# Concepts
# Introduction to Angles – Part 2

Based on power point presentations by Pearson Education, Inc.

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# Learning Objectives

1. Define "special" angles and some of their integer multiples in degrees and radians. Memorize some of their integer multiples.
2. Define quadrantal angles in degrees and radians. Memorize how to find their integer multiples.
3. Define coterminal angles in degrees and radians. Memorize how to find them.
4. Find the location of angles in degrees and radians in the rectangular coordinate system.
5. Graph angles in a Rectangular Coordinate System.

NOTE: This lesson contains some examples. You can find more examples in the “Examples” document also located in the appropriate MOM Learning Materials folder.

# 1. "Special" Angles (1 of 4)

In trigonometry, three acute angles and their integer multiples show up over and over again. You will also find them in physics and engineering. They are often called "special" angles. They can be expressed in degrees and radians. These conversions must be memorized!

$30° \equiv \frac{\pi}{6}$ (radians) $45° \equiv \frac{\pi}{4}$ (radians) $60° \equiv \frac{\pi}{3}$ (radians)

All three angles are acute, that is they magnitudes are between $0^o$ and $90^o$.

# Special Angles (2 of 4)

Next, we are going to study often used integer multiples of the special angles. The next nine (9) conversions must be memorized!

Integer multiples of $30^\circ \equiv \frac{\pi}{6}$

**5(30°) = 150°** please note that $150^\circ \equiv 5\left(\frac{\pi}{6}\right) = \frac{5\pi}{6}$ (obtuse angle)

**7(30°) = 210°** please note that $210^\circ \equiv 7\left(\frac{\pi}{6}\right) = \frac{7\pi}{6}$ (reflex angle)

**11(30°) = 330°** please note that $330^\circ \equiv 11\left(\frac{\pi}{6}\right) = \frac{11\pi}{6}$ (reflex angle)

# Special Angles (3 of 4)

Integer multiples of $45° \equiv \frac{\pi}{4}$ :

**3(45$^o$) = 135$^o$** please note that $135° \equiv 3\left(\frac{\pi}{4}\right) = \frac{3\pi}{4}$ (obtuse angle)

**5(45$^o$) = 225$^o$** please note that $225° \equiv 5\left(\frac{\pi}{4}\right) = \frac{5\pi}{4}$ (reflex angle)

**7(45$^o$) = 315$^o$** please note that $315° \equiv 7\left(\frac{\pi}{4}\right) = \frac{7\pi}{4}$ (reflex angle)

# Special Angles (4 of 4)

Integer multiples of $60^\circ \equiv \frac{\pi}{3}$ :

**$2(60^o) = 120^o$** please note that $120^\circ \equiv 2\left(\frac{\pi}{3}\right) = \frac{2\pi}{3}$ . This is an obtuse angle.

NOTE: While $4(30^o)$ also equals $120^o$, the $120^o$ angle is NOT considered an integer multiple of $30^o$.

**$4(60^o) = 240^o$** please note that $240^\circ \equiv 4\left(\frac{\pi}{3}\right) = \frac{4\pi}{3}$ . This is a reflex angle.

NOTE: While $8(30^o)$ also equals $240^o$, the $240^o$ angle is NOT considered an integer multiple of $30^o$.

**$5(60^o) = 300^o$** please note that $300^\circ \equiv 5\left(\frac{\pi}{3}\right) = \frac{5\pi}{3}$ . This is a reflex angle.

NOTE: While $10(30^o)$ also equals $300^o$, the $300^o$ angle is NOT considered an integer multiple of $30^o$.

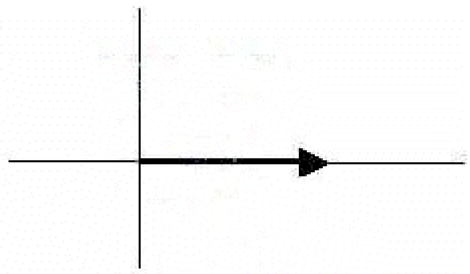
# 2. Quadrantal Angles (1 of 3)

When the terminal side of angles lies along a coordinate axis, they are called **Quadrantal Angles**. They can be expressed in degrees and radians.

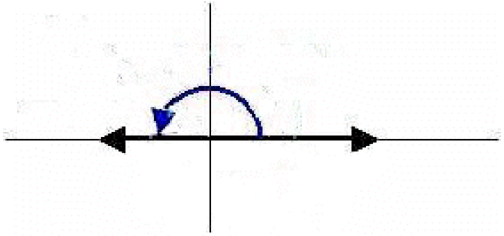
All angles that are integer multiples of $90^\circ \equiv \frac{\pi}{2}$ are *Quadrantal Angles*.

