Concepts and Examples Introduction to Arithmetic Whole and Decimal Numbers

Based on power point presentations by Pearson Education, Inc.

Revised by Ingrid Stewart, Ph.D.

Learning Objectives

1. Use the vocabulary and symbols of arithmetic.
2. Define natural and whole numbers.
3. Operations on Natural and Whole Numbers.
4. Define Decimal Numbers.
5. Operations on Decimal Numbers.

NOTE: This lesson contains some examples. You can find more examples in the "Examples" document also located in the appropriate MOM Learning Materials folder.

What is Arithmetic?

Arithmetic is an elementary branch of mathematics that studies numerical operations like addition, subtraction, multiplication, and division on whole numbers, decimal numbers, fractions, negative numbers, and irrational number. It also includes exponentiation, extraction of roots, and taking logarithms.

Arithmetic operations form the basis of many branches of mathematics. They also play a role in the sciences, like physics and economics, and are present in many aspects of daily life. For example, we use arithmetic to calculate change while shopping, to manage personal finances, to cook and bake, or to build.

The practice of arithmetic is thousands of years old. Ancient civilizations like the Egyptians and the Sumerians invented numeral systems to solve practical arithmetic problems in about 3000 BC. The ancient Indians developed the concept of zero and the decimal system, which Arab mathematicians further refined and spread to the Western world during the medieval period (about 500 to 1500 AD).

1. Some Vocabulary and Symbols of Arithmetic

The Equal Sign (=)

The sign **=** indicates that the values to the right of it are equal to the values to the left of it. Visually, the sign consist of two short horizontal parallel lines.

For example, 8 + 2 = 5 + 5

Addition

Given 8 + 2 = 10, the 8 and the 2 are called **Addend 1 and Addend 2**. The 10 is called **Sum**.

Subtraction

Given 8 – 2 = 6, the 8 is called the **Minuend** and the 2 is called the **Subtrahend**. The 6 is called **Difference**.

Some Vocabulary and Symbols of Arithmetic

Division

$8 \div 2 = 4$. The 8 is called **Dividend** and the 2 is called the **Divisor**. The 4 is called **Quotient**.

Multiplication

$8 \cdot 2$ or 8×2 equals 16. The 8 is called **Multiplicand** and the 2 is called the **Multiplier**. The 16 is called **Product**.

NOTE:

In higher mathematics we usually express multiplication using parentheses ( ) and not the multiplication symbols $\cdot$ or $\times$.

That is, instead of $8 \cdot 2$ or 8×2 we write 8(2), which is pronounced “8 times 2”.

Some Vocabulary and Symbols of Arithmetic (3 of 3)

"Approximately Equal" Sign ($\cong$)

The sign $\cong$ indicates that the values to the right of it are similar the the values to the left of it but the two values are not exactly equal. Visually, the sign is a "squiggly" over the equal sign. Often, the sign $\approx$ is used instead which means "almost equal."

For example, anyone who has heard of the number π (pi) knows that it is approximately equal to 3.14. That is, $\pi \cong 3.14$.

Factors

Numbers that divide into other numbers without leaving a remainder.

For example, 2 and 3 divide into 6 without a remainder, but 4 and 5 do not. Therefore, only 2 and 3 are factors of 6.

Furthermore, when we see, for example, 5(2)(6), we call each number a factor of the product.

2. Natural and Whole Numbers (1 of 2)

The first set of numbers we will examine is the set of *Natural Numbers* sometimes called counting numbers.

We write this set as *{1, 2, 3, 4, 5, ...}*. Please note that sets of numbers are always enclosed in braces.

The next set is the set of *Whole Numbers*. It includes all the *Natural Numbers* and the number 0.

We write this set as *{0, 1, 2, 3, 4, 5, ...}*.

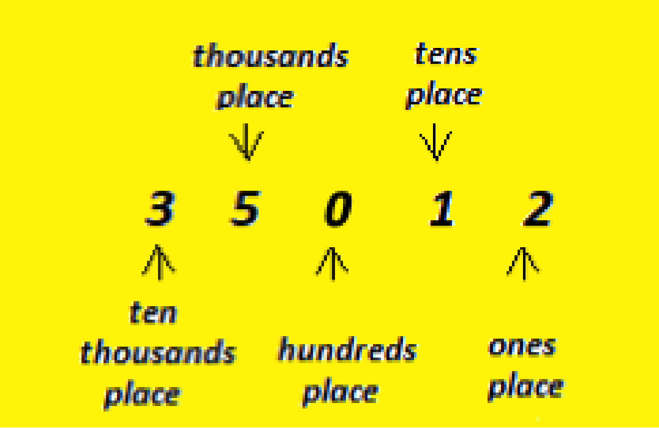
Note: The three dots indicate that this set does not have a final element and that the listing goes on forever. They are called an "ellipsis". The sets of *Natural Numbers* and *Whole Numbers* are infinite sets!

