Concepts and Examples
Systems of Two Linear Equations in Two Variables

Based on power point presentations by Pearson Education, Inc.

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Learning Objectives

1. Memorize the characteristics of systems of two linear equations in two variables.
2. Solve systems of two linear equations in two variables using the *Substitution Method*.
3. Solve systems of two linear equations in two variables using the *Elimination Method*.

1. Introduction to Systems of Two Linear Equations in Two Variables (1 of 4)

We learned that $Ax + By + C = 0$ is the general form of a linear equation in two variables, namely *x* and *y*. Many application problems can be modeled by these equations but many more can be modeled by two linear equations used simultaneously. When this happens, we talk about a *System of Equations*.

For example, $\begin{cases} 3x + 2y = 4 \\ 2x + y = 1 \end{cases}$

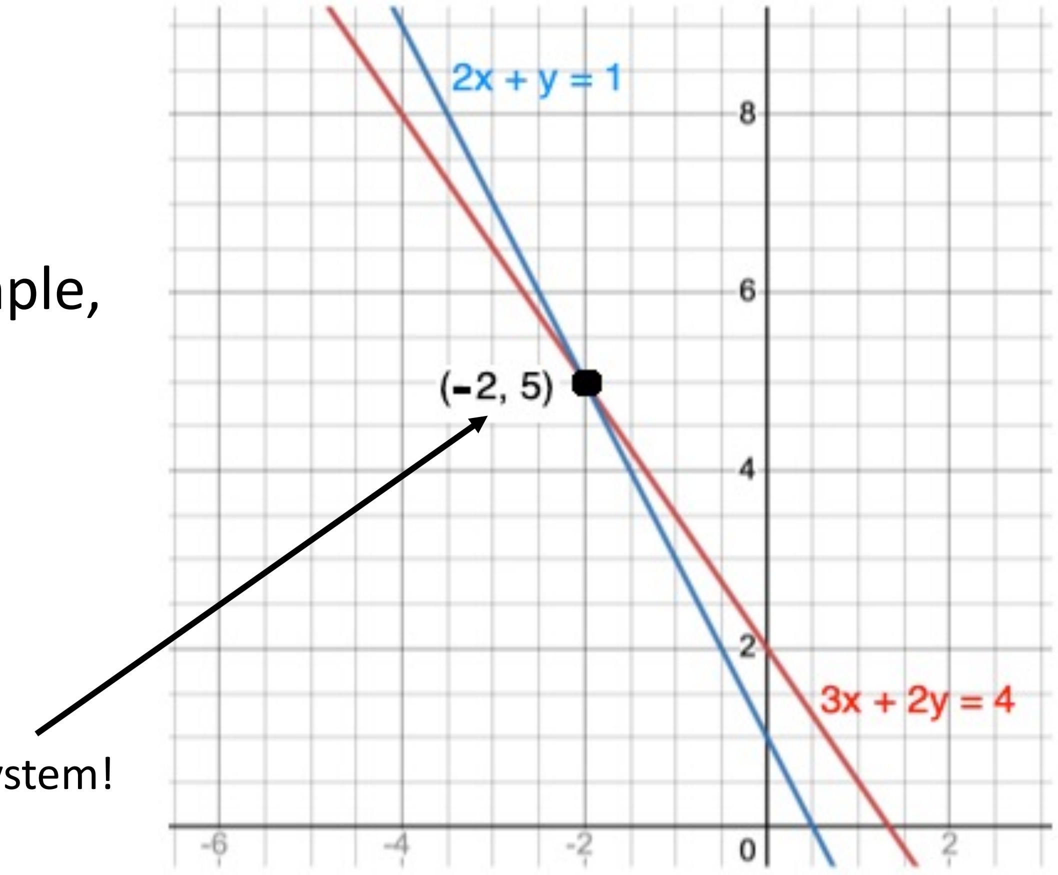
We indicate a system by drawing a left brace {.

NOTE: The linear equations in a system are usually not written in general form.

Introduction to Systems of Two Linear Equations in Two Variables (2 of 4)

When solving a system of two linear equations in two variables, we are actually attempting to find the ordered pair of the point at which their graphs intersect.

