Concepts
Tangent, Cotangent, Cosecant, and Secant Functions

Based on power point presentations by Pearson Education, Inc.

Revised by Ingrid Stewart, Ph.D.

Learning Objectives

1. Memorize the shape and characteristics of the graph of the basic tangent function.
2. Memorize the shape and characteristics of the graph of the basic cotangent function.
3. Know what transformations do to the basic sine and cosine functions.
4. Graph some tangent and cotangent functions by hand.
5. Memorize the shape and characteristics of the graph of the basic cosecant function.
6. Memorize the shape and characteristics of the graphs of the basic secant function.

NOTE: This lesson contains some examples. You can find more examples in the "Examples" document also located in the appropriate MOM Learning Materials folder.

In the previous lesson, we looked at the basic sine and cosine functions and their graphs. We also examined some transformations. In this lesson we will study the basic tangent and cotangent functions, their graphs, and some transformations. Lastly, there will be a brief overview of the basic secant and cosecant functions.

Please note that you must memorize the graphs and the characteristics of the tangent and cotangent functions and those of their transformations! You must also Memorize the shape and characteristics of the graph of the basic cosecant and secant functions.

1. The Basic Tangent Function and its Graph

The definition of the basic tangent function is

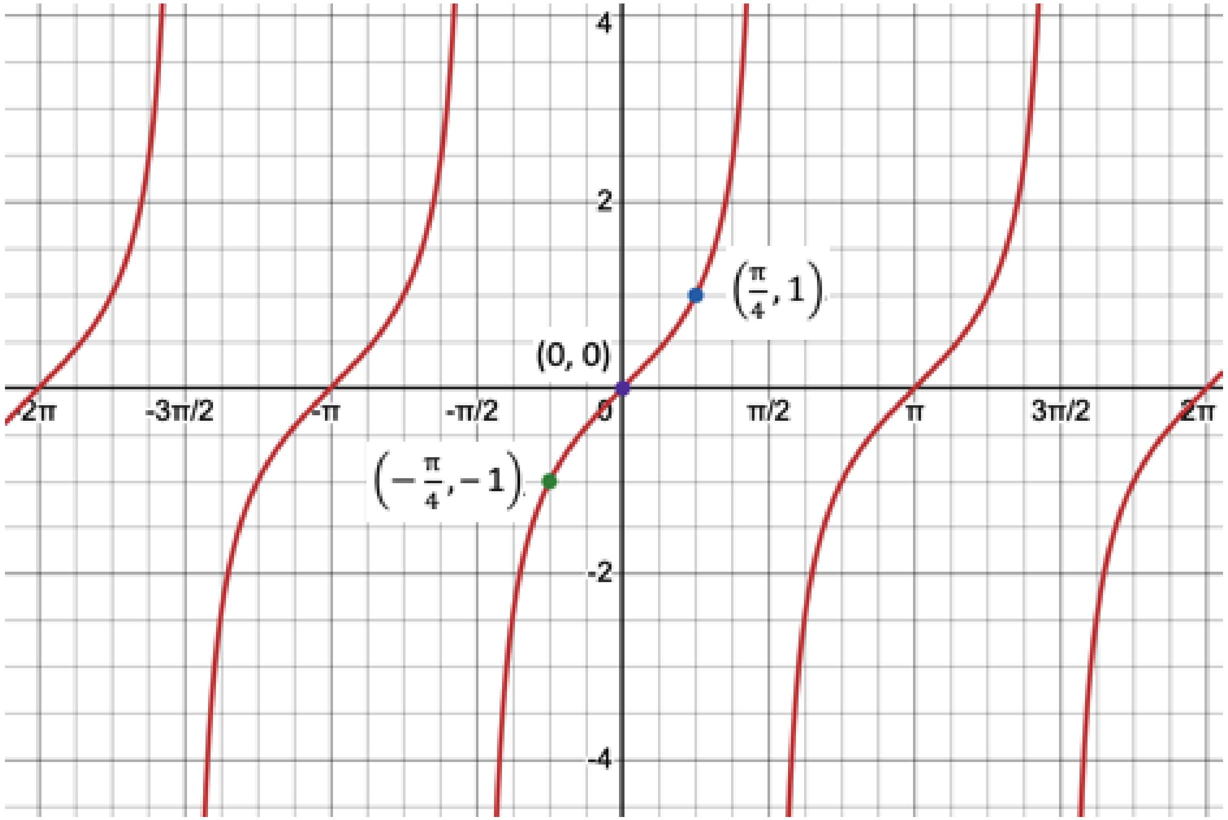
f(x) = tan x

Domain: *All real numbers except all numbers of the form* $\frac{\pi}{2} + k\pi$, where ***k*** is any integer. There the tangent function is undefined. Please note that the numbers in the domain of ALL trigonometric functions are radians. NO degree measures can be used in their domain.

Range: *All real numbers* or $(-\infty, \infty)$ in Interval Notation.

The Basic Tangent Function and its Graph

We will now use the Desmos online graphing calculator to create a picture of ***f(x) = tan x***. You must memorize the graph!



Please note that the graph is concave down  to the left of its *x*-intercepts and concave up  to the right of its *x*-intercepts. That is, the branches are NOT straight lines!

The Basic Tangent Function and its Graph (3 of 7)

NOTE: You can find instructions on how to use Desmos at http://profstewartmath.com/Math127/A_CONTENTS/desmos.htm .

Some of the points on the graph are labeled. Specifically, $\left(-\frac{\pi}{4}, -1\right)$, (0, 0), and $\left(\frac{\pi}{4}, 1\right)$. How were these points found?

