Examples
Polynomial Equations in One Variable

Based on power point presentations by Pearson Education, Inc.
Revised by Ingrid Stewart, Ph.D.

Learning Objectives

1. Memorize the definition of a polynomial expression.
2. Memorize the definition of a polynomial equation.
3. Solve polynomial equations.

Example 1: Solve a Polynomial Equation (1 of 3)

Find all real solutions of $\boldsymbol{x^3 + 5x^2 - 2x - 10 = 0}$.

There is already a 0 on the right side of the equal sign.

This polynomial equation is a perfect candidate for trying factoring by grouping because there are an even number of terms. Let's try the following grouping. The goal is to end up with two like factors in each group.

$\boldsymbol{(x^3 + 5x^2) + (-2x - 10) = 0}$

The greatest common factor in the first group is x^2 and in the second group it is – 2. Let's factor them out. Remember, ”factoring” is the same as “dividing” each term by a common factor!

Example 1: Solve a Polynomial Equation (2 of3)

$\boldsymbol{x^2(x + 5) + (-2)(x + 5) = 0}$

Notice that we achieved our goal. We ended up with a factor of $(x + 5)$ in the first group and in the second group.

We will now factor it out of each group as follows:

$\boldsymbol{(x + 5)(x^2 - 2) = 0}$

Using the *Zero Product Principle*, we find

$\boldsymbol{x + 5 = 0}$ and $\boldsymbol{x^2 - 2 = 0}$

The first equation gives us $\boldsymbol{x = -5}$ which is a real solution.

Example 1: Solve a Polynomial Equation (3 of 3)

Now let's use the *Square Root Property* to find the solution of the second equation $\boldsymbol{x^2 - 2 = 0}$.

$\boldsymbol{x^2 = 2}$

then $\boldsymbol{x = \pm\sqrt{2}}$ which are two real solutions.

In summary, the real solutions of this polynomial equation are $\boldsymbol{-5}$, $\boldsymbol{-\sqrt{2}}$, and $\boldsymbol{\sqrt{2}}$.

Example 2: Solve a Polynomial Equation (1 of 2)

Find all real solutions of $2x^3 + 16x^2 + 30x = 0$.

There is already a 0 on the right side of the equal sign.

First, we will factor out the common factor $2x$ as follows:

$2x\,(x^2 + 8x + 15) = 0$

Next, we will try to factor the trinomial in the parentheses.

We get the following: $2x\,(x + 3)(x + 5) = 0$

Using the *Zero Product Principle*, we find

$2x = 0$ and $x + 3 = 0$ and $x + 5 = 0$

Example 2: Solve a Polynomial Equation (2 of 2)

Solving the three equations, we end up with $\boldsymbol{x} = \mathbf{0}$ and $\boldsymbol{x} = \mathbf{-3}$ and $\boldsymbol{x} = \mathbf{-5}$ which are all real solutions.

In summary, the real solutions of this polynomial equation are **– 5**, **– 3,** and **0**.

Example 3: Solve a Polynomial Equation (1 of 2)

Find all real solutions of $\boldsymbol{x^4 - 8x^2 - 9 = 0}$.

There is already a 0 on the right side of the equal sign.

Sometimes, you can encounter polynomial equations that are quadratic in form. That is, one exponent on the variable is exactly twice as large as the exponent of the other variable! Note $\boldsymbol{(x^2)^2 - 8\,(x^1)^2 - 9 = 0}$!

Therefore, we can factor it “like” a polynomial of degree 2.

Then $\boldsymbol{(x^2 - 9)\,(x^2 + 1) = 0}$.

Now let's use the *Zero Product Principle* and set each factor equal to ***0***.

$\boldsymbol{x^2 - 9 = 0}$ and $\boldsymbol{x^2 + 1 = 0}$

Example 3: Solve a Polynomial Equation (2 of 2)

Next, let's use the *Square Root Property* to solve the first equation $x^2 - 9 = 0$.

$x^2 = 9$

$x = \pm\sqrt{9}$

$x = \pm 3$ which are two real solutions.

Finally, let's use the *Square Root Property* to solve the second equation $x^2 + 1 = 0$.

$x^2 = -1$

$x = \pm\sqrt{-1}$

$x = \pm i$ which are two imaginary solutions.

In summary, the polynomial equation has only two real solutions, namely ***–3*** and ***3***.

Example 4: Solve a Polynomial Equation (1 of 3)

Find all real solutions of $\boldsymbol{x^3 - 4x^2 = -2x + 8}$.

First, we will change the right side of the equation to 0. That is,

$\boldsymbol{x^3 - 4x^2 + 2x - 8 = 0}$.

Usually when the equation has four (4) terms we will try factoring by grouping. We will start by placing the first two terms and the last two terms in a group as follows:

$\boldsymbol{(x^3 - 4x^2) + (2x - 8) = 0}$.

Next, we will factor out preferably the greatest common factor from each group.

Example 4: Solve a Polynomial Equation (2 of 3)

We observe that the greatest common factor in the first group is x^2 and in the second group it is 2. We get the following:

$$x^2(x - 4) + 2(x - 4) = 0$$

We notice that the product in the first group and in the second group have a factor of $(x - 4)$ in common. This is the goal of factoring by grouping because now we can factor it out as follows:

$$(x - 4)(x^2 + 2) = 0$$

Please note that only now is the equation written as a product of factors.

Example 4: Solve a Polynomial Equation (3 of 3)

Now let's use the *Zero Product Principle* and set each factor equal to ***0***.

$\boldsymbol{x - 4 = 0}$ and $\boldsymbol{x^2 + 2 = 0}$

$\boldsymbol{x - 4 = 0}$ yields $\boldsymbol{x = 4}$ which is a real solution.

Next, let's use the *Square Root Property* to solve $\boldsymbol{x^2 + 2 = 0}$.

$\boldsymbol{x^2 = -2}$

and $x = \pm\sqrt{-2} = \pm i\sqrt{2}$ which are two imaginary solutions.

Therefore, the polynomial equation has one real solution, namely ***4***.

Example 5: Solve a Polynomial Equation (1 of 2)

Find all real solutions of $3x^3 - 30x^2 + 75x = 0$.

There is already a 0 on the right side of the equal sign.

First, we will factor out the common factor $3x$ as follows:

$$3x(x^2 - 10x + 25) = 0$$

Next, we will try to factor the trinomial in the parentheses.

We get the following: $3x(x - 5)(x - 5) = 0$

Example 5: Solve a Polynomial Equation (2 of 2)

Now let's use the *Zero Product Principle* and set each factor equal to ***0***.

3*x* = 0 and ***x* – 5 = 0**

Then, ***x* = 0** and ***x* = 5** which are two real solutions.

In summary, we find that this polynomial equation has two real solutions, namely **0** and **5**.