Natural and Whole Numbers (2 of 2)

In our number system, the value of a digit depends on its place, or position, in the number. Each place has a value of 10 times the place to its right. A whole number in standard form is separated into groups of three digits from right to left using commas.

Place Value of Whole Numbers

For example, given the number ***35,012***, we say it has five digits whose respective places or place values are as follows:



3. Operations on Natural and Whole Numbers (1 of 12)

a. Rule for Adding Two or More Natural and Whole Numbers:

1. Align the numbers in columns by place value.
2. Add the digits in each column starting on the bottom right.
3. Regroup whenever the sum of a column is more than one digit.

Operations on Natural and Whole Numbers

Example 1:

Find the sum of ***180 + 87 + 15*** without a calculator.

```
  181
   87
+  15
 _1_1___
  283
```

The sum of the digits in the *ones* column is ***5 + 7 + 0 = 12***. Record the ***2*** in the *ones* place of the sum and regroup the ***1*** to the *tens* place.

The sum of the digits in the *tens* column is ***1 + 1 + 8 + 8 = 18***. Record the ***8*** in the *tens* place of the sum and regroup another ***1*** to the *hundreds* place.

The sum of the digits in the *hundreds* column is ***1 + 0 + 0 + 1 = 2***. Record the 2 in the *hundreds* place of the sum.

Operations on Natural and Whole Numbers

b. Rule for Subtracting Two Natural and Whole Numbers:

1. Align the numbers in columns by place value.
2. Subtract the digits in each column starting on the bottom right.
3. Regroup whenever subtracting a larger digit from a smaller one.

Operations on Natural and Whole Numbers (4 of 12)

Example 2:

Find the difference of ***135 – 27*** without a calculator.

$$\begin{array}{r} {\scriptstyle 2}\phantom{^{1}5} \\ 1\not{3}\,{}^{1}5 \\ -\ 2\ 7 \\ \hline 108 \end{array}$$

In the *ones* column, we are subtracting a larger number from a smaller one. Therefore, we borrow 10 from the tens column. Now we subtract 7 from 15 to get 8. Record the ***8*** in the *ones* place of the difference. Notice that the 3 in the tens place became a 2!

In the *tens* column, we are subtracting a 2 from a 2 to get 0. Record the ***0*** in the *tens* place of the difference.

In the *hundreds* column, we are subtracting a 0 from a 1 to get 1. Record the ***1*** in the *hundreds* place of the difference.

Operations on Natural and Whole Numbers

c. Rule for Multiplying Two Natural and Whole Numbers:

1. Place the two numbers one under the other.
2. Multiply the multiplicand, in turn, by each digit of the multiplier starting with the ones place.
3. Align the ones digit of each of these "partial products" with their multiplier digit.
4. Add the "partial products" as they are aligned.

Operations on Natural and Whole Numbers (6 of 12)

Example 3:

Find the product of ***528*** $\times$ ***203*** without a calculator.

```
    528
  × 203
  -----
   1584
   000
+ 1056
 ------
 107184
```

- We get 1584 as follows:

  Multiply ***3(8) = 24,*** align the 4 with the *ones* column. Multiply ***3(2) = 6*** *a*nd add the 2 from 24, which equals 8. Align 8 with the tens column. Multiply ***3(5) = 15*** and write the product to the left of 84 aligning with the hundreds and thousands column.

- We get the 000 because we are multiplying 528 by 0. Notice that we wrote the number 000 starting at the tens column of 1584!

- We get 1056 as follows:

  Multiply ***2(8) = 16,*** align the 6 with the *hundreds* column. Multiply ***2(2) = 4*** *a*nd add the 1 from 16, which equals 5. Align 5 with the thousands column. Multiply ***2(5) = 10*** and write the product to the left of 56 aligning with the ten-thousands and hundred-thousands column.

- Lastly, we add the three partial sums to get 107,184.

Operations on Natural and Whole Numbers

Example 4:

Find the product of $\boldsymbol{567 \times 1000}$ without a calculator.

A quick way to multiply a whole number by a multiple of 10 without a calculator is to add the number of zeros in the multiplier to the multiplicand.

Since the multiplier 1000 has three zeros, we simply add them to the multiplicand.

We find that $\boldsymbol{567 \times 1000 = 567000}$

Operations on Natural and Whole Numbers (8 of 12)

d. Rule for Dividing Two Natural and Whole Numbers:

Note: There are six steps!

1. Determine how many times the divisor (the number you're dividing by) divides into the first digit(s) of the dividend (the number being divided). Write this quotient above the division symbol, aligned with the last digit used in the dividend.
2. Multiply the quotient digit by the divisor. Write the result below the corresponding digits of the dividend.
3. Subtract the product from the corresponding digits of the dividend.

