Concepts
The Slope of a Line

Based on power point presentations by Pearson Education, Inc.
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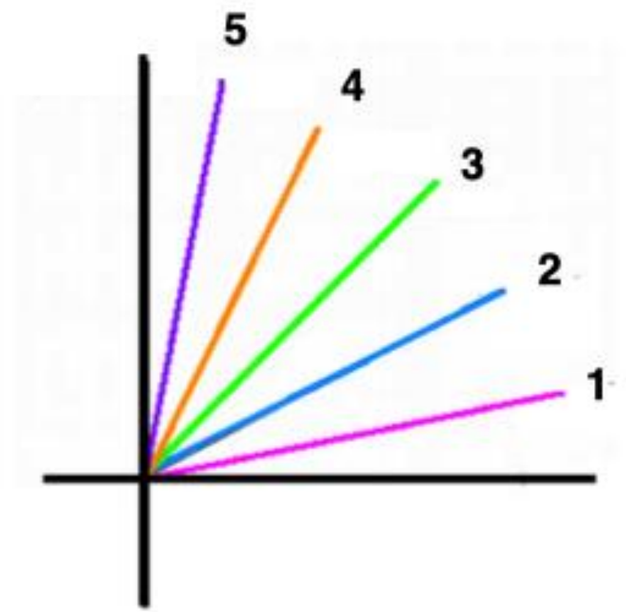
Learning Objectives

1. Define and find the slope of a line.
2. Define the slope-intercept form of a linear equations in two variables.
3. Graph linear equations in slope-intercept form by hand in the rectangular coordinate system using the *Point-by-Point Plotting Method* and the *Intercept Method*.

NOTE: This lesson contains some examples. You can find more examples in the “Examples” document also located in the appropriate MOM Learning Materials folder.

1. Definition of the Slope of a Line

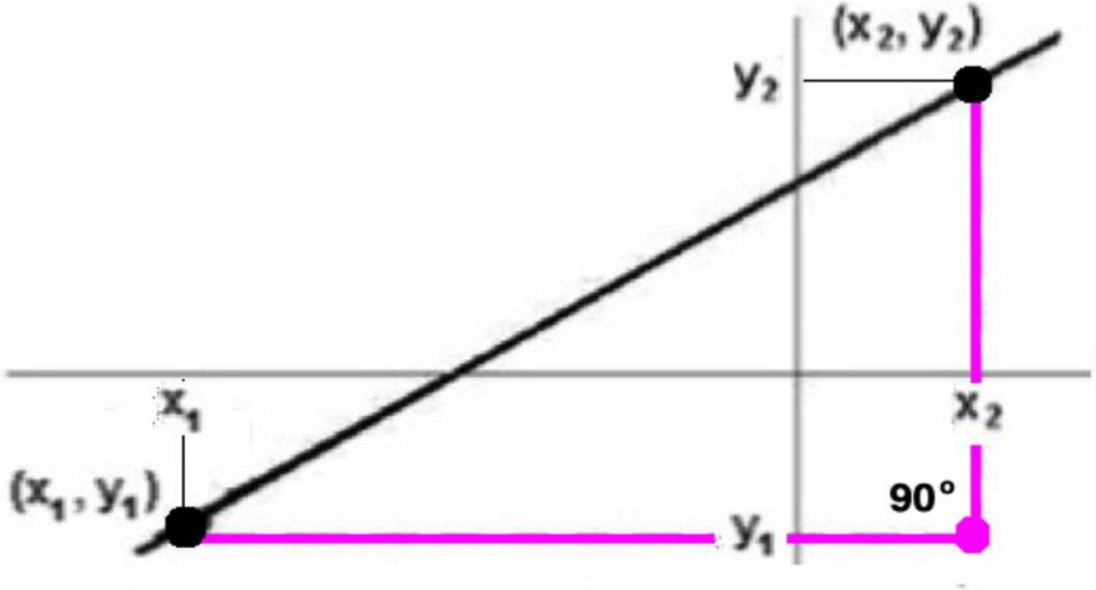
In the last lesson, we discussed the graphs of linear equations in two variables. We found out that their graphs are either increasing or decreasing lines. Let's look at a picture of several increasing lines.



