Concepts
The Slope of a Line

Based on power point presentations by Pearson Education, Inc.
Revised by Ingrid Stewart, Ph.D.

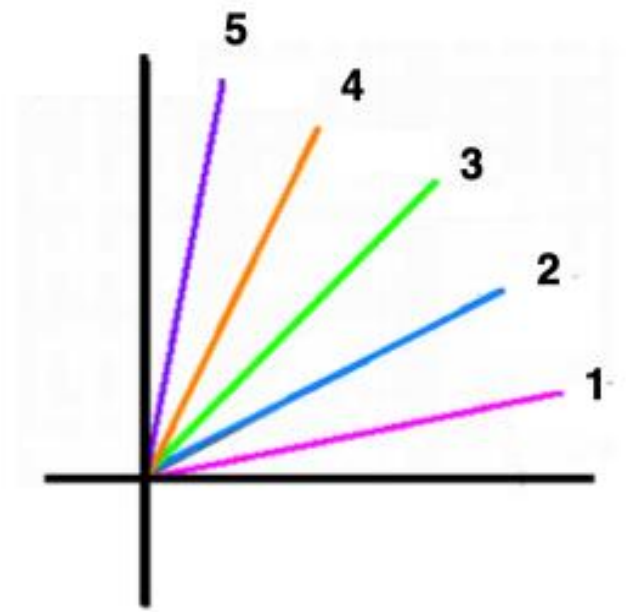
Learning Objectives

1. Define and find the slope of a line.
2. Define the slope-intercept form of a linear equations in two variables.

NOTE: This lesson contains some examples. You can find more examples in the “Examples” document also located in the appropriate MOM Learning Materials folder.

1. Definition of the Slope of a Line (1 of 6)

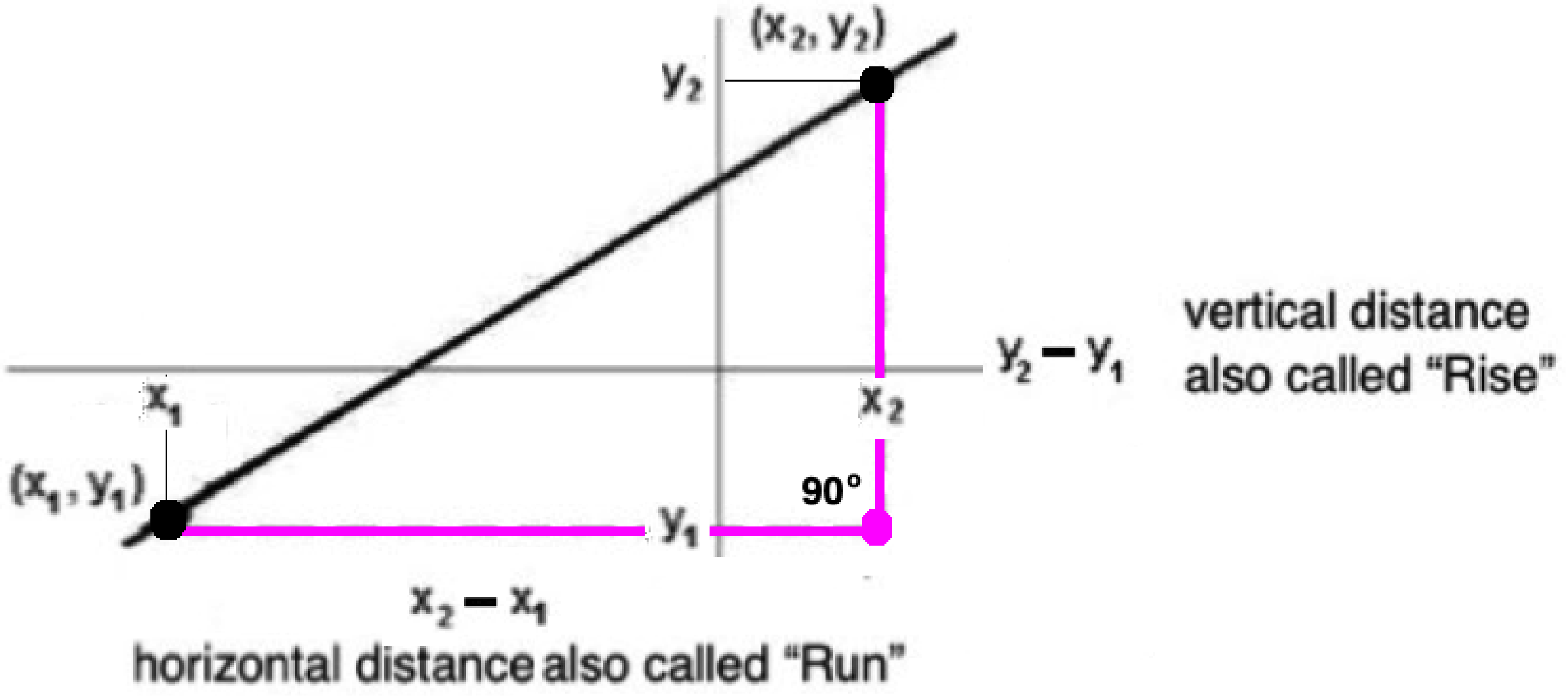
In the last lesson, we discussed the graphs of linear equations in two variables. We found out that their graphs are either increasing or decreasing lines. Let’s look at a picture of several increasing lines.