Given ***f(x) = tan x***, use the calculator in radian mode and find the following:

$\boldsymbol{f}\left(-\frac{\pi}{4}\right) = \boldsymbol{tan} -\frac{\pi}{4} = -1$ $\boldsymbol{f}(0) = \boldsymbol{tan}\, 0 = 0$ $\boldsymbol{f}\left(\frac{\pi}{4}\right) = \boldsymbol{tan}\, \frac{\pi}{4} = 1$

The Basic Tangent Function and its Graph (4 of 7)

Please note the following: $\boldsymbol{f}\left(-\frac{\pi}{2}\right) = \boldsymbol{tan} -\frac{\pi}{2}$ and $\boldsymbol{f}\left(\frac{\pi}{2}\right) = \boldsymbol{tan}\,\frac{\pi}{2}$ are undefined.

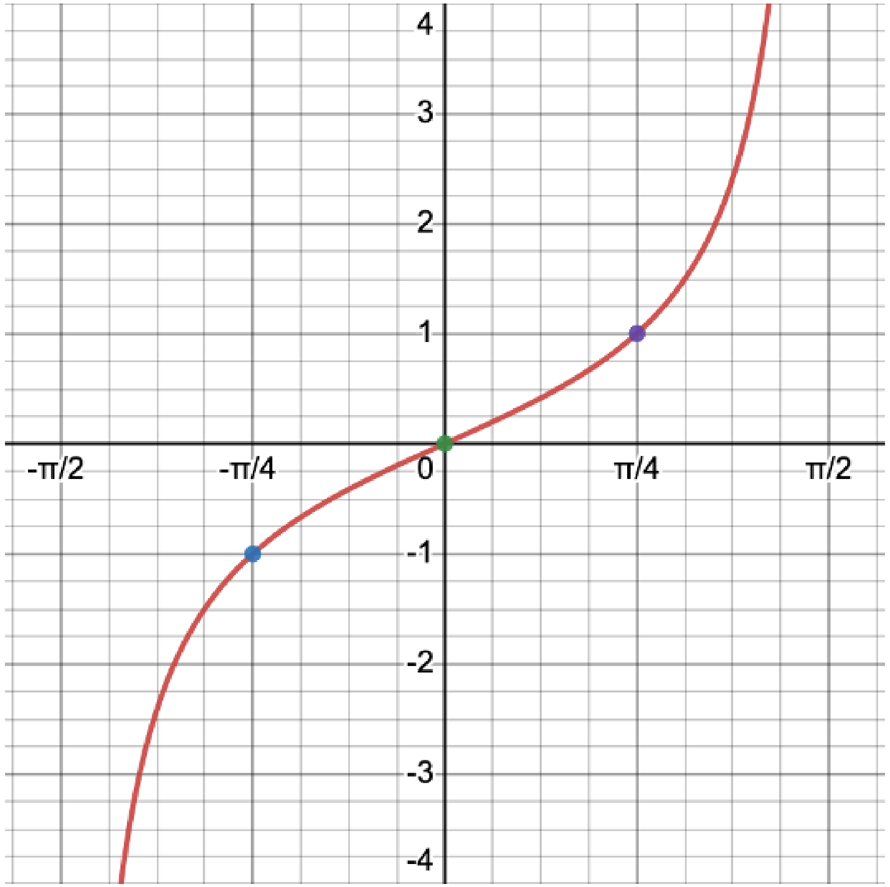
There, the graph of the basic tangent function has *vertical asymptotes* dividing the graph into infinitely many branches. The branches never touch the *vertical asymptote* but are also never parallel to it.

From algebra we know that *vertical asymptotes* are invisible vertical lines that separate a graph into two or more branches. A graphing utility will not show the asymptotes. We have to know that they exist. When we graph the basic tangent function by hand, we indicate *vertical asymptotes* by dashed vertical lines.

We will now only plot the 3 points from the previous slide into the coordinate system and then connect them keeping in mind the shape of the graph of the tangent function. You must memorize this portion of the graph!

The Basic Tangent Function and its Graph

The equations of the *vertical asymptotes* are $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$.



We call this portion of the graph of the tangent function the **representative picture**. Note the concavities to the right and left of the *x*-intercept!

It lies on the open interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ which is called the **period** of the tangent graph. Clearly, the period is **π** in length.

The Basic Tangent Function and its Graph

The representative picture repeats "forever" along the positive and negative *x*-axis and creates the graph of the basic tangent function. Therefore, the tangent function is called "periodic".

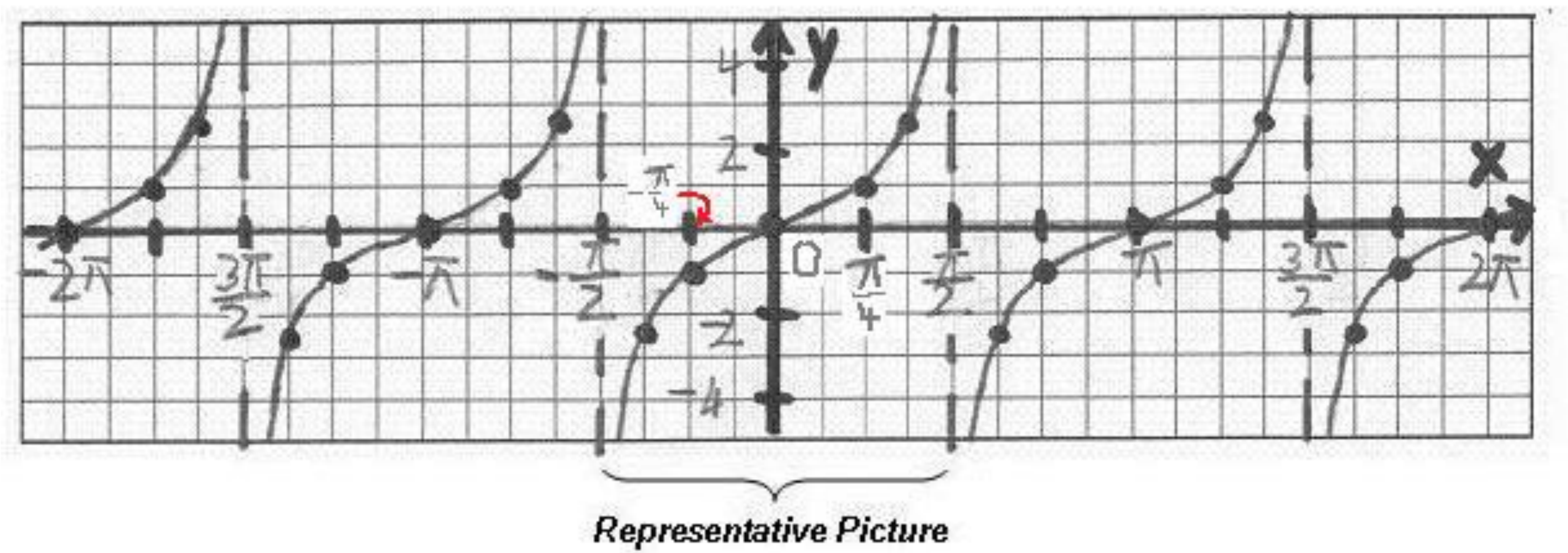
Note that the representative picture is divided into four (4) equal intervals.

