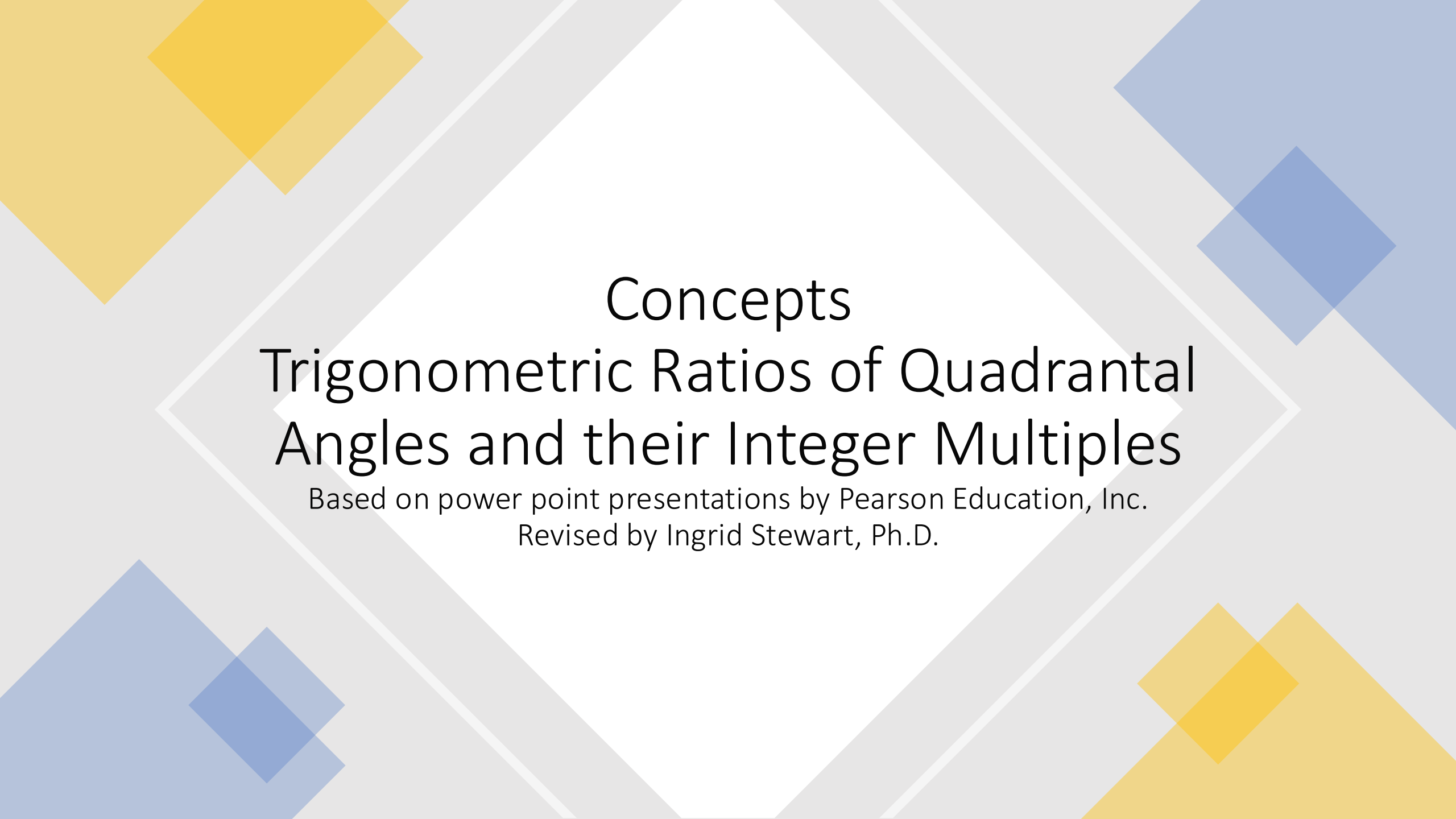




Concepts Trigonometric Ratios of Quadrantal Angles and their Integer Multiples

Based on power point presentations by Pearson Education, Inc.
Revised by Ingrid Stewart, Ph.D.

Learning Objectives

1. Memorize the values of trigonometric ratios of quadrantal angles.
2. Find the exact values of trigonometric ratios of integer multiples of quadrantal angles without a calculator.