We notice that each line has a different steepness. For example, the purple line #5 is much steeper than the pink line #1.

Definition of the Slope of a Line (2 of 6)

In mathematics, the steepness of a line is measured by an entirely man-made **Slope Formula**. Specifically, at one point it was defined to be the change in vertical distance divided by the change in horizontal distance as we "travel" from one point on a line in a rectangular coordinate system to another point. Usually, these two points are defined by the ordered pairs (x_1, y_1) and (x_2, y_2).



Definition of the Slope of a Line (3 of 6)

The slope of a line is indicated by the lower-case letter *m*. Why *m*? No one knows for sure. Some mathematicians claim the *m* comes from the French word “monter” which means “to climb”.

The slope of the line through two distinct ordered pairs (x_1, y_1) and (x_2, y_2) is defined by the following formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{rise}{run}$$

Regardless of the sign of the *x*-coordinates or the *y*-coordinates, the minus sign between the *y*-values and the *x*-values in the slope calculation must always be there.

NOTE: The steeper the line, the larger is the ***m***.

Definition of the Slope of a Line

Example 1:

Find the slope of the line passing through the points determined by the ordered pairs (4, −2) and (−1, 5).

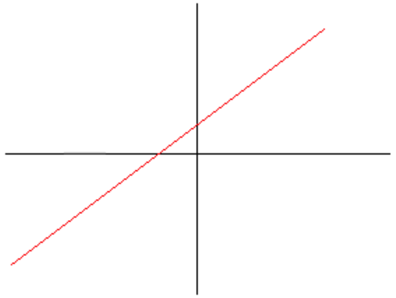
We will let (4, −2) equal (x_1, y_1) and (−1, 5) equal (x_2, y_2). However, you can also let (−1, 5) equal (x_1, y_1) and (4, −2) equal (x_2, y_2). In either case, you will get the same answer.

Let's say that (4, −2) equals (x_1, y_1) and (−1, 5) equals (x_2, y_2). Be sure not to get confused!

Then $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{5-(-2)}{-1-4} = \dfrac{5+2}{-5} = \dfrac{7}{-5} = -\dfrac{7}{5}$

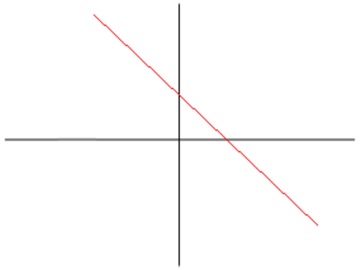
2. Definition of the Slope of a Line (5 of 6)

We will not state how certain slope properties affect the characteristics of a line.



POSITIVE SLOPE

All increasing lines have a positive slope.