We notice that each line has a different steepness. For example, the purple line #5 is much steeper than the pink line #1.

Definition of the Slope of a Line (2 of 6)

In mathematics, the steepness of a line, usually called the **slope**, is measured by an entirely man-made formula. Specifically, it was defined to be the change in vertical distance divided by the change in horizontal distance as we "travel" from one point on a line in a rectangular coordinate system to another point. Usually, these two points are defined by the ordered pairs (x_1, y_1) and (x_2, y_2).



$y_2 - y_1$ (vertical distance called the “rise”)

$x_2 - x_1$ (horizontal distance called the “run”)

Definition of the Slope of a Line (3 of 6)

The **slope** of a line is indicated by the lower-case letter *m*. Why *m*? No one knows for sure. Some mathematicians claim the *m* comes from the French word “monter” which means “to climb”.

Given two distinct ordered pairs (x_1, y_1) and (x_2, y_2) lying on a line, the **slope** of that line is defined by the following formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{rise}{run}$$

Regardless of the sign of the *x*-coordinates or the *y*-coordinates, the minus sign between the *y*-values and the *x*-values in the slope calculation must always be there.

Definition of the Slope of a Line

Example 1:

Find the slope of the line passing through the points determined by the ordered pairs (4, −2) and (−1, 5).

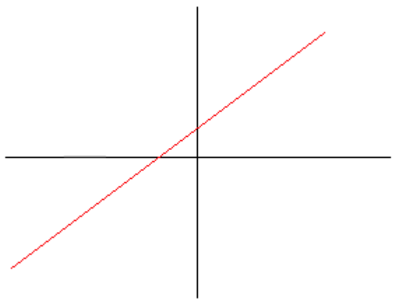
We will let (4, −2) equal (x_1, y_1) and (−1, 5) equal (x_2, y_2). However, you can also let (−1, 5) equal (x_1, y_1) and (4, −2) equal (x_2, y_2). In either case, you will get the same answer.

Let’s say that (4, −2) equals (x_1, y_1) and (−1, 5) equals (x_2, y_2). Be sure not to get confused!

Then $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-(-2)}{-1-4} = \frac{5+2}{-5} = \frac{7}{-5} = -\frac{7}{5}$

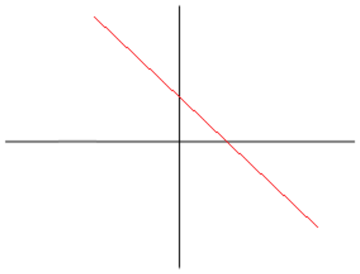
2. Definition of the Slope of a Line

We will not state how certain slope properties affect the characteristics of a line.



POSITIVE SLOPE

All increasing lines have a positive slope.



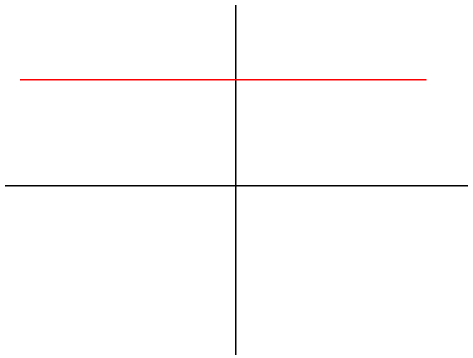
NEGATIVE SLOPE

ALL decreasing lines have a negative slope.

Definition of the Slope of a Line

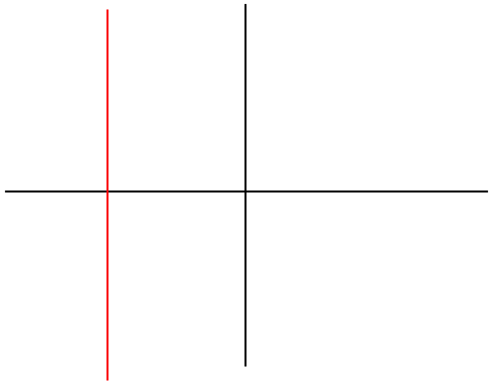
SLOPE OF 0

All horizontal lines have a slope of 0.
Slopes of 0 occur when the numerator in the slope formula is 0.



UNDEFINED SLOPE

All vertical lines have an undefined slope.
Undefined slopes occur when the denominator in the slope formula is 0.