- The graph starts to the right of a *vertical asymptote*.
- At the end of the first interval, we find an ordered pair and plot it.
- At the end of the second interval the graph has an *x*-intercept.
- At the end of the third interval, we find an ordered pair and plot it.
- At the end of the fourth interval there is a *vertical asymptote*.

You must memorize these characteristics!

The Basic Tangent Function and its Graph (7 of 7)

Below is a hand-drawn graph of the basic tangent function ***f(x) = tan x***. Note that we plotted the vertical asymptotes as dashed vertical lines.



2. The Basic Cotangent Function and its Graph (1 of 7)

The definition of the basic cotangent function is

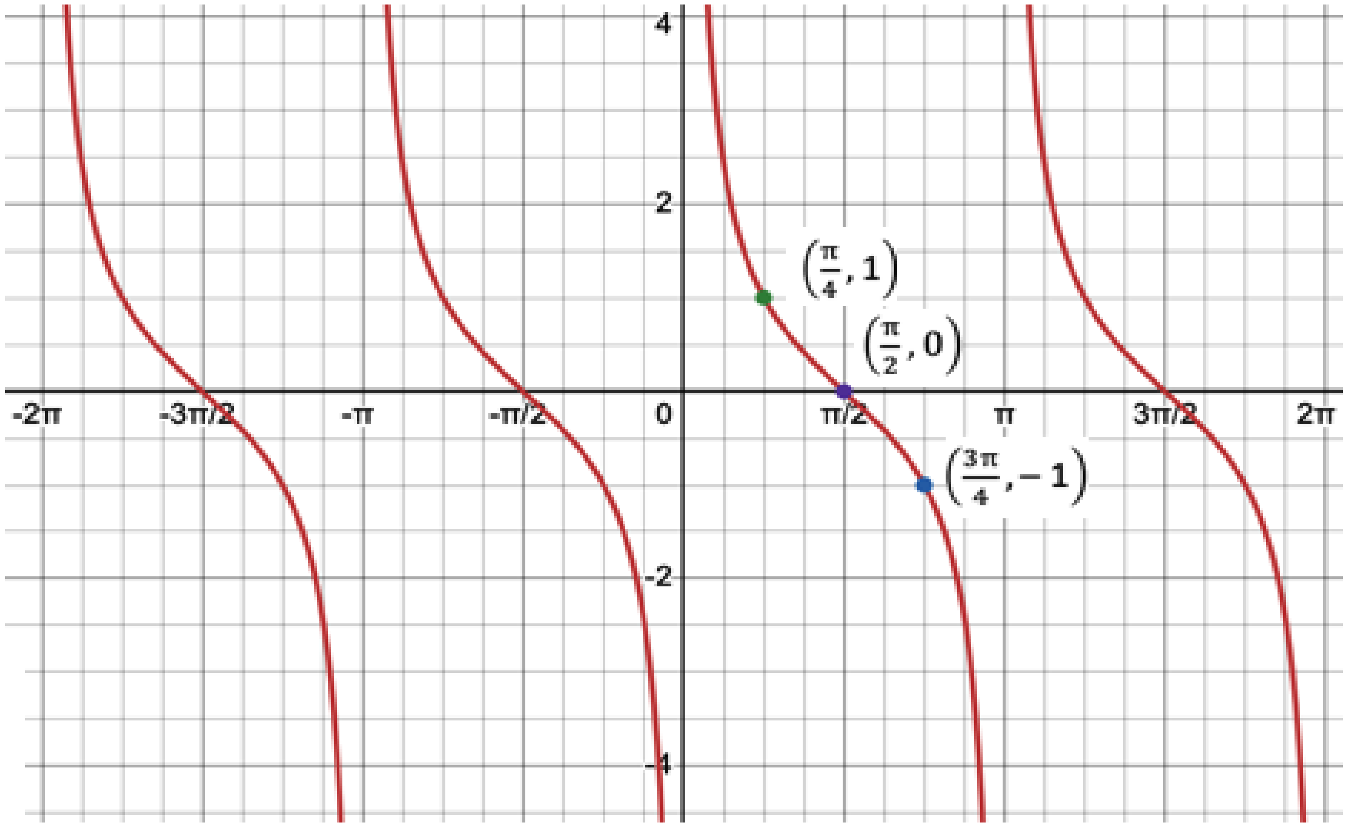
p(x) = cot x

Domain: *All real numbers except all numbers of the form **k**π,* where ***k*** is any integer. There the cotangent function is undefined. Please note that the numbers in the domain of ALL trigonometric functions are radians. NO degree measures can be used in their domain.

