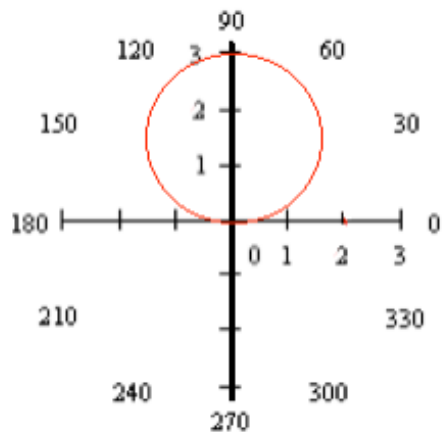
We are going to discuss three types of circles. Following are their pictures in a polar coordinate system:



Circle Equations in Polar Form (2 of 4)

You must recognize the equation of the circle together with its location in the polar coordinate system..

$r = a \sin \theta$ where **$a > 0$**



Specifically, the equation of this graph is $r = 3 \sin \theta$.

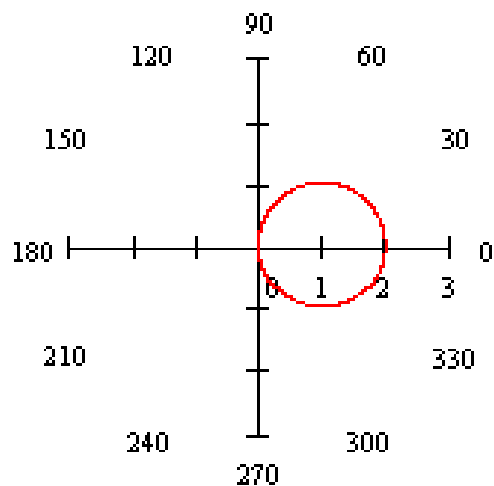
It has radius $\frac{3}{2}$.

The graphs of **$r = a \sin \theta$** have their center somewhere along a vertical r -axis. The radius is $\frac{a}{2}$. You must know this.

Circle Equations in Polar Form (3 of 4)

You must recognize the equation of the circle together with its location in the polar coordinate system..

$r = a \cos \theta$ where $a > 0$



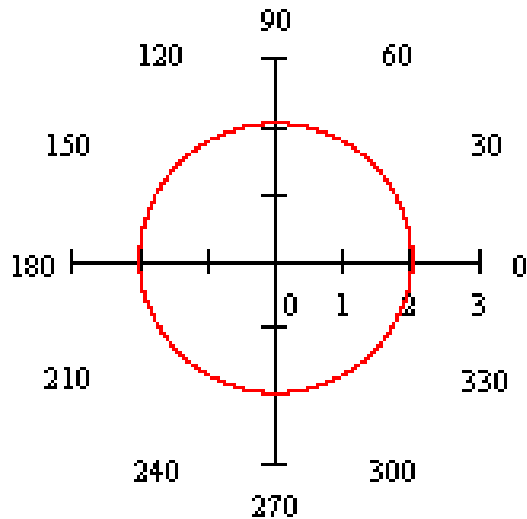
Specifically, the equation of this graph is $r = 2 \cos \theta$.
It has radius $\frac{2}{2} = 1$.

The graphs of **$r = a \cos \theta$** have their center somewhere along a horizontal polar axis. The radius is $\frac{a}{2}$. **You must know this.**

Circle Equations in Polar Form (4 of 4)

You must recognize the equation of the circle together with its location in the polar coordinate system.

$r = a$ where $a > 0$



**Specifically, the equation of this graph is $r = 2$.
It has radius 2.**

The graphs of **$r = a$** have their center at the pole. The radius is **a** . You must know this.

2. Limaçons (1 of 3)

Limaçons are pronounced lim-uh-sons which is French for "snail". The ç (pronounced c cedilla) is a special French letter. It is a "c" with a hook (cedilla) on the bottom.

The equations of limaçons can be of the following forms:

$$r = a \pm b \sin \theta \text{ where } a \text{ and } b > 0$$

$$r = a \pm b \cos \theta \text{ where } a \text{ and } b > 0$$

Limaçons (2 of 3)

We are going to discuss four types of limaçons. Following are their pictures, which you must recognize them together with their names.

