Concepts
Polar Equations

Based on power point presentations by Pearson Education, Inc.
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Learning Objectives

1. Change rectangular equations into polar equations.
2. Change polar equations into rectangular equations.

NOTE: This lesson contains some examples. You can find more examples in the “Examples” document also located in the appropriate MOM Learning Materials folder.

It is often also necessary to transform equations from rectangular to polar form or vice versa.

1. Change Rectangular Equations to Polar Form (1 of 3)

Strategy for transforming Equations from Rectangular to Polar Form:

Step 1 – Use one or more of the conversions $x = r\cos\theta$, $y = r\sin\theta$, $r^2 = x^2 + y^2$, and $\tan\theta = \frac{y}{x}$ to change a rectangular equation to polar form.

Example 1:

Change the rectangular equation $x^2 + y^2 - 4y = 0$ to polar form. Write the polar equation as r in terms of θ.

Knowing that $y = r\sin\theta$ and $r^2 = x^2 + y^2$, we can convert as follows:

$r^2 - 4r\sin\theta = 0$

Change Rectangular Equations to Polar Form (2 of 3)

Step 2 - Write the polar equation from Step 1 as ***r*** in terms of θ. NOTE: Writing ***r*** in terms of θ means that we must isolate ***r*** on one side of the equal sign with a coefficient 1.

Example 1 continued:

Given $r^2 - 4r\sin\theta = 0$, we will now factor ***r*** out of the two terms:

$$r(r - 4\sin\theta) = 0$$

Using the *Zero Product Principle* from algebra, we will set each factor equal to 0 to get the following:

$r = 0$ and $r - 4\sin\theta = 0$

Please note, that $r = 0$ is the equation of the pole!

Change Rectangular Equations to Polar Form (3 of 3)

Example 1 continued:

Since the pole is already included in $\boldsymbol{r - 4\sin\theta = 0}$ (Hint: Let θ equal 0 !!), we don't need $\boldsymbol{r = 0}$.

Therefore, all we need to do is write $\boldsymbol{r - 4\sin\theta = 0}$ in terms of θ.

That is, $\boldsymbol{r = 4\sin\theta}$.

In summary, the rectangular equation $\boldsymbol{x^2 + y^2 - 4y = 0}$ is $\boldsymbol{r = 4\sin\theta}$ in polar form.

2. Change Polar Equations to Rectangular Form (1 of 3)

Strategy for transforming Equations from Polar to Rectangular Form:

Use one or more of the conversions $x = r\cos\theta$, $y = r\sin\theta$, $r^2 = x^2 + y^2$, and $\tan\theta = \frac{y}{x}$ to change a polar equation to rectangular form.

IF a conversion is not readily apparent, you must try one or more of the following:

- Multiply both sides of the equation by ***r***.
- Square both sides of the equation.
- Use the *Reciprocal* or *Quotient Identities* to change to sines and cosines.

Change Polar Equations to Rectangular Form (2 of 3)

Example 2:

Change the polar equation $r = -4\cos\theta$ to any rectangular form.

Since a conversion is not readily apparent, we will multiply both sides of the equation by r to get the following:

$$r^2 = -4r\sin\theta$$

Knowing that $x = r\cos\theta$ and $r^2 = x^2 + y^2$, we can now convert as follows:

$$x^2 + y^2 = -4x$$

In summary, the polar equation $r = -4\cos\theta$ is $x^2 + y^2 = -4x$ in rectangular form. If we wish, we could also state $x^2 + y^2 + 4x = 0$.

Change Polar Equations to Rectangular Form (3 of 3)

Example 2:

Change the polar equation $r = 2$ to any rectangular form.

Since a conversion is not readily apparent, we will square both sides of the equation to get the following:

$$r^2 = 4$$

Knowing that $r^2 = x^2 + y^2$, we can now convert as follows:

$$x^2 + y^2 = 4$$

In summary, the polar equation $r = 2$ is $x^2 + y^2 = 4$ in rectangular form.