Concepts and Examples
Radical Expressions

Based on power point presentations by Pearson Education, Inc.
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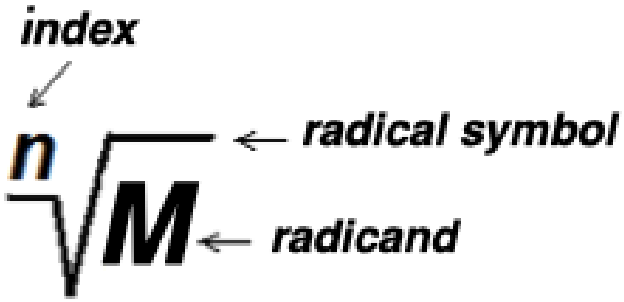
Learning Objectives

1. Evaluate radical expressions.
2. Perform arithmetic operations on radical expressions.
3. Convert radical expressions to exponential expressions and vice versa.

1. Evaluate Radical Expressions (1 of 10)

Radical expressions or simply “radicals” are related to exponential expressions in that they reverse the operation of raising a number to a power. The radical symbol $\sqrt{}$ indicates this process. Evaluating a radical expression is also called “finding a root.” Please examine the general radical expression carefully!



This is expressed as “the *nth* root of *M*” where *M* can be a real number or a mathematical expression and *n* is a positive integer.

Some radicals can be evaluated by hand. However, we use calculators most often. They are programmed with the calculus concepts that allow us to approximate the value of all radical expressions.

Evaluate Radical Expressions (2 of 10)

Example 1:

Explain what $\sqrt{16}$ means.

Here we do not see an index. We must automatically know that it is **2**. The 2 is usually not written.

This exponential expression asks us to find "the 2nd root of 16". HOWEVER, since the index 2 appears most often in mathematics and the sciences, we overwhelmingly prefer to state, "**find the square root of 16.**"

Evaluate Radical Expressions (3 of 10)

Example 2:

Evaluate $\sqrt{16}$ (“the square root of 16”) without a calculator.

In order to evaluate a radical with index **2**, we must find a number that, when multiplied **two times**, equals 16. We know that $4 \cdot 4 = 16$. Therefore, $\sqrt{16}$ equals 4.

Incidentally, $-4 \cdot -4$ also equals 16. However, we always use positive numbers when evaluating radicals.

Evaluate Radical Expressions (4 of 10)

Radical expressions can have rational, irrational, or imaginary values.

- **Rational values** are of the form $\boldsymbol{\frac{a}{b}}$ where ***a*** and ***b*** are integers with ***b*** not equal to 0. Don't forget that integers can also be written in this form.

Example 3:

Evaluate $\sqrt[4]{16}$ ("the fourth root of 16") without a calculator. The index is **4**.

In order to evaluate a radical with index **4**, we must find a number that, when multiplied **four times**, equals 16. We know that (2) · (2) · (2) · (2) = 16. Therefore, $\sqrt[4]{16}$ equals 2.

Evaluate Radical Expressions (5 of 10)

Example 4:

Evaluate $\sqrt[3]{-8}$ ("the third root or cube root of negative 8") without a calculator.

