# Concepts Introduction to Conic Sections — Ellipse and Circle

Based on power point presentations by Pearson Education, Inc. Revised by Ingrid Stewart, Ph.D.

# Learning Objectives

- 1. Memorize which standard equation pertains to what graph of conic ellipses (vertical major axis, horizontal major axis).
- 2. Memorize the characteristics of the vertices, the center, the foci, the major axis, and the minor axis.
- Change given equations of conic ellipses to their standard form.
- 4. Memorize the standard equation pertains conic circles.
- 5. Change given equations of conic circles to their standard form.

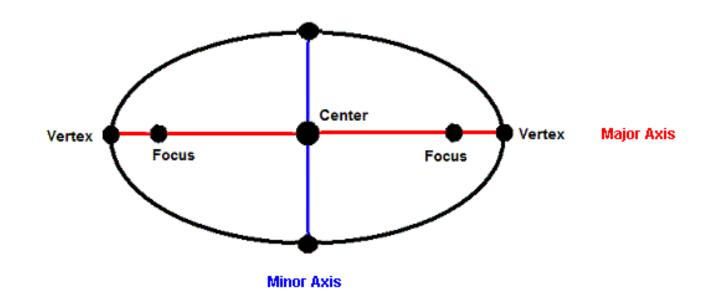
NOTE: This lesson only contains pictures and standard equations of ellipses. It contains NO examples.

Our task will be to change given equations of ellipses to standard form.

You can find examples in the "Examples" document also located in the appropriate MOM Learning Materials folder.

# 1. The Ellipse (1 of 3)

Ellipses resulting from conic sections can have a horizontal or a vertical major axis. Following is a picture of an ellipse with a horizontal major axis.

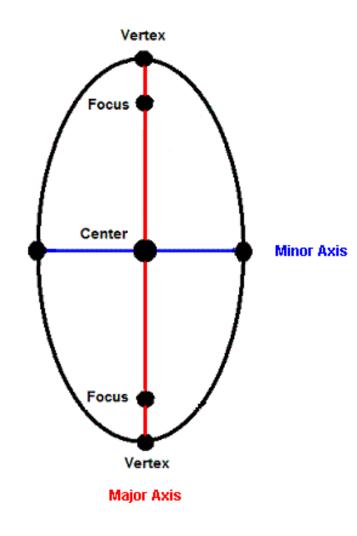


Geometrically, the standard equation of an ellipse with a horizontal major axis is  $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$  where 0 < b < a.

This equation must be memorized. The coordinates of the Center are (h, k).

#### The Ellipse (2 of 3)

Following is the picture of an ellipse with a vertical major axis.



Geometrically, the standard equation of an ellipse with a vertical major axis is  $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$  where 0 < b < a.

This equation must be memorized. The coordinates of the Center are (h, k).

# The Ellipse (3 of 3)

#### Some Vocabulary

Please refer to the pictures in the previous two slides!

**Foci (singular: foci)** – Two points that lie on the major axis above and below or to the right and left of the vertex.

**Vertices (singular: verte)** – Two points where the line through the two foci intersects with the ellipse.

**Major Axis** – It is a line segment that joins the two vertices.

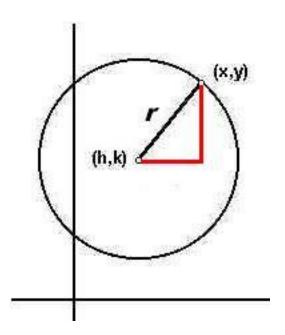
**Center** – The midpoint of the major axis.

**Minor Axis** – The line segment perpendicular to the major axis intersecting at its midpoint and ending at the ellipse.

#### 2. The Circle (1 of 2)

In algebra, we already discussed circles and their graphs. We were told that their standard equations are  $(x - h)^2 + (y - k)^2 = r^2$ . Just like all the other conic sections, a circle can also be described as the intersection of a plane with two cones stacked tip to tip.

Geometrically, the standard equation of a circle is also  $(x - h)^2 + (y - k)^2 = r^2$ . This equation must be memorized. The coordinates of the Center are (h, k).



#### The Circle (2 of 2)

The standard equation of the circle is almost identical to the one for the ellipse. The only difference is that the denominators a and b are identical. Unlike the ellipse where 0 < b < a.

We can see this when we divide both sides of the standard equation of the circle by  $r^2$ . Then we get the following:

$$\frac{(x-h)^2}{r^2} + \frac{(y-k)^2}{r^2} = 1$$

We can see that the denominators are identical!