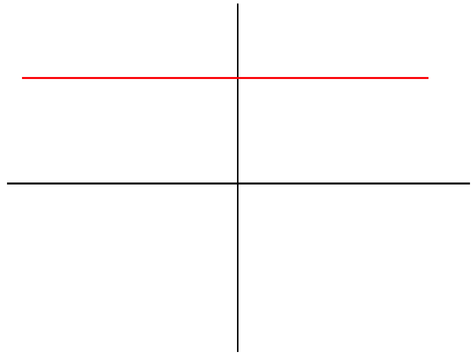


NEGATIVE SLOPE

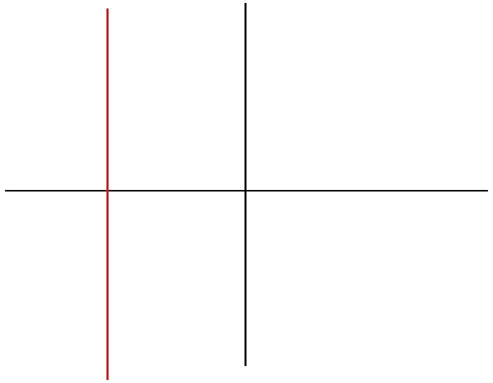
ALL decreasing lines have a negative slope.

Definition of the Slope of a Line (6 of 6)

SLOPE OF 0
All horizontal lines have a slope of 0.
Slopes of 0 occur when the numerator in the slope formula is 0.



UNDEFINED SLOPE
All vertical lines have an undefined slope.
Undefined slopes occur when the denominator in the slope formula is 0.



2. The Slope-Intercept Form of Linear Equations in Two Variables (1 of 4)

We have already discussed the **general form** of a linear equation in two variables. It is $\boldsymbol{Ax + By + C = 0}$, where $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$ are real numbers, but $\boldsymbol{A}$ and $\boldsymbol{B}$ cannot be 0.

We are now going to discuss the **slope-intercept form** of a linear equation in two variables. It is as follows:

> $\boldsymbol{y = mx + b}$ where $\boldsymbol{m}$ is the slope of the line, and the real number $\boldsymbol{b}$ is the y-intercept.

NOTE: The slope-intercept form of a linear equation is simply a mathematical "manipulation" of the general form with the introduction of the letter m.

The Slope-Intercept Form of Linear Equations in Two Variables (2 of 4)

Example 1:

Write the general form of the linear equation $-18x - 2y + 11 = 0$ in slope-intercept form.

We need to write the equation in the form $y = mx + b$. Note that the y-term is "isolated" on the left of the equal sign. The x-term and the constant are on the right side of the equal sign with the x-term in front of the constant.

Let's now do the following manipulations:

$-2y + 11 = 18x$ (added $18x$ to both sides)

$-2y = 18x - 11$ (subtracted 11 from both sides)

The Slope-Intercept Form of Linear Equations in Two Variables (3 of 4)

Example 1 continued with – 2*y* = 18*x* – 11 :

$y = \frac{18x - 11}{-2}$ (divided both sides by – 2)

As you can see, *y* is now isolated on one side. But we are not quite done!

We will now distribute – 2 to every term in the numerator on the right side of the equal sign as follows:

$y = \frac{18x}{-2} - \frac{11}{-2}$

and $y = -9x + \frac{11}{2}$

We find that $y = -9x + \frac{11}{2}$ can be written as $-18x - 2y + 11 = 0$ in slope-intercept form.

The Slope-Intercept Equation of a Line (4 of 4)

Examples of linear equations written in *slope-intercept form*:

$y = -18x + 11$ (where $m = -18$ is the slope of the line and $b = 11$ is the y-intercept)

$y = 2x$ (where $m = 2$ is the slope of the line and $b = 0$ is the y-intercept)

$y = -x - 1$ (this is still considered slope-intercept form) because the equation can be written as $y = -x + (-1)$. Th

erefore, $m = -1$ is the slope of the line and $b = -1$ is the y-intercept.

Note: The *Slope-Intercept Form* requires a plus sign between the terms. Plus signs make no changes to the equations unlike minus signs.