For example,



The solution of the system!

which is a picture of the system $\begin{cases} 3x+2y=4 \\ 2x+y=1 \end{cases}$

Introduction to Systems of Two Linear Equations in Two Variables (3 of 4)

In the previous example, we solved the systems of two linear equations in two variables by graphing the two equations. Unfortunately, this often results in very inexact solutions. Therefore, we prefer to use algebraic methods, namely the *Substitution Method* or the *Elimination Method* also called *Elimination by Addition Method*.

Incidentally, it is not always possible to find a point of intersection. For example, when the two lines in the system are parallel. In that case, the system has no solutions. It is then called an **inconsistent system**.

Introduction to Systems of Two Linear Equations in Two Variables (4 of 4)

Also, the two lines in the system could be one and the same line. We just don’t notice it right away because this fact is often ”disguised.” In that case, we say that the system has infinitely many solutions. It is then called a **dependent and consistent system**.

To be complete, if the two lines in the system do intersect in exactly one point, we call it a **consistent system**.

2. Solve Systems of Two Linear Equations in Two Variables Using the Substitution Method (1 of 4)

Substitution Method Strategy

Step 1 – Unless already done, "isolate" one of the variables of one of the equations on one side of the equal sign.

Example 2:

Solve the system $\begin{cases} 3x + 2y = 4 \\ 2x + y = 1 \end{cases}$ Equation 1, Equation 2

Note, it is recommended that you always name the equations, such as 1, 2, 3, etc.!

We decide to isolate y in Equation 2 on the left side of its equal sign.

$2x + y = 1$ (Equation 2)

$y = -2x + 1$ (new Equation 2)

Solve Systems of Two Linear Equations in Two Variables Using the Substitution Method (2 of 4)

Step 2 – Substitute the “appropriate” variable in the equation NOT used in Step 1 with the expression equaling the isolated variable in Step 1. Solve the equation.

Example 2 continued:

We will use Equation 1 and substitute its y with $-2x+1$.

$3x + 2y = 4$ (Equation 1)

$3x + 2(-2x + 1) = 4$

$3x - 4x + 2 = 4$

$-x + 2 = 4$

$-x = 2$

$x = -2$ This is the x-coordinate of the point of intersection!

Solve Systems of Two Linear Equations in Two Variables Using the Substitution Method (3 of 4)

Step 3 – Substitute the "appropriate" variable in either one of the original equations with the value of the coordinate found in Step 2. Solve the equation.

Example 2 continued:

We will use the original Equation 1 and substitute its x with $x = -2$ found in Step 2.

$3x + 2y = 4$ (Equation 1)

$3(-2) + 2y = 4$

$-6 + 2y = 4$

$2y = 10$

$y = 5$ This is the y-coordinate of the point of intersection!

Solve Systems of Two Linear Equations in Two Variables Using the Substitution Method (4 of 4)

Step 4 - Write the solution of the system as an ordered pair.

Example 2 continued:

Given $x = -2$ (Step 2) and $y = 5$ (Step 3), the solution of the system is $(-2, 5)$.

3. Solve Systems of Two Linear Equations in Two Variables Using the Elimination Method (1 of 5)

Elimination Method Strategy

Step 1 – Unless already done, rewrite both equations in the system so that like terms are directly below each other with variable terms appearing on the left and constants appearing on the right side of the equal sign.

Example 3:

Solve the system

$$\begin{cases} 4x + 5y = 3 & \text{Equation 1} \\ 2x - 3y = 7 & \text{Equation 2} \end{cases}$$

Step 1 is already done!

Solve Systems of Two Linear Equations in Two Variables Using the Elimination Method (2 of 5)

Step 2 - Find the value of any variable by eliminating the other one using addition. We may have to first "appropriately" change the coefficients of one or both equations!

Example 3 continued:

$$\begin{cases} 4x + 5y = 3 & \text{Equation 1} \\ 2x - 3y = 7 & \text{Equation 2} \end{cases}$$

We decide to eliminate the *x*-terms by addition to find the value of *y*.