Following are the five most often used *Quadrantal Angles*. These conversions must be memorized!

$0^\circ \equiv 0$ (radians)

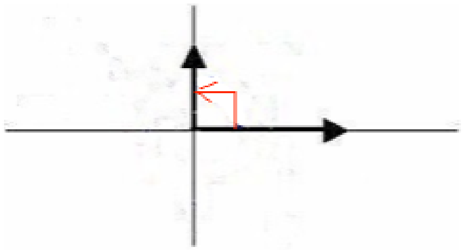


$180^\circ \equiv \pi$ (radians)

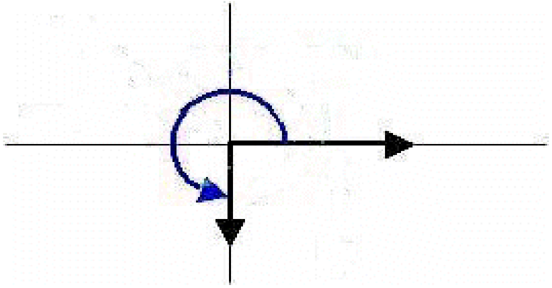


# Quadrantal Angles 

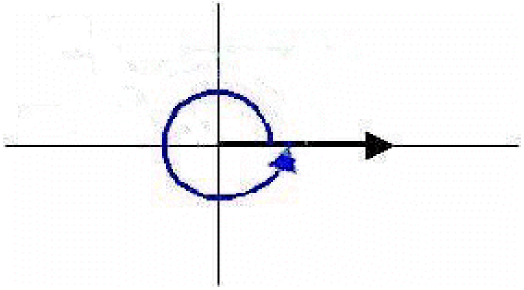
$90° \equiv \frac{\pi}{2}$ (radians)



$270° \equiv \frac{3\pi}{2}$ (radians)



$360° \equiv 2\pi$ (radians)



# Quadrantal Angles (3 of 3)

When we begin to use calculators, you will find that most do not give answers in the form of π. For example, your answer would be 6.28 instead of 2π.

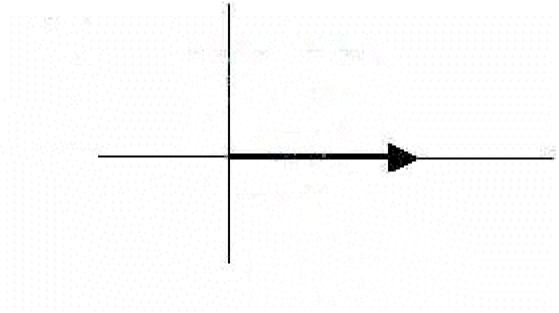
The following decimal approximations sometimes come in handy when using a calculator later on. They should be memorized now!

$\frac{\pi}{2} \approx 1.57$ $\quad\pi \approx 3.14$ $\quad\frac{3\pi}{2} \approx 4.71$ $\quad 2\pi \approx 6.28$

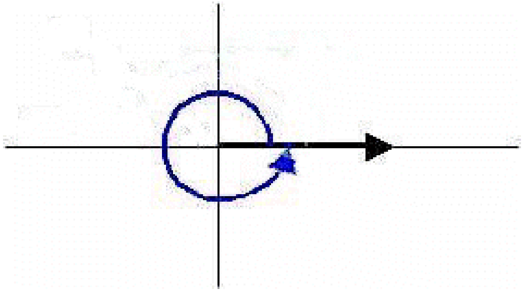
# 3. Coterminal Angles (1 of 2)

Angles with the same initial and terminal side are called coterminal angles. For example, the angles with measure $0^\circ \equiv 0$ and $360^\circ \equiv 2\pi$ are coterminal.

$0^\circ \equiv 0$ (radians)



$360^\circ \equiv 2\pi$ (radians)



We find the coterminal angles of an angle $\theta$ by adding to it any integer multiples of $360^o$ or depending on the angle measure integer multiples of $2\pi$. Following is the general expressions of angles coterminal to some angle $\theta$:

$\theta + 360^o$ ***k*** or $\theta + 2\pi$ ***k*** where ***k*** represents any integer.

# Coterminal Angles (2 of 2)

Example 1:

Is the angle with measure of $495^o$ coterminal with an angle of measure $135^o$?

To determine the coterminality of two angles, we take out of the larger angle all multiples of $360^o$ (or $-360^o$) and then look at the remainder.