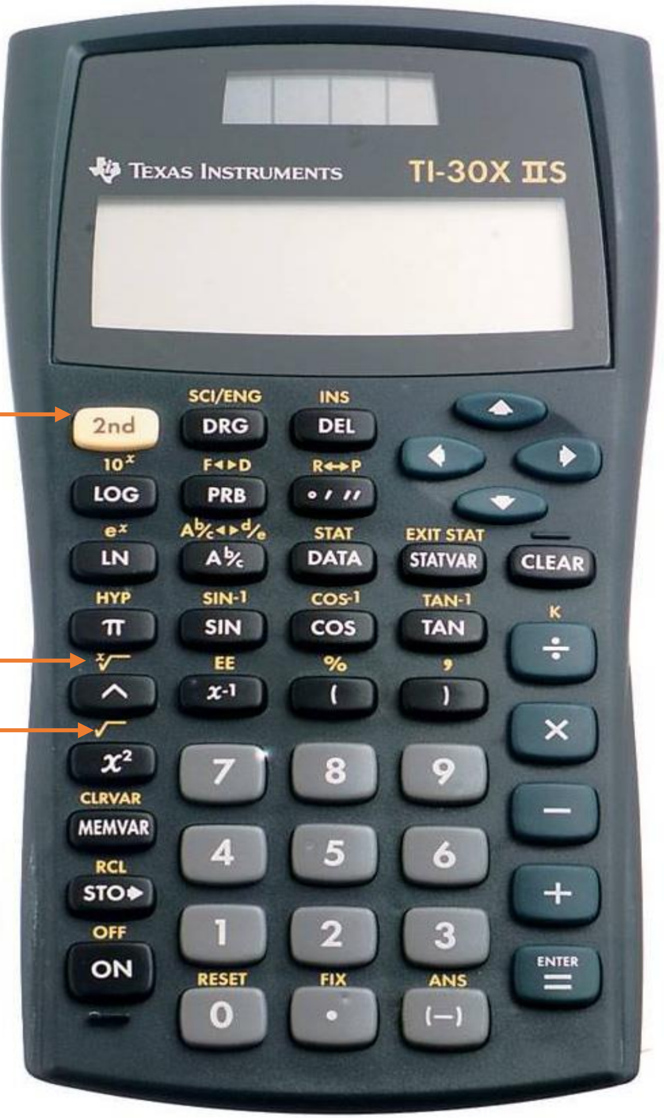
In order to evaluate a radical with index **3**, we must find a number that, when multiplied **three times**, equals – 8. We know that $(-2) \cdot (-2) \cdot (-2) = -8$. Therefore, $\sqrt[3]{-8}$ equals – 2.

Evaluate Radical Expressions (6 of 10)

To invoke the radical functions, press the 2nd button!

$\sqrt[x]{\ }$ is used for any index

$\sqrt{\ }$ is used only for index 2



Example 5:

Evaluate $\sqrt{9}$ with a calculator.

We will use the TI-30X IIS:

- Press the 2nd button and then the x^2 button. You will see $\sqrt{\ }($.
- Type **9** and press the right parentheses **)** button to close the parentheses.
- Press the ENTER button.

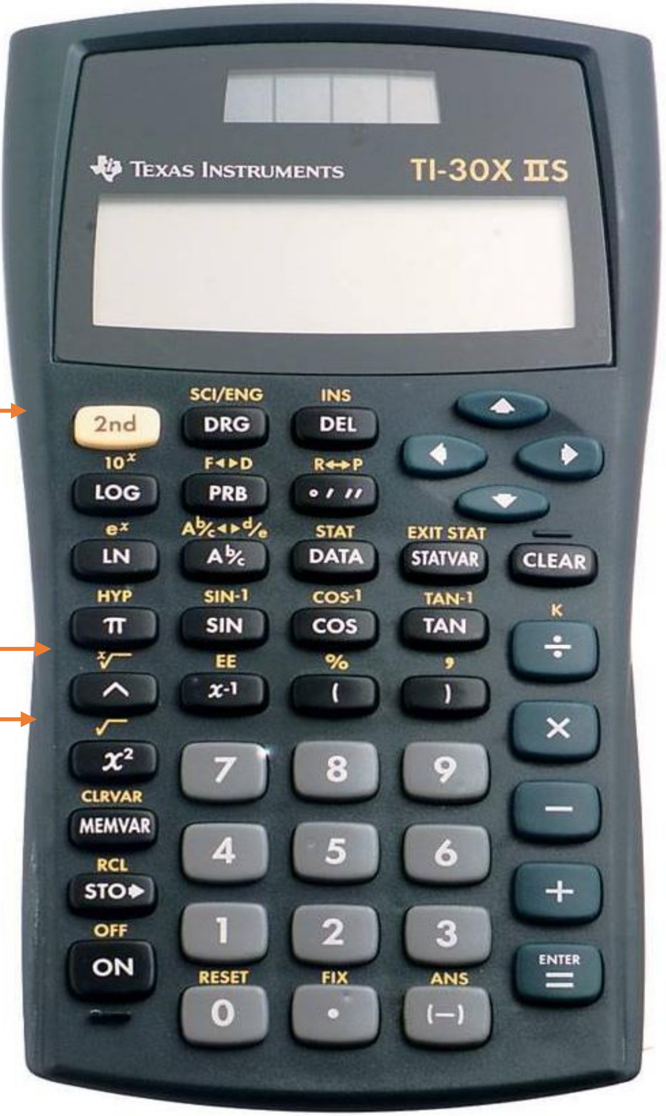
The answer displayed is **3** which is a rational number but more specifically an **integer**.