Convex Limaçon



$\frac{a}{b}$ in the equation is greater than or equal to 2. It is “almost” a circle!

Dimpled Limaçon



$\frac{a}{b}$ in the equation is in between 1 and 2 not including 1 and 2.

Limaçons (3 of 3)

Heart-Shaped Limaçon (also called Cardioid)



$\frac{a}{b}$ in the equation equals 1.

When compared to the dimpled limaçon, the "dimple" is more pronounced in the Cardioid.

Limaçon with Inner Loop



$\frac{a}{b}$ in the equation is less than 1 not including 1.

3. Rose Curves (1 of 2)

The equations of rose curves can be of the following forms:

$r = a \sin n\theta$ where a and $b \neq 0$ and n is NOT equal to 1

$r = a \cos n\theta$ where a and $b \neq 0$ and n is NOT equal to 1

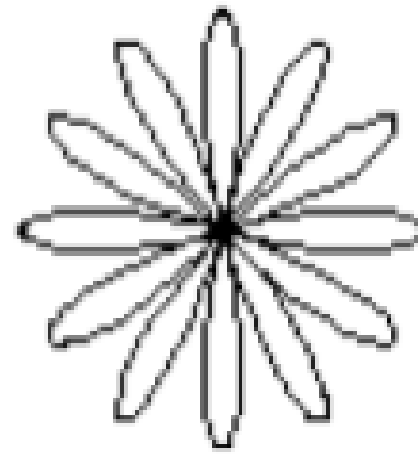
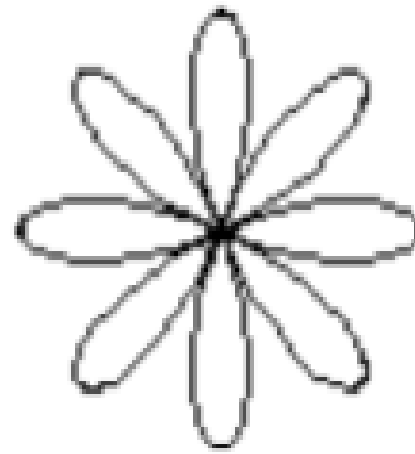
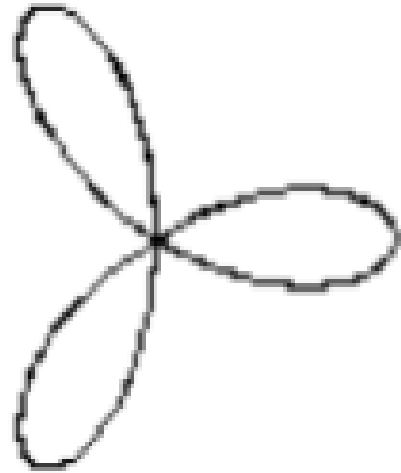
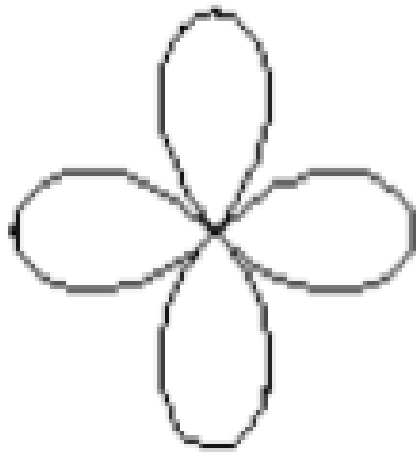
We are going to discuss two types of rose curves. One type of rose curve has an odd number of petals and in the other type, has an even number of petals.

The number of rose petals depends on the angle multiplier n .

When n is odd, the rose curve has n petals. When n is even, the rose curve has $2n$ petals. You must memorize this!

Rose Curves (2 of 2)

Following are some pictures of rose curves. You must recognize pictures of rose curves!



4 petals
 $n = 2$ in the equation

3 petals
 $n = 3$ in the equation

8 petals
 $n = 4$ in the equation

12 petals
 $n = 6$ in the equation