Operations on Natural and Whole Numbers (9 of 12)

4. Bring down the next digit of the dividend next to the remainder.
5. Repeat steps 1-4 with the new number until there are no more digits to bring down.
6. If there is a non-zero number remaining after the last subtraction, it is the remainder and is written next to the quotient with "R" preceding it.

Operations on Natural and Whole Numbers

Example 5:

Find the quotient $7\overline{)2135}$ without a calculator.

Starting on the left, find the first group of digits of the dividend that the divisor divides into. This is the number ***21***.

$$\begin{array}{r} 3 \\ 7\overline{)2135} \end{array}$$

Divide ***21*** by **7** and write ***3*** above the rightmost digit of ***21***. This is the position of the quotient.

$$\begin{array}{r} 3 \\ 7\overline{)2135} \\ 21 \end{array}$$

Multiply ***3*** by ***7*** and write this product below the digits ***21*** of the dividend. Align places!

$$\begin{array}{r} 3 \\ 7\overline{)2135} \\ -21 \\ \hline 0 \end{array}$$

Subtract the product from ***21*** and write the difference ***0*** below the product aligning places.

Operations on Natural and Whole Numbers

$$
\begin{array}{r}
3 \\
7\,\overline{)\,2135} \\
-\underline{21} \\
03
\end{array}
$$

To the right of the difference ***0*** we write the next digit of the dividend, which is ***3***.

$$
\begin{array}{r}
30 \\
7\,\overline{)\,2135} \\
-\underline{21} \\
03 \\
-\underline{0} \\
3
\end{array}
$$

Note that ***7*** divides into ***3*** zero times! But we must write a 0 into the quotient next to the 3.

Operations on Natural and Whole Numbers (12 of 12)

```
    30
7 ) 2135
  − 21
    ——
    03
   − 0
    ——
     35
```

To the right of the difference ***03*** we write the last digit of the dividend, which is ***5***.

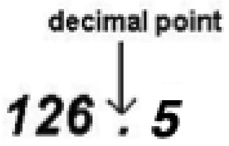
```
    305
7 ) 2135
  − 21
    ——
    03
   − 0
    ——
     35
   − 35
     ——
      0
```

Divide ***35*** by **7** and write ***5*** next to the ***0*** in the quotient. Multiply ***5*** by ***7*** and write this product below the difference ***35***. Align places! Subtract the product from the difference of ***35*** and write ***0*** below the product aligning places.

Since there are no more digits in the dividend, we are done. The quotient is ***305*** and the remainder is ***0***.

4. Decimal Numbers (1 of 3)

Our counting system lets us write partial amounts of whole numbers using a clever symbol called the **decimal point**. When a number contains a decimal point, we call it a **decimal number** or simply a **decimal**.



Please note that the 5 to the right of the decimal point, indicates that the number is exactly halfway in between 126 and 127.

Whole numbers can be written as decimal numbers by using zeros in the decimal places. We can add as many decimal places as needed depending on a particular situation.

For example, the number ***45*** can be written as ***45.0*** or ***45.00*** or ***45.000***, and so on.

Decimal Numbers (2 of 3)

Decimal numbers can be (a) terminating; (b) non-terminating and non-repeating; or (c) non-terminating and repeating.

(a) Terminating, Non-Repeating Decimal Number – The number *0.75* is a terminating, non-repeating decimal number.

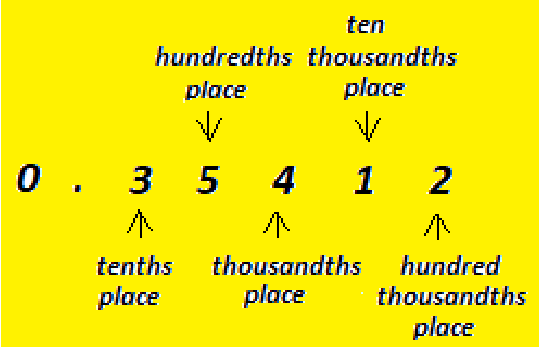
(b) Non-Terminating, Repeating Decimal Number – The number $0.\overline{81}$ is a non-terminating, repeating decimal. It is equal to *0.81 81 81 81 81 81 81…* with *81* repeating infinitely.

(c) Non-Terminating, Non-Repeating Decimal Number – The value of the Greek number Pi, whose symbol is π, is a non-terminating, non-repeating decimal number approximately equal to *3.141592654 …* or better known as *3.14*. We encounter this number often in circle calculations.

Decimal Numbers (3 of 3)

Place Value of Decimal Numbers

For example, given the number ***0.35412***, we say it has five decimal places whose respective places or place values are as follows:



Please note that when there is no number to the right of the decimal place, we can express this as either ***0.35412*** or simply as ***.35412*** leaving off the ***0*** in the ones place.