Range: *All real numbers* or $(-\infty, \infty)$ in Interval Notation.

The Basic Cotangent Function and its Graph

We will now use the Desmos online graphing calculator to create a picture of ***p(x) = cot x***. You must memorize the graph!



Please note that the graph is concave up  to the left of its *x*-intercepts and concave down  to the left of its *x*-intercepts. That is, the branches are NOT straight lines!

The Basic Cotangent Function and its Graph

NOTE: You can find instructions on how to use Desmos at http://profstewartmath.com/Math127/A_CONTENTS/desmos.htm .

Some of the points on the graph are labeled. Specifically, $\left(\frac{\pi}{4}, 1\right)$, $\left(\frac{\pi}{2}, 0\right)$, and $\left(\frac{3\pi}{4}, -1\right)$. How were these points found?

Given ***p(x) = cot x***, use the calculator in radian mode and find the following:

$\boldsymbol{p}\left(\frac{\pi}{4}\right) = \boldsymbol{cot}\,\frac{\pi}{4} = 1$ $\boldsymbol{p}\left(\frac{\pi}{2}\right) = \boldsymbol{cot}\,\frac{\pi}{2} = 0$ $\boldsymbol{p}\left(\frac{3\pi}{4}\right) = \boldsymbol{cot}\,\frac{3\pi}{4} = -1$

The Basic Cotangent Function and its Graph (4 of 7)

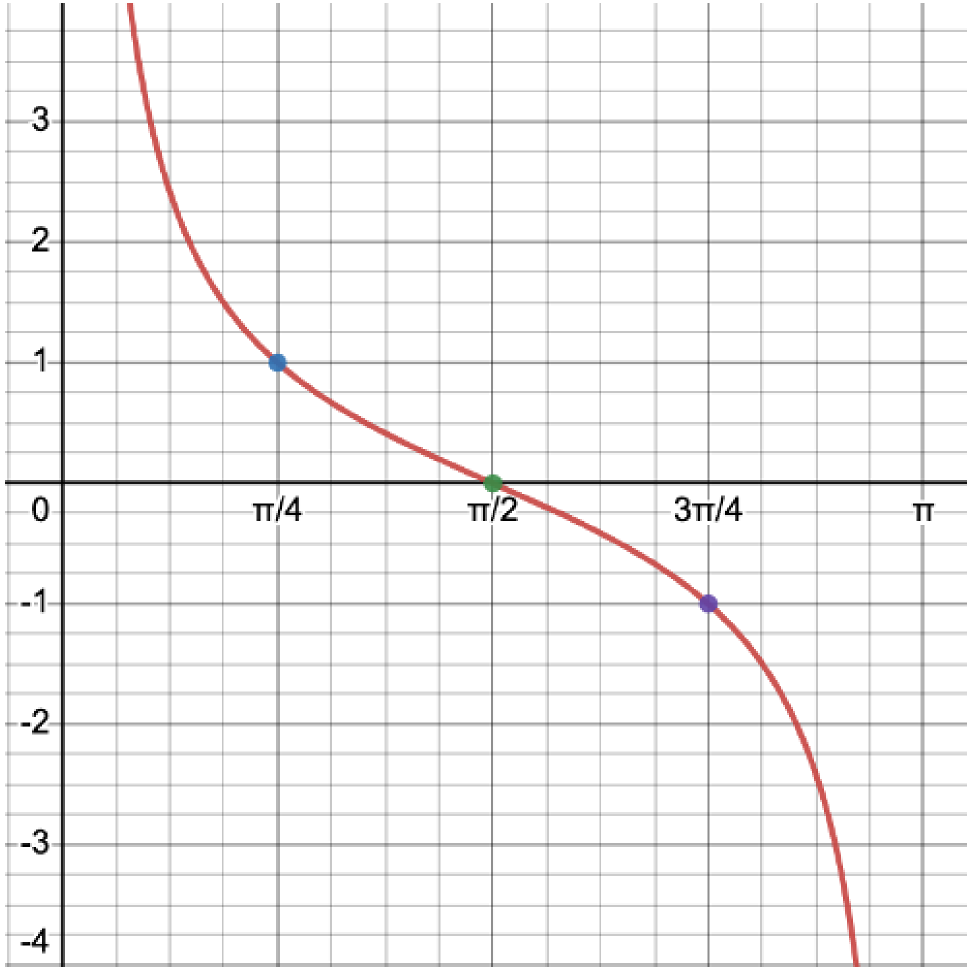
Please note the following: $\boldsymbol{p}(0)$ = ***cot*** 0 and $\boldsymbol{p}(\pi)$ = ***cot*** π are undefined.

There, the graph of the basic cotangent function has *vertical asymptotes* dividing the graph into infinitely many branches. The branches never touch the *vertical asymptote* but are also never parallel to it.

We will now only plot the 3 points from the previous slide into the coordinate system and then connect them keeping in mind the shape of the graph of the cotangent function. You must memorize this portion of the graph!

The Basic Cotangent Function and its Graph

The equations of the *vertical asymptotes* are $\boldsymbol{x} = 0$ and $\boldsymbol{x} = \pi$.



We call this portion of the graph of the tangent function the **representative picture**. Note the concavities to the right and left of the *x*-intercept!

It lies on the open interval $(0, \pi)$ which is called the **period** of the tangent graph. Clearly, the period is $\boldsymbol{\pi}$ in length.

The Basic Cotangent Function and its Graph

The representative picture repeats "forever" along the positive and negative *x*-axis and creates the graph of the basic cotangent function. Therefore, the tangent function is called "periodic”.