NOTE: This lesson contains some examples. You can find more examples in the “Examples” document also located in the appropriate MOM Learning Materials folder.

1. Find the Values of Trigonometric Ratios of Quadrantal Angles (1 of 5)

In this lesson, we will find the EXACT values of their trigonometric ratios of quadrantal angles. We already discussed them in Lesson 1! **NOTE: We will NOT use a calculator for homework and on the Module 1 Quiz!**

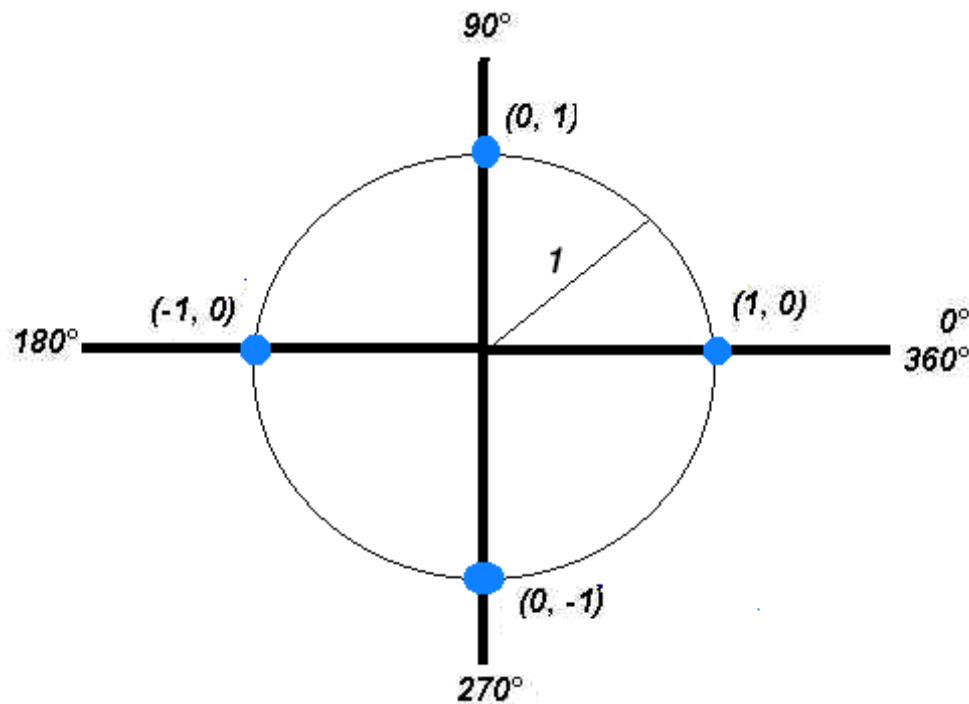
First, let's derive the values of the six trigonometric ratios of the 0° , 90° , 180° , 270° , and 360° angles. Most cannot exist in a right triangle because the sum of any triangle is always 180° .

Therefore, let's derive the values of the sine, cosine, and tangent ratios of the angles with measures 0° , 90° , 180° , 270° , and 360° using a unit circle.

Find the Values of Trigonometric Ratios of Quadrantal Angles

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Following is a picture of what was described in the previous slide.



As we can see, there are four points on the unit circle, namely $(1, 0)$, $(0, 1)$, $(-1, 0)$, and $(0, -1)$.