However, to do this, the coefficients of *x* in both equations must be opposite numbers. For example, 4 and – 4!

We can accomplish opposite numbers by multiplying every term in Equation 2 by – 2. This preserves equality! But the coefficient of *x* will then be – 4.

Solve Systems of Two Linear Equations in Two Variables Using the Elimination Method (3 of 5)

Example 3 continued:

We get the following equivalent system!

$$\begin{cases} 4x + 5y = 3 \\ -4x + 6y = -14 \end{cases}$$

Equation 1

– 2 · Equation 2

Now we can eliminate the *x*-terms because 4*x* + (– 4*x*) = 0 and solve for *y*!

$$\begin{array}{rr} & 4x + 5y = 3 \\ + & -4x + 6y = -14 \\ \hline & 11y = -11 \\ & y = -1 \end{array}$$

This is the *y*-coordinate of the point of intersection!

Solve Systems of Two Linear Equations in Two Variables Using the Elimination Method (4 of 5)

Step 3 - Substitute the “appropriate” variable in either one of the original equations with the value of the coordinate found in Step 2. Solve the equation.

Example 3 continued:

We will use the original Equation 1 and substitute its y with $y = -1$ found in Step 2.

$4x + 5y = 3$ (Equation 1)

$4x + 5(-1) = 3$

$4x - 5 = 3$

$4x = 8$

$x = 2$ This is the x-coordinate of the point of intersection!

Solve Systems of Two Linear Equations in Two Variables Using the Elimination Method (5 of 5)

Step 4 - Write the solution as an ordered pair.

Example 3 continued:

Given $x = 2$ (Step 3) and $y = -1$ (Step 2), the solution of the system is $(2, -1)$.

Example 4: Use the Elimination Method (1 of 2)

Solve the following system by the Elimination Method.

$$\begin{cases} 5x - 2y = 4 & \text{Equation 1} \\ -10x + 4y = 7 & \text{Equation 2} \end{cases}$$

All like terms are directly below each other. Let's eliminate the *x*-terms by addition. This allows us to find the value of the *y*-coordinate of the point of intersection.

To do this, we will multiply every term in Equation 1 by the number 2. This preserves equality! The coefficient of *x* will then be 10.

Example 4: Use the Elimination Method (2 of 2)

We get the following equivalent system!

$$\begin{cases} 10x - 4y = 8 & 2 \cdot \text{Equation 1} \\ -10x + 4y = 7 & \text{Equation 2} \end{cases}$$

Now we can eliminate the *x*-terms because 10*x* + (– 10*x*) = 0 and solve for *y*!

$$\begin{array}{rr} & 10x - 4y = 8 \\ + & -10x + 4y = 7 \\ \hline & 0 = 15 \end{array}$$

STOP: 0 never equals 15. We can therefore conclude that the system has NO solutions. Graphically, this indicates that the two lines in the system are parallel to each other.

Example 5: Use the Elimination Method (1 of 2)

Solve the following system by the Elimination Method.

$$\begin{cases} x - 4y = -8 & \text{Equation 1} \\ 5x - 20y = -40 & \text{Equation 2} \end{cases}$$

All like terms are directly below each other. Let's eliminate the *y*-terms by addition. This allows us to find the value of the *x*-coordinate of the point of intersection.

To do this, we will multiply every term in Equation 1 by the number – 5. This preserves equality! The coefficient of *y* will then be + 20.

Example 5: Use the Elimination Method (2 of 2)

We get the following equivalent system!

$$\begin{cases} -5x+20y=40 & -5 \cdot \text{Equation 1} \\ 5x-20y=-40 & \text{Equation 2} \end{cases}$$

Now we can eliminate the *y*-terms because – 20*y* + 20*y* = 0 and find *x*!

$$\begin{array}{rr} & -5x+20y=40 \\ + & 5x-20y=-40 \\ \hline & 0=0 \end{array}$$

0 = 0 is a true statement. However, no variables are left. This indicates that the two linear equations in the system are identical. Graphically, we are looking at one and the same line. We say that this system has infinitely many solutions.