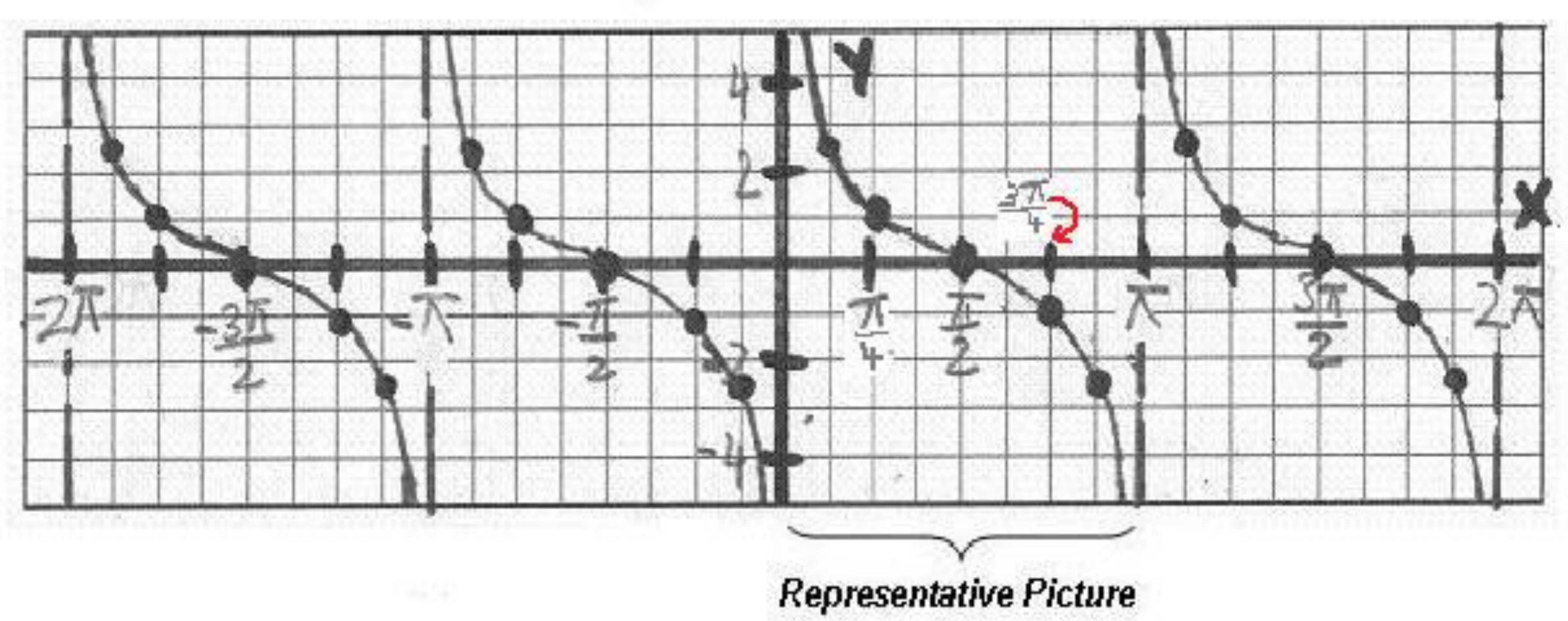
Note that the representative picture is divided into four (4) equal intervals.

- The graph starts to the right of a *vertical asymptote*.
- At the end of the first interval, we find an ordered pair and plot it.
- At the end of the second interval the graph has an *x*-intercept.
- At the end of the third interval, we find an ordered pair and plot it.
- At the end of the fourth interval there is a *vertical asymptote*.

You must memorize these characteristics!

The Basic Cotangent Function and its Graph

Below is a hand-drawn graph of the basic cotangent function ***p(x) = cot x***. Note that we plotted the vertical asymptotes as dashed vertical lines.



3. Transformations of the Basic Tangent and Cotangent Functions (1 of 4)

We will now investigate some transformations of the basic tangent and cotangent functions. Specifically, we will only discuss transformations of the form ***y = a tan (bx)*** and ***y = a cot (bx)***.

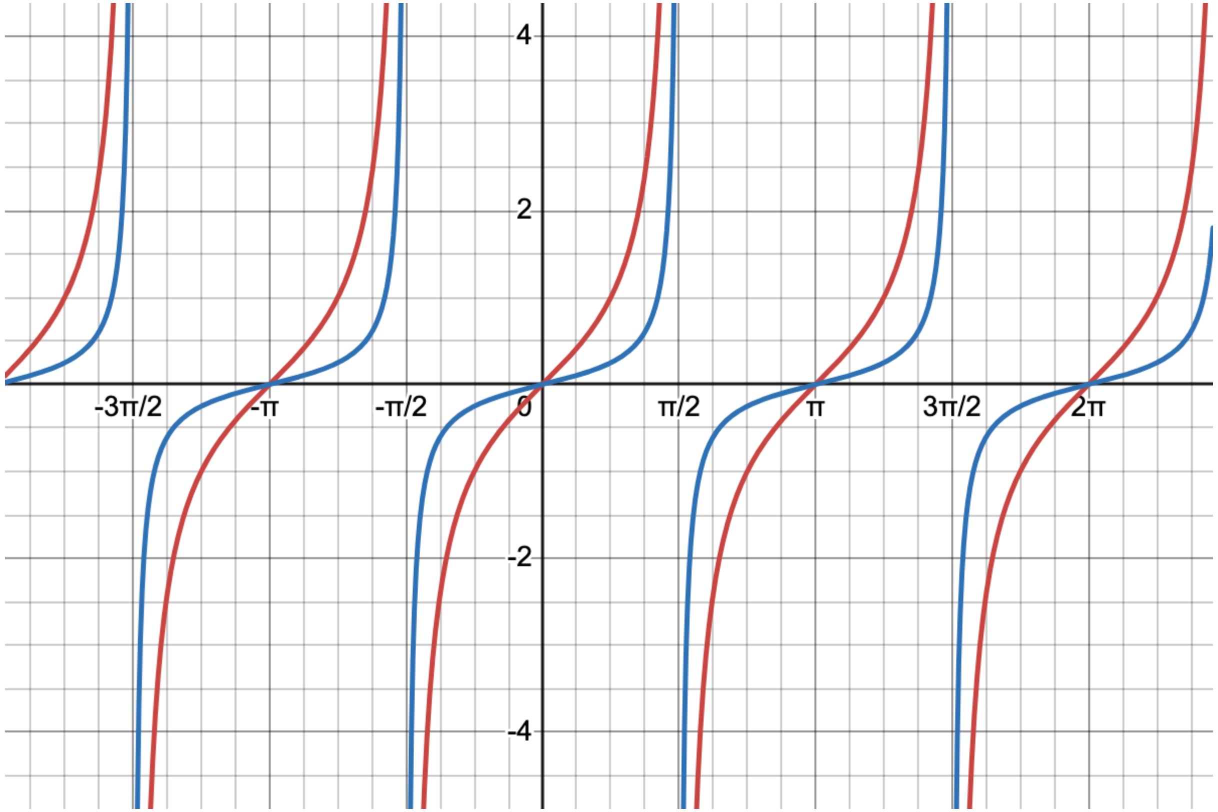
The number ***a***:

It indicates a vertical stretch or compression of the graph of the basic tangent and cotangent functions. Please note that there is NO amplitude involved as there is in the sine and cosine functions.

Transformations of the Basic Tangent and Cotangent Functions (2 of 4)

Example 1:

Graph $\boldsymbol{y = \frac{1}{4} \tan x}$ using Desmos together with $\boldsymbol{y = \tan x}$



red graph: $\boldsymbol{y = \tan x}$

blue graph: $\boldsymbol{y = \frac{1}{4} \tan x}$

Note that the transformed graph has more pronounced concavities.

Transformations of the Basic Tangent and Cotangent Functions (3 of 4)

The number ***b***:

It indicates a horizontal stretch or compression of the graph of the basic tangent and cotangent functions. This will affect the length of the **period *P***.

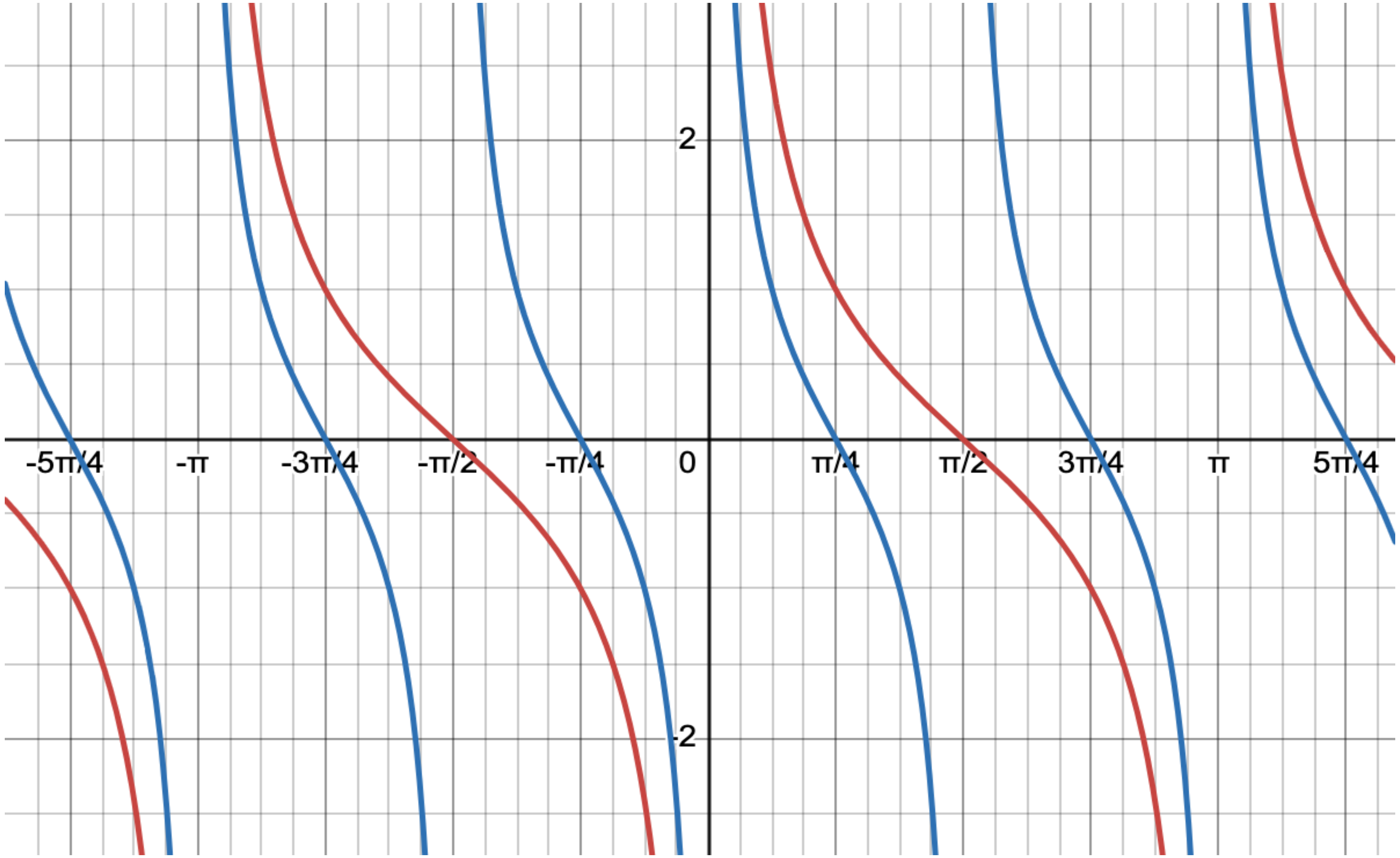
The formula $P = \frac{\pi}{b}$ calculates the period of the graph of the tangent and cotangent functions.