Evaluate Radical Expressions (7 of 10)

To invoke the radical functions, press the 2nd button!

$\sqrt[x]{\ }$ is used for any index

$\sqrt{\ }$ is used only for index 2



Example 6:

Evaluate $\sqrt[3]{8}$ with a calculator.

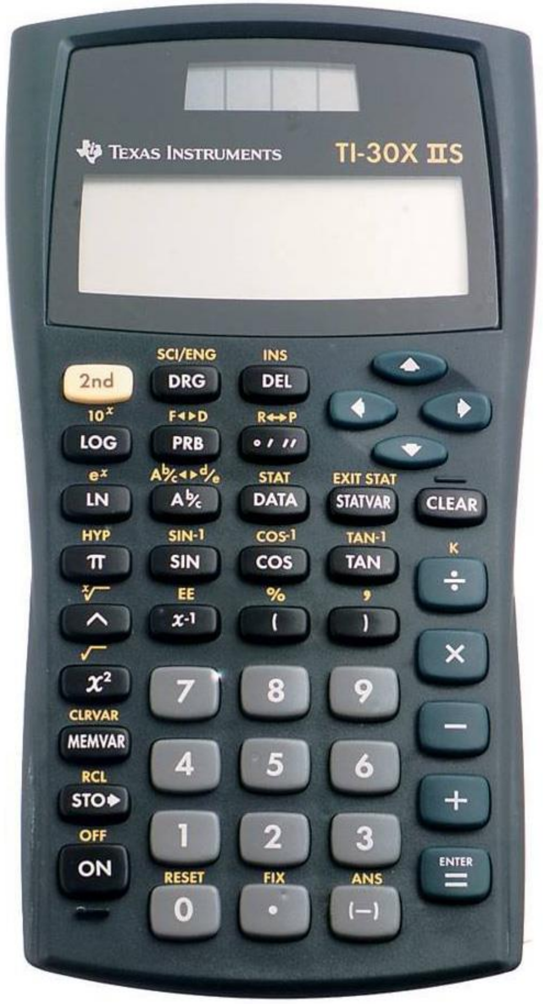
We will use the TI-30X IIS:

- Type the index **3**.
- Press the 2nd button and then the ^ (caret) button. You will see 3$\sqrt[x]{\ }$.
- Type **8**.
- Press the ENTER button.

The answer displayed is **2** which is a rational number but more specifically an integer.

Evaluate Radical Expressions (8 of 10)

- **Irrational values** are those that cannot be converted to the form $\frac{a}{b}$.



Example 7:

Evaluate $\sqrt{10}$ with a calculator.

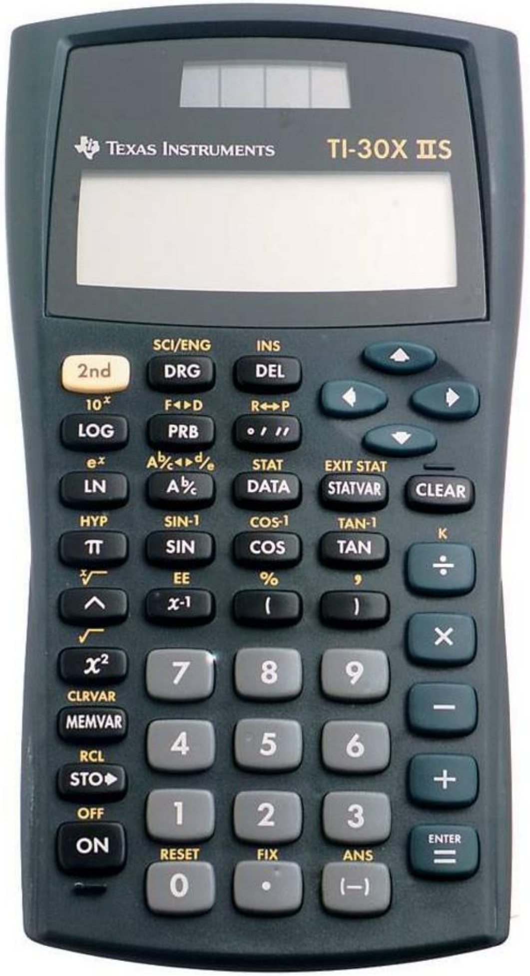
We will use the TI-30X IIS calculator:

- Press the 2nd button and then the x^2 button. You will see $\sqrt{(}$.
- Type **10** and press the right parenthesis **)** button to close the parentheses.
- Press the ENTER button.