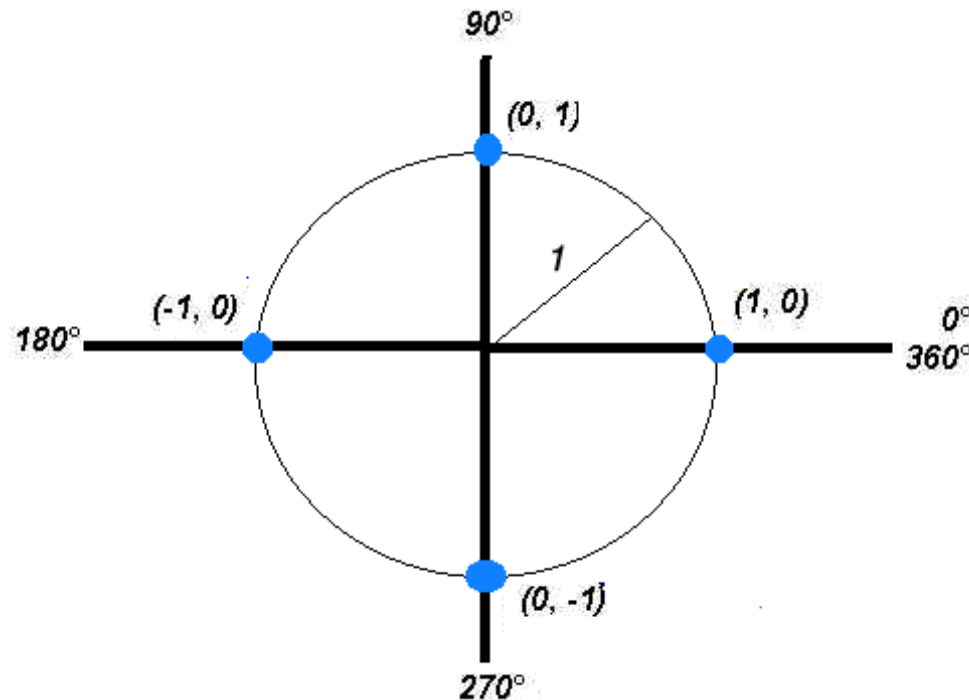
Now, we will define the values of the cosine, sine, and tangent ratios using the four points stated above.

NOTE: Their **x-coordinates** will be equivalent to the sides adjacent (**adj**), and their **y-coordinates** will be equivalent to the sides opposite (**opp**).

Find the Values of Trigonometric Ratios of Quadrantal Angles

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Following are the values of the trigonometric ratios of sine, cosine, and tangent paired with 0° and 90° . The other values can be derived in a similar manner. NOTE: We will use the radius **1** as the “hypotenuse”.



$$\sin 0^\circ = \frac{\text{opp}}{\text{hyp}} = \frac{0}{1} = 0$$

$$\cos 0^\circ = \frac{\text{adj}}{\text{hyp}} = \frac{1}{1} = 1$$

$$\tan 0^\circ = \frac{\text{opp}}{\text{adj}} = \frac{0}{1} = 0$$

$$\sin 90^\circ = \frac{\text{opp}}{\text{hyp}} = \frac{1}{1} = 1$$

$$\cos 90^\circ = \frac{\text{adj}}{\text{hyp}} = \frac{0}{1} = 0$$

$$\tan 90^\circ = \frac{\text{opp}}{\text{adj}} = \frac{1}{0}$$

Note that the value of $\tan 90^\circ$ is undefined.

Find the Values of Trigonometric Ratios of Quadrantal Angles

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The following table is a summary of what we just found in the previous slides. **The EXACT values of trigonometric ratios of the quadrantal angles 0° , 90° , 180° , 270° , and 360° MUST be memorized because they are used frequently in mathematics, physics, and engineering. We will NOT use a calculator to find these values for homework and the Module 1 Quiz!**