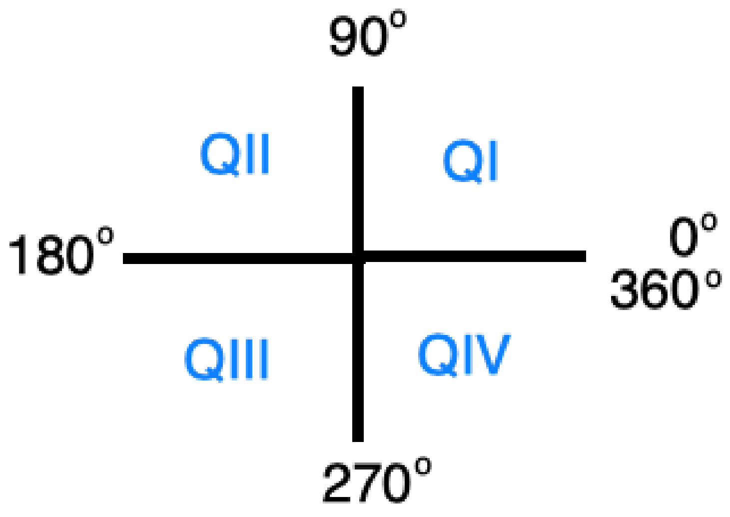
In our case, $495^o = 360^o(1) - 135^o$

The remainder is $135^o$. This tells us that the angle with measure of $495^o$ is coterminal with an angle of measure $135^o$.

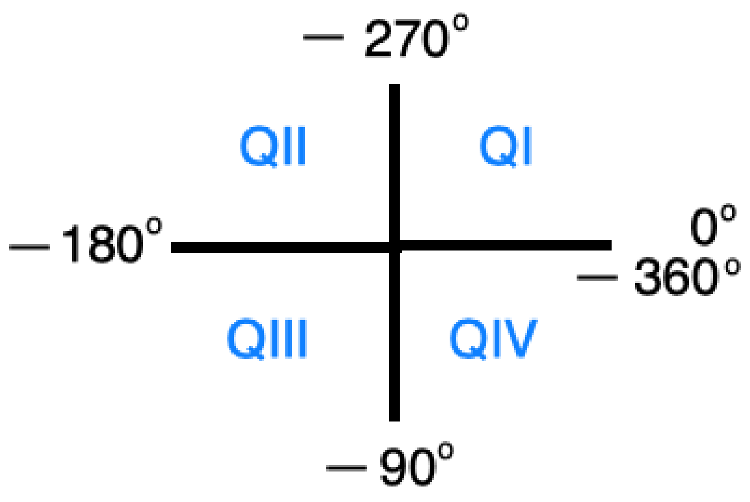
# 4. The "Location" of Angles in the Rectangular Coordinate System (1 of 4)

Since we often place angles into a rectangular coordinate system, it is a good idea to know their "location." The "location" of an angle is determined by the location of its terminal side. **Please study the following pictures and use them as a guide in the next two slides.**

Positive Angles:




Negative Angles:



# The “Location” of Angles in the Rectangular Coordinate System (2 of 4)

Using the pictures on the previous slides please memorize the following four (4) rules:

1. Angles with terminal side in Quadrant I are called first-quadrant angles. Remember, positive angles with terminal side in QI are between $0^{o}$ and $90^{o}$. Negative angles with terminal side in QI are between $-270^{o}$ and $-360^{o}$.

2. Angles with terminal side in Quadrant II are called second-quadrant angles. Remember, positive angles with terminal side in QII are between $90^{o}$ and $180^{o}$. Negative angles with terminal side in QII are between $-180^{o}$ and $-270^{o}$.

# The “Location” of Angles in the Rectangular Coordinate System (3 of 4)

3. Angles with terminal side in Quadrant III are called third-quadrant angles. Remember, positive angles with terminal side in QIII are between $180^o$ and $270^o$. Negative angles with terminal side in QIII are between $-90^o$ and $-180^o$.

4. Angles with terminal side in Quadrant IV are called fourth-quadrant angles. Remember, positive angles with terminal side in QIV are between $270^o$ and $360^o$. Negative angles with terminal side in QIV are between $0^o$ and $-90^o$.

# The “Location” of Angles in the Rectangular Coordinate System (4 of 4)

Example 2:

Find the location of the terminal side of the following angles. Hint: Graph them and/or use the pictures on slide 13!

a. $77^o$ QI angle
b. $125^o$ QII angle
c. $216^o$ QIII angle
d. $330^o$ QIV angle

e. $-77^o$ QIV angle
f. $-125^o$ QIII angle
g. $-216^o$ QII angle
h. $-330^o$ QI angle

# 5. Graph Angles in the Rectangular Coordinate System (1 of 7)

In a previous lesson, we were told that in trigonometry we place angles into a *Rectangular Coordinate System* with their Vertex at the Origin (0, 0), and their initial side along the positive horizontal axis. We then say that the angle is in **standard position**.