5. Operations on Decimal Numbers (1 of 11)

a. Rule for Adding Two or More Decimal Numbers:

1. Align the numbers in columns by place value with the decimal points directly under each other.
2. Then add the digits of each column starting on the right. If there is no digit, use 0.
3. Regroup whenever the sum of a column is more than one digit.
4. Place the decimal point in the sum directly under the other decimal points.

Operations on Decimal Numbers (2 of 11)

Example 6:

Find the sum of ***67.9 + 23 + 0.34*** without a calculator.

$$
\begin{array}{r}
67.90 \\
23.00 \\
+\ \ 0.34 \\
\underline{\ \ ^{1}\,^{1}\ \ \ \ \ } \\
91.24
\end{array}
$$

Operations on Decimal Numbers (3 of 11)

b. Rule for Subtracting Two Decimal Numbers:

1. Align the numbers in columns by place value with the decimal points directly under each other. Then subtract the digits in each column starting on the right.
2. Regroup whenever subtracting a larger digit from a smaller one.
3. Place the decimal point in the difference directly under the other decimal points.

Example 7:

Find the difference of ***201 – 72.35*** without a calculator.

$$\begin{array}{r} {}^{1}\ \ {}^{9}\ {}^{10}\ {}^{9}\ \ \\ \not{2}\,{}^{1}\not{0}\,{}^{1}\not{1}.{}^{1}\not{0}\,{}^{1}0 \\ -\quad 7\,2.3\,5 \\ \hline 1\,2\,8.6\,5 \end{array}$$

Operations on Decimal Numbers

c. Rule for Multiplying Two Decimal Numbers:

1. Ignore the decimal point and multiply just like it was done with whole numbers.
2. Count the number of decimal places in the multiplicand and in the multiplier.
3. In the product, count from the right the number of digits equal to the sum of the decimal places of the multiplicand and the multiplier. This is where the decimal point will be placed.

Operations on Decimal Numbers (6 of 11)

Example 8:

Find the product of ***0.291*** × ***0.14*** without a calculator.

$$
\begin{array}{r}
0.291 \\
\times \;\; 0.14 \\
\hline
1164 \\
+ \;\; 291 \;\; \\
\hline
0.04074
\end{array}
$$

Three (3) decimal places plus two (2) decimal places equals five (5) decimal places.

There are not enough digits in the final product to accommodated five (5) decimal places. Therefore, we had to insert an extra ***0*** to the left of the product!

Operations on Decimal Numbers (7 of 11)

Example 9:

Find the product of $\boldsymbol{24.5 \times 100}$ without a calculator.

A quick way to multiply a whole number by a multiple of 10 without a calculator is to move the decimal point in the multiplicand the number of zeros in the multiplier.

Since the multiplier 100 has two zeros, we simply move the decimal point of the multiplicand two places to the right.

We find that $\boldsymbol{24.5 \times 100 = 2450}$

Note: We had to add a 0 to the end to satisfy the requirement of TWO decimal places.

Operations on Decimal Numbers (8 of 11)

d. Rule for Dividing Two Decimal Numbers:

The division process for decimal numbers is very similar to that of whole numbers.

However, the first step is always to place a decimal point into the quotient position aligning with the decimal point of the dividend.

Additionally, if the divisor is a decimal number, it must always be changed to a whole number. We do this by multiplying both the dividend and the divisor by a multiple of 10.

Operations on Natural and Whole Numbers

Example 10:

Find the quotient $2.3\overline{)3.68}$ without a calculator.

Since the divisor is NOT a whole number, the first thing we will do is multiply both the divisor and the dividend by 10 to get the following:

$$23\overline{)36.8}$$

Since the divisor is now a whole number, we'll insert a decimal point into the quotient. Remember that it must be placed right above the decimal point of the dividend.

$$\begin{array}{r} . \\ 23\overline{)36.8} \end{array}$$

Operations on Natural and Whole Numbers (10 of 11)

Since ***23*** is larger than the whole number ***36*** of the dividend and therefore divides into it once, the whole number part of the quotient will be ***1*** with a remainder of 13.

```
       1.
23 ) 36.8
   - 23
     --
     13
```

To continue the division process, we now have to use a digit from the fractional part of the dividend.

We will ignore the decimal point in the dividend and divide ***23*** into ***138***. However, all digits of the quotient MUST now be placed to the right of the decimal point.

Operations on Natural and Whole Numbers (11 of 11)

$$
\begin{array}{r}
1.6 \\
23\overline{)\ 36.8} \\
-\ 23 \\
\hline
13\ 8 \\
-\ 13\ 8 \\
\hline
0
\end{array}
$$

We find the quotient to be ***1.6***. There is NO remainder.