2. The Slope-Intercept Equation of a Line

We have already discussed the **general form** of a linear equation in two variables. It is ***ax + by + c* = 0**, where ***a***, ***b***, and ***c*** are numbers, but ***a*** and ***b*** cannot be 0.

We are now going to discuss the **slope-intercept form** of a linear equation in two variables. It is as follows:

> ***y* = *mx* + *b*** where the number ***m*** is the slope of the line, and the number ***b*** is the *y*-intercept.

NOTE: The slope-intercept equation of a line is simply a mathematical "manipulation" of the general form with the introduction of the letter *m*.

The Slope-Intercept Equation of a Line (2 of 5)

Example 2:

Write the general form of the linear equation $-18x - 2y + 11 = 0$ in slope-intercept form $\boldsymbol{y = mx + b}$.

We need to write the equation in the form $y = mx + b$. Note that the y-term is "isolated" on the left of the equal sign. The x-term and the constant are on the right side of the equal sign with the x-term in front of the constant.

Let's now do the following manipulations:

$-2y + 11 = 18x$ (added $18x$ to both sides)

$-2y = 18x - 11$ (subtracted 11 from both sides)

The Slope-Intercept Equation of a Line (3 of 5)

Example 2 continued with – 2y = 18x – 11 :

$y = \frac{18x - 11}{-2}$ (divided both sides by – 2)

As you can see, y is now isolated on one side. But we are not quite done!

We will now distribute – 2 to every term in the numerator on the right side of the equal sign as follows:

$y = \frac{18x}{-2} - \frac{11}{-2}$

and $y = -9x + \frac{11}{2}$

We find that **$-18x - 2y + 11 = 0$** can be written as **$y = -9x + \frac{11}{2}$** in slope-intercept form.

The Slope-Intercept Equation of a Line (4 of 5)

Examples of linear equations written in *slope-intercept form* $\mathbf{y = mx + b}$:

a. $y = -x + 11$ (where $m = -1$ is the slope of the line and $b = 11$ is the y-intercept)

b. $y = 2x$ (where $m = 2$ is the slope of the line and $b = 0$ is the y-intercept)

The Slope-Intercept Equation of a Line (5 of 5)

Examples of linear equations written in *slope-intercept form* $\boldsymbol{y = mx + b}$:

c. $y = -3x - 1$

This looks almost like the slope-intercept form, but there is no plus sign between the terms which is a requirement of the slope-intercept form. We know that $m = -3$. What about b?

Please note that this equation can be written as $y = -3x + (-1)$. We inserted a plus sign between the terms, which is absolutely allowed. Now the equation is in slope-intercept form, and we can state that $b = -1$.

Please note that b is NOT equal to 1!

3. The Graphs of Linear Equations in Slope-Intercept Form

We will graph linear equations in slope-intercept form the same way we graphed linear equation in the previous lesson. That is, we will use either the *Point-by-Point Plotting Method* or the *Intercept Method*.

NOTE: In a previous algebra courses, you may have learned a method of graphing called the “Slope-Intercept” Method. We will not use this method in our course.

The Graphs of Linear Equations in Slope-Intercept Form (2 of 4)

Example 3:

Graph the linear equation $y = -3x - 6$ by hand. This linear equation is in *slope-intercept form*!

Since we are not told which graphing method to use, let's try to use the *Intercept Method*.

Find the ordered pair associated with the *y*-intercept.

Since the linear equation is in slope- intercept form, we know that *b* is the *y*-intercept, therefore, the *y*-intercept is -6

The ordered pair associated with the *y*-intercept is $(0, -6)$.

The Graphs of Linear Equations in Slope-Intercept Form

Example 3 continued with $y = -3x - 6$:

Find the ordered pair associated with the x-intercept.

Let $y = 0$ and solve for x.

$0 = -3x - 6$ (this is a linear equation in one variable)

$3x = -6$

$x = -2$

The x-intercept is -2, so the ordered pair associated with it is $(-2, 0)$.

The Graphs of Linear Equations in Slope-Intercept Form (4 of 4)

Example 3 continued:

Graph the linear equation by drawing a line through the points created by the ordered pairs associated with the *y*- and *x*-intercepts.