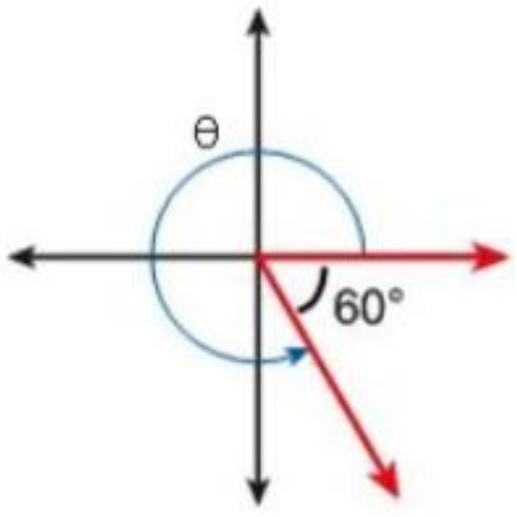
When we draw angles, we then make an arc, either in clockwise or counter-clockwise direction, indicating the size of the angle.

# Graph Angles in the Rectangular Coordinate System (2 of 7)

Example 3:

Graph angle θ with measure $300^o$ in a rectangular coordinate system.

This is a positive angle with terminal side in QIV since it is larger than $270^o$ and smaller than $360^o$. Remember, when given a positive angle, the arc is drawn in **counter-clockwise direction**!

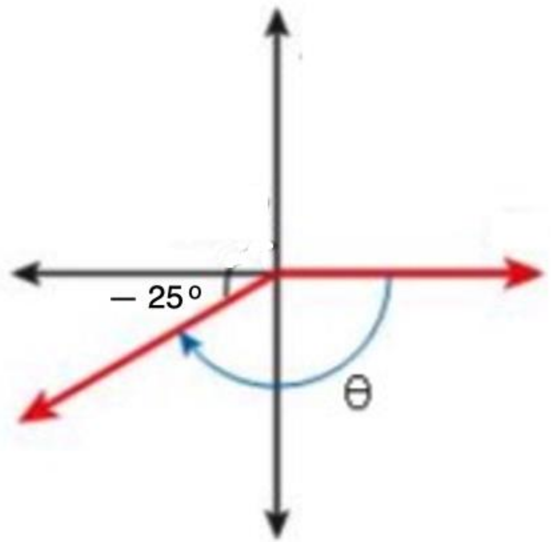


# Graph Angles in the Rectangular Coordinate System (3 of 7)

Example 4:

Graph angle θ with measure – 155° in a rectangular coordinate system.

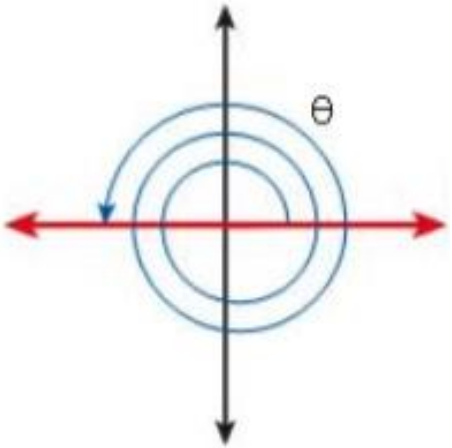
This is a negative angle with terminal side in QIII angle since it is larger than – 90° and smaller than – 180°. Remember, when given a negative angle, the arc is drawn in **clockwise direction**!



# Graph Angles in the Rectangular Coordinate System 

Example 5:

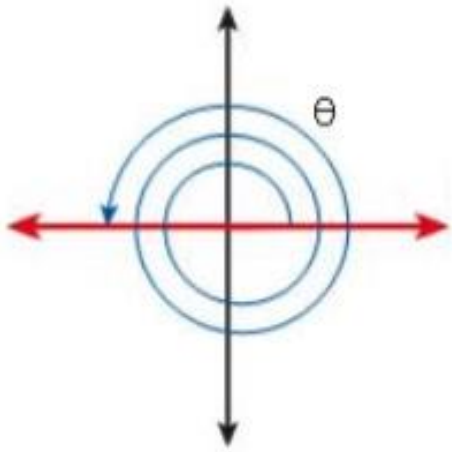
Determine the measure of the angle θ drawn in a rectangular coordinate system.




We notice that the angle arc is drawn in counter-clockwise direction. Therefore, we are dealing with a positive angle.

# Graph Angles in the Rectangular Coordinate System (5 of 7)

Example 5 continued:




We see two (2) 360$^{o}$ arcs. So, we have an angle of 2(360$^{o}$) = 720$^{o}$. But we are not done. The arc continues after 720$^{o}$, and ends at the horizontal axis reserved for 180$^{o}$ (see pictures in slide 13).

We add this to 720$^{o}$ to get 900$^{o}$. This is the angle we see in the picture above.