- If ***b*** is between 0 and 1, the period of the basic tangent and cotangent function gets larger. This is a horizontal stretch.
- If ***b*** is greater than 1, the period of the basic tangent and cotangent function gets smaller. This is a horizontal compression.

Transformations of the Basic Tangent and Cotangent Functions

Example 2:

Graph ***y = cot 2x*** using Desmos together with ***y = cot x***.



red graph: ***y = cot x***

blue graph: ***y = cot 2x***

Indicates a change of the period of the graph of ***y = cot x*** to $\frac{\pi}{2}$.

4. Graph Some Tangent and Cotangent Functions by Hand
(1 of 2)

We will only graph functions of the form ***y = a tan bx*** and ***y = a cot bx*** by hand. Following are the eight (8) steps necessary to properly graph these functions by hand. Please study and then rework the examples in the corresponding "Examples" document.

1. Determine the period using $P = \frac{\pi}{b}$.
2. Keep in mind the graph and characteristics of the basic function.
3. Mark off a distance along the *x*-axis to represent the period of the representative picture.
4. Divide the period into four equal intervals.

Steps continued on next slide!

Graph Some Sine and Cosine Functions by Hand (2 of 2)

Graphing strategy continued:

5. Draw dashed vertical lines at the beginning and end of the period to represent the *vertical asymptotes*. Note: Do not draw the *y*-axis as a dashed line!
6. Create the appropriate representative picture by using the beginning/ending point of each interval. Keep in mind the amplitude. Be mindful of ***a*** being positive or negative!
7. Connect the points found in (6) to form the representative picture being mindful of the *vertical asymptotes*.
8. Copy the representative picture several more times along the negative and positive *x*-axis.

5. The Basic Cosecant Function and its Graph

The definition of the basic cosecant function is

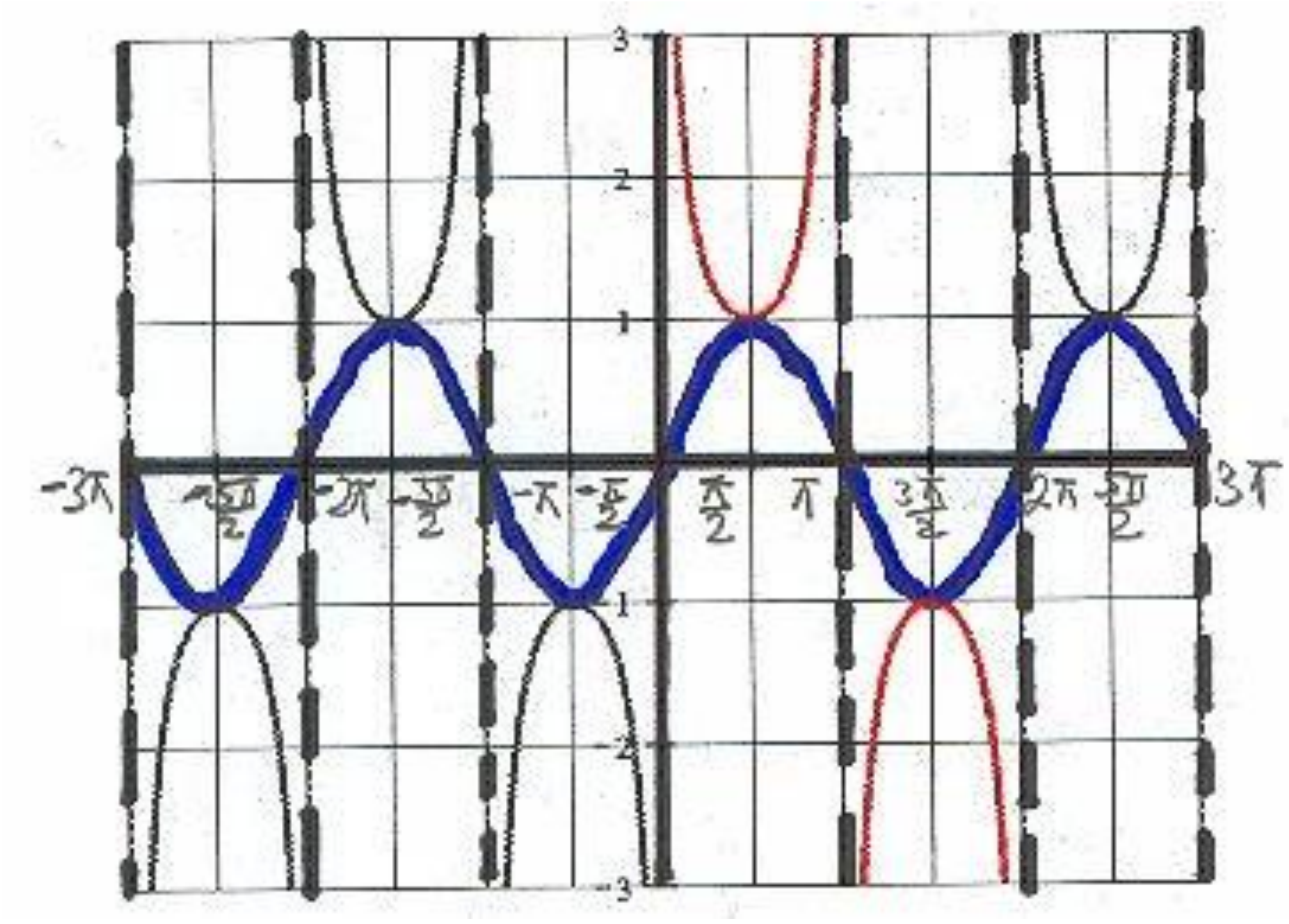
$$g(x) = \csc x$$

Domain: *All real numbers except all numbers of the form **k**π,* where ***k*** is any integer. There the cosecant function is undefined. Please note that the numbers in the domain of ALL trigonometric functions are radians. NO degree measures can be used in their domain.

