The Slope of a Line

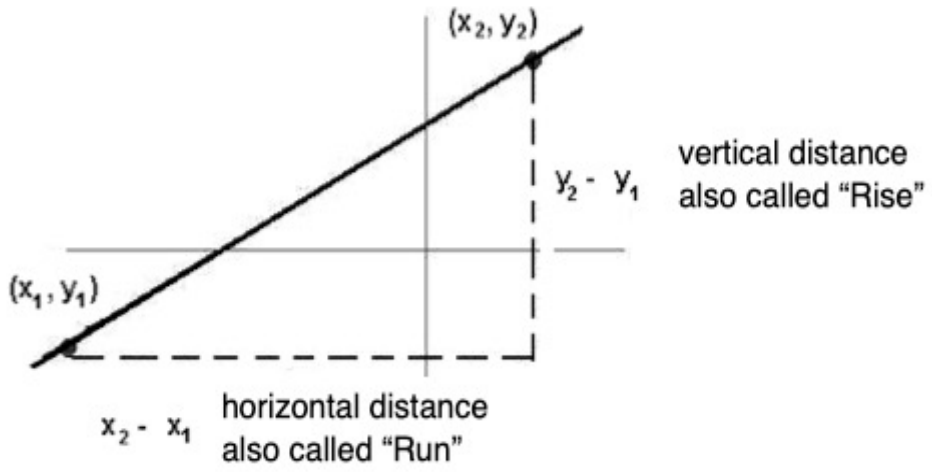
Based on power point presentations by Pearson Education, Inc.
Revised by Ingrid Stewart, Ph.D.

Learning Objectives

Define and calculate the slope of a line.

Definition of the Slope of a Line (1 of 2)

In layman's terms, the slope is a measure of the **steepness of a line**. It is said to be the change in vertical distance divided by the change in horizontal distance as we "travel" from one point, say (x_1, y_1), to another point, say (x_2, y_2) lying on the same line in a rectangular coordinate system.



Definition of the Slope of a Line (2 of 2)

In mathematics, the slope of a line is indicated by using the lower-case letter *m*. Why *m*? No one knows for sure. Some mathematicians claim the *m* comes from the French word "monter" which means "to climb".

The slope of the line through two distinct points (x_1, y_1) and (x_2, y_2) lying in a coordinate system is formally defined as

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

NOTE: Regardless of the sign of the *x*-coordinates or the *y*-coordinates, the minus sign between the *y*-values and the *x*-values in the slope calculation must always be there.

We can also say $m = \frac{\text{Rise}}{\text{Run}}$ or $m = \frac{\text{change in } y}{\text{change in } x}$

Example 1: Calculate the Slope of a Line

Find the slope of the line passing through the points (4, −2) and (−1, 5).

Here you can say that (4, −2) is (x_1, y_1) and (−1, 5) is (x_2, y_2). However, you can also state that (−1, 5) is (x_1, y_1) and (4, −2) is (x_2, y_2). In either case, you will get the same answer.

Let's say that (4, −2) is (x_1, y_1) and (−1, 5) is (x_2, y_2). Be sure not to get confused! Then

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-(-2)}{-1-4} = \frac{5+2}{-5} = \frac{7}{-5} = -\frac{7}{5}$$

NOTE: Regardless of the sign of the *x*-coordinates or the *y*-coordinates, the minus sign between the *y*-values and the *x*-values in the slope calculation must always be there.

Example 2: Calculate the Slope of a Line

Find the slope of the line passing through the points (– 1, 3) and (– 4, – 6).

Here you can say that (– 1, 3) is (x_1, y_1) and (– 4, – 6) is (x_2, y_2). However, you can also state that (– 4, – 6) is (x_1, y_1) and (– 1, 3) is (x_2, y_2). In either case, you will get the same answer.

Let's say that (– 4, – 6) is (x_1, y_1) and (– 1, 3) is (x_2, y_2). Be sure not to get confused! Then

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-6)}{-1-(-4)} = \frac{3+6}{-1+4} = \frac{9}{3} = 3$$

NOTE: Regardless of the sign of the *x*-coordinates or the *y*-coordinates, the minus sign between the *y*-values and the *x*-values in the slope calculation must always be there.

Example 3: Calculate the Slope of a Line

Find the slope of the line passing through the points (6, 3) and (6, 4).

Let's say that (6, 3) is (x_1, y_1) and (6, 4) is (x_2, y_2). Be sure not to get confused! Then

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-3}{6-6} = \frac{1}{0}$$

Since there is a 0 in the denominator, this particular slope is undefined.

Example 4: Calculate the Slope of a Line

Find the slope of the line passing through the points (1, 5) and (− 9, 5).

Let's say that (1, 5) is (x_1, y_1) and (− 9, 5) is (x_2, y_2). Be sure not to get confused! Then

$$m = \frac{y_2 - y_1}{x_2 - y_{x1}} = \frac{5-5}{-9-1} = \frac{0}{-10} = 0$$

Since there is a 0 in the numerator, this particular slope equals 0. Please note the difference between Example 3 and Example 4!