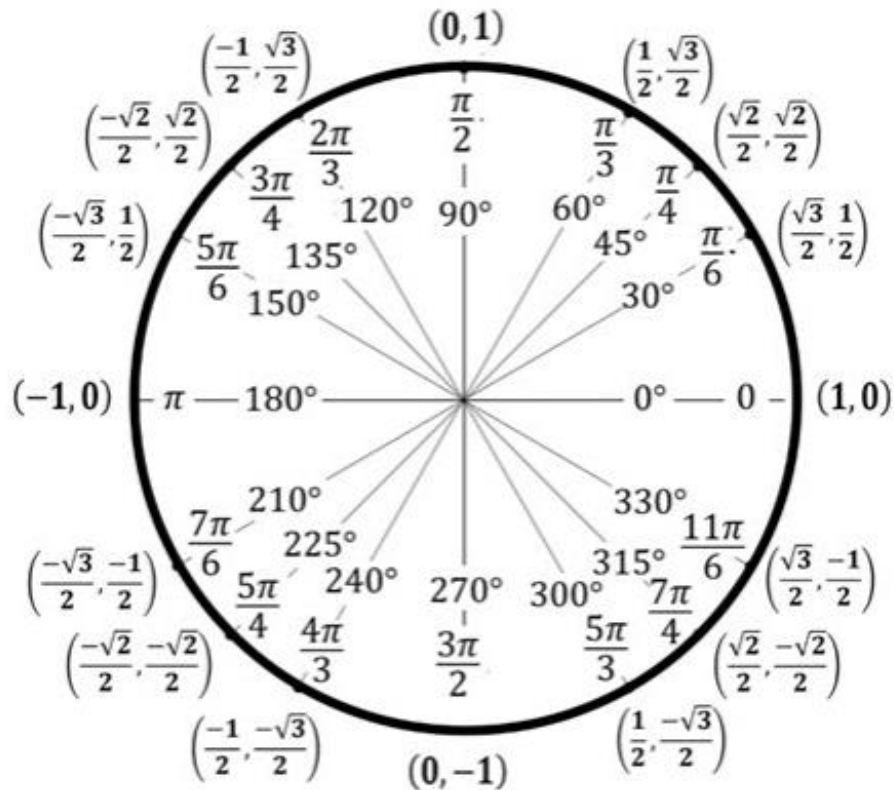
x	$\sin x$	$\cos x$	$\tan x$	$\csc x$	$\sec x$	$\cot x$
$0^\circ \equiv 0$	0	1	0	undefined	1	undefined
$90^\circ \equiv \frac{\pi}{2}$	1	0	undefined	1	undefined	0
$180^\circ \equiv \pi$	0	-1	0	undefined	-1	undefined
$270^\circ \equiv \frac{3\pi}{2}$	-1	0	undefined	-1	undefined	0
$360^\circ \equiv 2\pi$	0	1	0	undefined	1	undefined

Memorization Hint: Only memorize the values for sine and cosine. Then use the *Reciprocal* and/or *Quotient Identities* to find the remaining values.

Find the Values of Trigonometric Ratios of Quadrantal Angles

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NOTE: Some students who have had trigonometry in the past might have memorized the EXACT values of the trigonometric ratios of multiples of the "special" and quadrantal angles using the following *unit circle*.



It is NOT a good idea to memorize the coordinates of the points in the picture and then use them to find the values of trigonometric ratios of certain angles.

Using *reference angles* is necessary for later concepts as well, and anyone who refuses to learn it now will be in trouble later on! So please, do not memorize the unit circle coordinates.

2. Find the Values of Trigonometric Ratios of Integer Multiples of Quadrantal Angles (1 of 5)

NOTE: You **MUST** find the values of trigonometric ratios of integer multiples of special angles without a calculator in the homework and on the Module 1 Quiz.

Strategy for finding values of trigonometric ratios of integer multiples of quadrantal angles in degrees and radians. **Note that Quadrantal Angles DO NOT have reference angles!**

Step 1 - Find the angle β coterminal with angle θ where $0^\circ < \theta < 360^\circ$ or $0 < \theta < 2\pi$. NOTE: In the process, you will may find $\beta = 0^\circ, 90^\circ, 180^\circ$, or 270° .

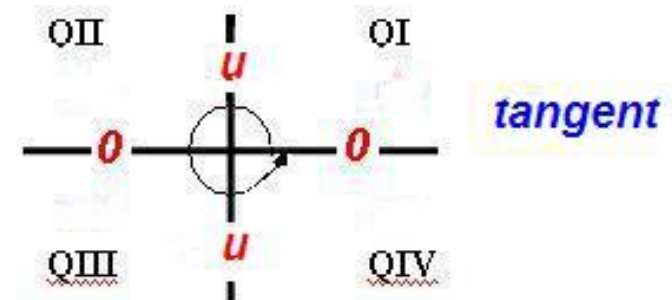
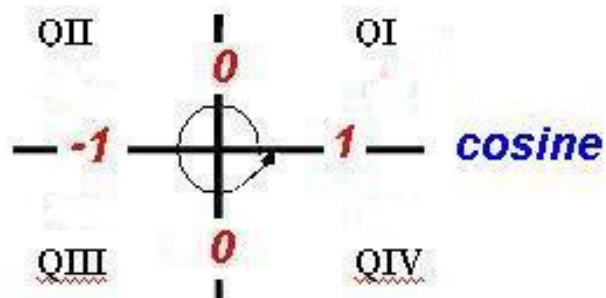
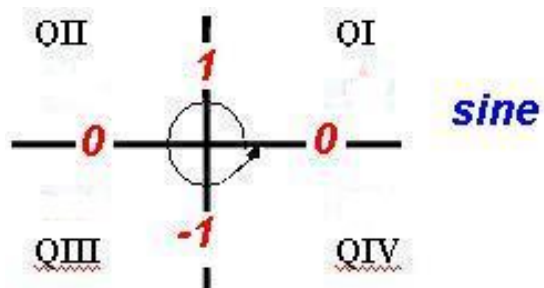
Example 1:

Find the EXACT value of $\sin 810^\circ$ without a calculator.