Range: *All real numbers except the ones between – 1 and 1.*

The Basic Cosecant Function and its Graph

Below is a hand-drawn graph of the basic cosecant function ***g*(*x*) = csc *x*** and the basic sine function ***y* = sin *x*** (dark blue). Note that we plotted the *vertical asymptotes* as dashed vertical lines (except for the *y*-axis).



You must memorize the picture of the cosecant function. Note that the graph has *vertical asymptotes* where the graph of ***y* = sin *x*** has *x*-intercepts.

The branches in red indicate the representative picture of the cosecant function!

6. The Basic Secant Function and its Graph (1 of 2)

The definition of the basic secant function is

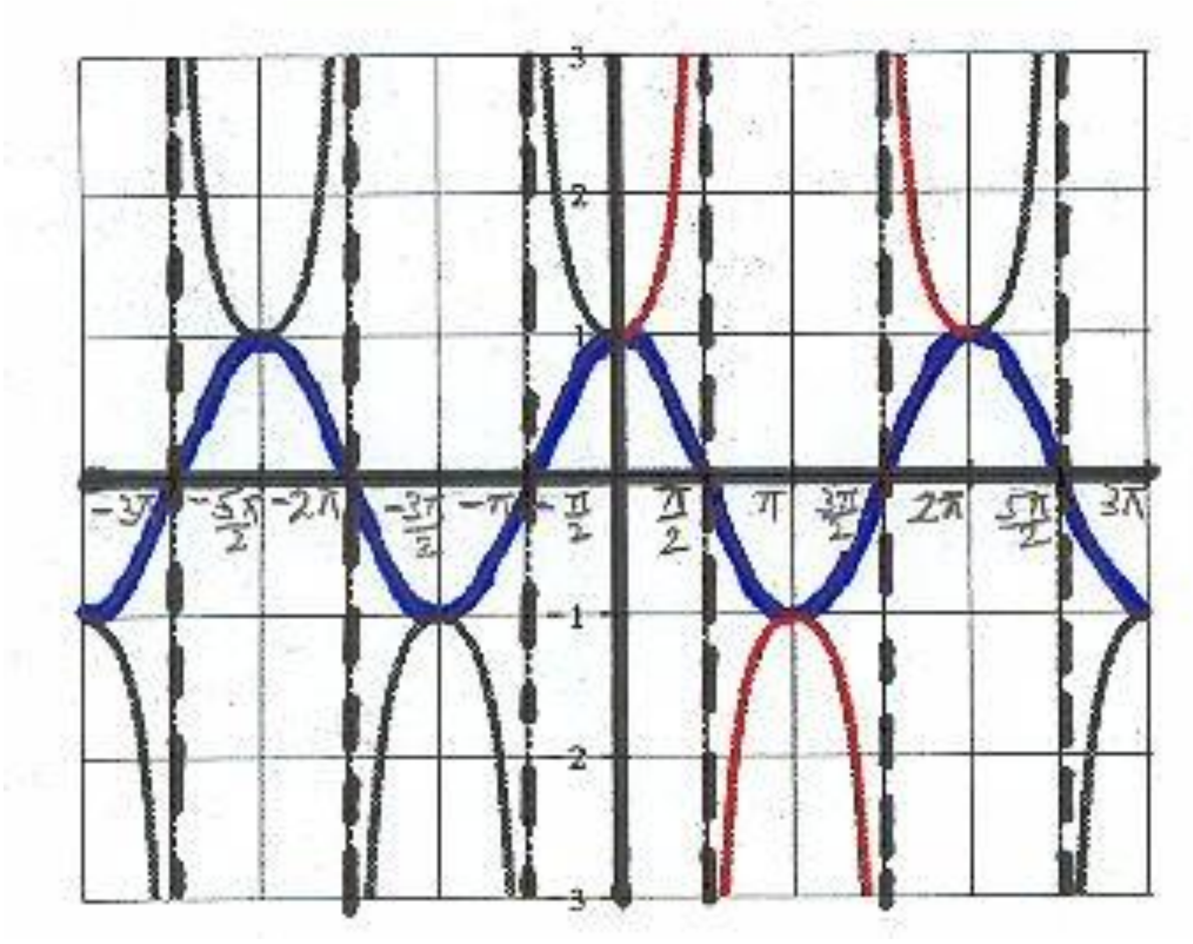
h(x) = sec x

Domain: *All real numbers except all numbers of the form* $\frac{\pi}{2} + k\pi$, where ***k*** is any integer. There the secant function is undefined. Please note that the numbers in the domain of ALL trigonometric functions are radians. NO degree measures can be used in their domain.

Range: *All real numbers except the ones between – 1 and 1.*

The Basic Secant Function and its Graph (2 of 2)

Below is a hand-drawn graph of the basic secant function ***h(x) = sec x*** and the basic cosine function ***y = cos x*** (dark blue). Note that we plotted the *vertical asymptotes* as dashed vertical lines.



You must memorize the picture of the secant function. Note that the graph has *vertical asymptotes* where the graph of ***y = cos x*** has *x*-intercepts.

The branches in red indicate the representative picture of the secant function!