Concepts
Polynomial Equations in One Variable

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Revised by Ingrid Stewart, Ph.D.

Learning Objectives

1. Memorize the definition of a polynomial expression.
2. Memorize the definition of a polynomial equation.
3. Solve polynomial equations.

1. Definition of a Polynomial Expression (1 of 3)

We have talked much about mathematical expressions in this course. Actually, there are infinitely many different types, and the one we are going to discuss in this lesson is a mathematical expression with a special name. It is called a **polynomial expression** or simply a **polynomial**. We have seen it before in this course, but then we didn't assign a specific name to it.

The **general form** of a polynomial expression in *x* is

$$\boldsymbol{a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0}, \text{ where}$$

$a_n, a_{n-1}, \ldots, a_2, a_1, a_0$ are the coefficients which are real numbers with $a_n \neq 0$.

$x^n, x^{n-1}, \ldots, x^2, x^1, x^0 = 1$ are their associated variables. Their exponents are strictly non-negative integers (positive integers and 0).

Definition of a Polynomial Expression (2 of 3)

Examples of mathematical expressions that are called polynomials:

$x^4 + 3x^2 + x - 5$ $\qquad$ $3x^5 - 4x^4 + x^3 - 9x^2 + 3x$

Please note, in mathematics we always try to write terms with the same variable in descending order of their exponent with the last term being the constant, if there is one!

Examples of mathematical expressions that are NOT polynomials:

$4x^3 + 3x^2 + 5x^{-1}$ is NOT a polynomial because the exponents -1 is not a positive integer.

$3x^5 + 5x^{\frac{2}{3}}$ is NOT a polynomial because the exponent $\frac{2}{3}$ is not an integer.

Definition of a Polynomial Expression (3 of 3)

Some Vocabulary:

Monomial – A special name for a polynomial containing one term.
For example, x^2 or $-x^9$ or 4, etc.

Binomial – A special name for a polynomial containing two terms.
For example, $-7x^{11} + 21$ or $x + 5$, etc.

Trinomial – A special name for a polynomial containing three terms.
For example, $4x^2 - x - 5$ or $x^4 + 2x^3 + 10$, etc.

2. Definition of a Polynomial Equation

We can turn a polynomial expression into a polynomial equation by setting the expression equal to 0.

A polynomial equation is usually written in standard form as follows:

$$a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_2 x^2 + a_1 x + a_0 = 0$$

3. Solve Polynomial Equations (1 of 5)

In this course we will only solve some special polynomial equations. Namely those that can be factored or the *quadratic formula* or the *square root property* can be applied. Please review “Factoring” in Lesson 11PRE and the entire Lesson 11.

Solution Strategy:

Step 1 - If necessary, bring all terms of the polynomial equation to one side so that the other side is equal to 0.

Example:

Solve the polynomial equation $2x^3 + 3x^2 = -4x - 6$. Only find real solutions.

First, we will change the right side of the equation to 0. That is,

$2x^3 + 3x^2 + 4x + 6 = 0$.

Solve Polynomial Equations (2 of 5)

Step 2 - Try to solve the polynomial equation by factoring, the *Quadratic Formula*, or the *Square Root Property*.

Example continued with $2x^3 + 3x^2 + 4x + 6 = 0$:

Usually when the equation has four (4) terms we will try factoring by grouping. We will start by placing the first two terms and the last two terms in a group as follows:

$(2x^3 + 3x^2) + (4x + 6) = 0$

Next, we will factor out preferably the greatest common factor from each group.

We observe that the greatest common factor in the first group is x^2 and in the second group it is 2. We get the following:

$x^2(2x + 3) + 2(2x + 3) = 0$

Solve Polynomial Equations (3 of 5)

Example continued with $x^2(2x + 3) + 2(2x + 3) = 0$

We notice that the product in the first group and in the second group have a factor of $(2x + 3)$ in common. This is the goal of factoring by grouping because now we can factor it out as follows:

$(2x + 3)(x^2 + 2) = 0$

Please note that only now is the equation written as a product of factors.

Solve Polynomial Equations

Step 3 - Apply the *Zero Product Principle* to the product in Step 2. Note, that more factoring, the *Quadratic Formula*, or the *Square Root Property* might have to be used.

Example continued with $(2x + 3)(x^2 + 2) = 0$:

Using the *Zero Product Principle*, we can now state $2x + 3 = 0$ and $x^2 + 2 = 0$, and both equations can easily be solved.

- Given the linear factor $2x + 3 = 0$, we can easily determine that $x = -\frac{3}{2}$ which is a real solution.

Solve Polynomial Equations (5 of 5)

Example continued:

- Given the quadratic factor $x^2 + 2 = 0$, we can use the *Quadratic Formula* or the *Square Root Property* to solve for *x*. Let’s use the later.

  We will write $x^2 = -2$, then $x = \pm\sqrt{-2} = \pm\, i\sqrt{2}$. We found two imaginary solutions.

In summary, we found only one real solution, namely $-\frac{3}{2}$.