$810^\circ = 360^\circ (2) + 90^\circ$. We find that 90° is coterminal with 810° .

Find the Values of Trigonometric Ratios of Integer Multiples of Quadrantal Angles (2 of 5)

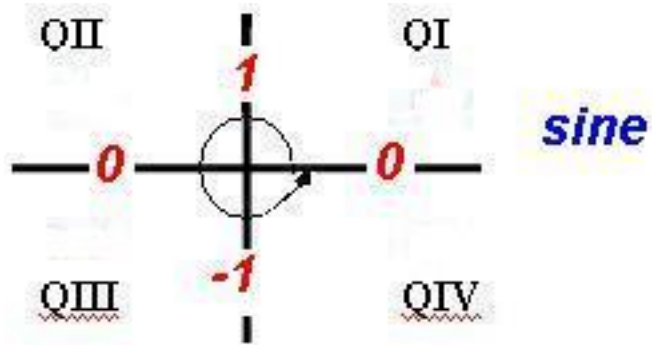
Step 2 – We already memorized the fact that the values of trigonometric ratios of coterminal angles are identical. We will use it now. **Memorize the following pictures taken from the table in Slide 7 which we also memorized.**



Find the Values of Trigonometric Ratios of Integer Multiples of Quadrantal Angles (3 of 5)

Example 1 continued:

We will use the following **memorized** picture to find the value of $\sin 90^\circ$:



We see that the EXACT value of $\sin 90^\circ$ is equal to 1. Therefore, the EXACT value of $\sin 810^\circ$ is also equal to 1.

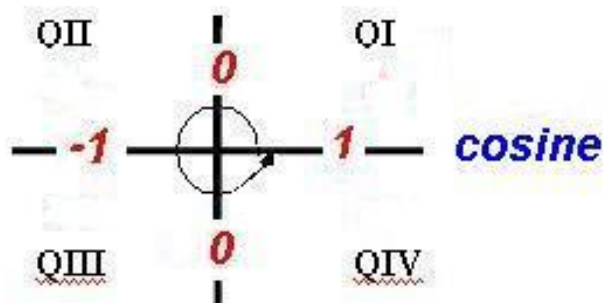
Find the Values of Trigonometric Ratios of Integer Multiples of Quadrantal Angles (4 of 5)

Example 2:

Find the EXACT value of $\cos 990^\circ$ without a calculator.

$990^\circ = 360^\circ (2) + 270^\circ$. We find that 270° is coterminal with 990° .

We will use the following **memorized** picture to find the value of $\cos 270^\circ$:



We see that the EXACT value of $\cos 270^\circ$ is equal to 0. Therefore, the EXACT value of $\cos 990^\circ$ is also equal to 0.

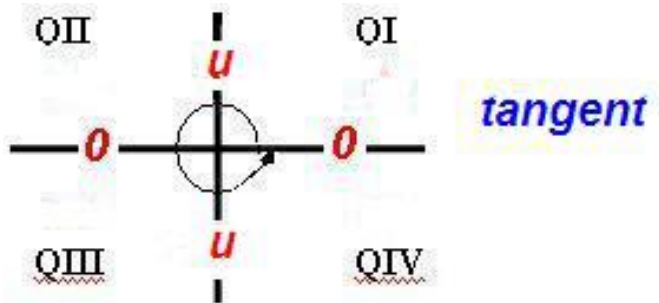
Find the Values of Trigonometric Ratios of Integer Multiples of Quadrantal Angles (5 of 5)

Example 3:

Find the EXACT value of $\tan 1620^\circ$ without a calculator.

$1620^\circ = 360^\circ (4) + 180^\circ$. We find that 180° is coterminal with 1620° .

We will use the following **memorized** picture to find the value of $\tan 180^\circ$:



We see that the EXACT value of $\tan 180^\circ$ is equal to 0. Therefore, the exact value of $\tan 1620^\circ$ is also equal to 0.