The answer displayed is **3.16227766**. Please note that the decimal portion is actually never-ending making the number an **irrational number**.

Evaluate Radical Expressions (9 of 10)

- **Imaginary values** occur ONLY when the index is even, and the radicand is negative. We will discuss imaginary numbers in detail in a later lesson!



Example 8:

Evaluate $\sqrt{-4}$ with a calculator.

We will use the TI-30X IIS calculator:

- Press the 2nd button and then the x^2 button. You will see $\sqrt{(}$.
- Press the (–) button.
- Type **4** and press the right parenthesis) button to close the parentheses.
- Press the ENTER button.

The answer displayed is “Domain Error”.

Evaluate Radical Expressions

Example 8 continued:

This is how the calculator informs you that $\sqrt{-4}$ is an imaginary number, which is a number that cannot be found on a number line.

Think about it!

In $\sqrt{-4}$ the index is 2 which is even. So, what number times itself equals – 4? We know 2 · 2 = 4 and – 2 · – 2 = 4. But there is no number that when multiplied by itself gives us – 4! Therefore, we say that $\sqrt{-4}$ is imaginary!

2. Arithmetic Operations on Radical Expressions (1 of 3)

Rule for Adding and Subtracting Radical Expressions:

We can add or subtract two or more radical expressions **if they have the SAME index and radicand**. We do so by adding and subtracting their "non-radical" factors.

For example, $6\sqrt{3} + 5\sqrt{3} = (6 + 5)\sqrt{3} = 11\sqrt{3}$

Note: There is an implied multiplication between the "non-radical" factors and the radical factors.

or $\sqrt{17} - 20\sqrt{17} = (1 - 20)\sqrt{17} = -19\sqrt{17}$

Note, $\sqrt{17}$ has a "non-radical" factor of 1.

Arithmetic Operations on Radical Expressions (2 of 3)

Rule for Multiplying Radical Expressions:

We can multiply two or more radical expressions **if they have the SAME index.** We do so by multiplying their radicands. If there are "non-radical" factors, they are multiplied separately.

For example, $6\sqrt{2} \cdot 5\sqrt{3} = (6 \cdot 5)\sqrt{2 \cdot 3} = 30\sqrt{6}$.

Note: There is an implied multiplication between the "non-radical" factors and the radical factors.

or $\sqrt{5} \cdot 20\sqrt{3} = (1 \cdot 20)\sqrt{5 \cdot 3} = 20\sqrt{15}$

Note, $\sqrt{5}$ has a "non-radical" factor of 1.

It is important to note that this rule will NOT work if the index is even and the radicand is negative

Arithmetic Operations on Radical Expressions (3 of 3)

Rule for Dividing Radical Expression:

We can divide two radical expressions **if they have the SAME index**. We do so by dividing their radicands. If there are “non-radical” factors, they are divided separately.

For example, $\frac{10\sqrt{6}}{5\sqrt{2}} = \frac{10}{5}\sqrt{\frac{6}{2}} = 2\sqrt{3}$

Note: There is an implied multiplication between the “non-radical” factors and the radical factors.

3. Convert Radical Expressions to Exponential Expressions and Vice Versa (1 of 4)

If ***a*** is a real number and $\frac{m}{n}$ is a positive rational number with $n \geq 2$ then

$$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m \quad \text{or} \quad a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

For example, we can write $x^{\frac{3}{7}}$ as $(\sqrt[7]{x})^3$ or as $\sqrt[7]{x^3}$.

Note, here the index *n* is 7 and the power *m* on *x* is 3!

Convert Radical Expressions to Exponential Expressions and Vice Versa (2 of 4)

Example 9:

Evaluate $27^{\frac{1}{3}}$ by hand and with a calculator.

By hand:

$27^{\frac{1}{3}} = \sqrt[3]{27} = 3$ because $27 = 3(3)(3) = 3^3$.

Note, here the index *n* is 3 and the power *m* on 27 is 1!

Convert Radical Expressions to Exponential Expressions and Vice Versa (3 of 4)

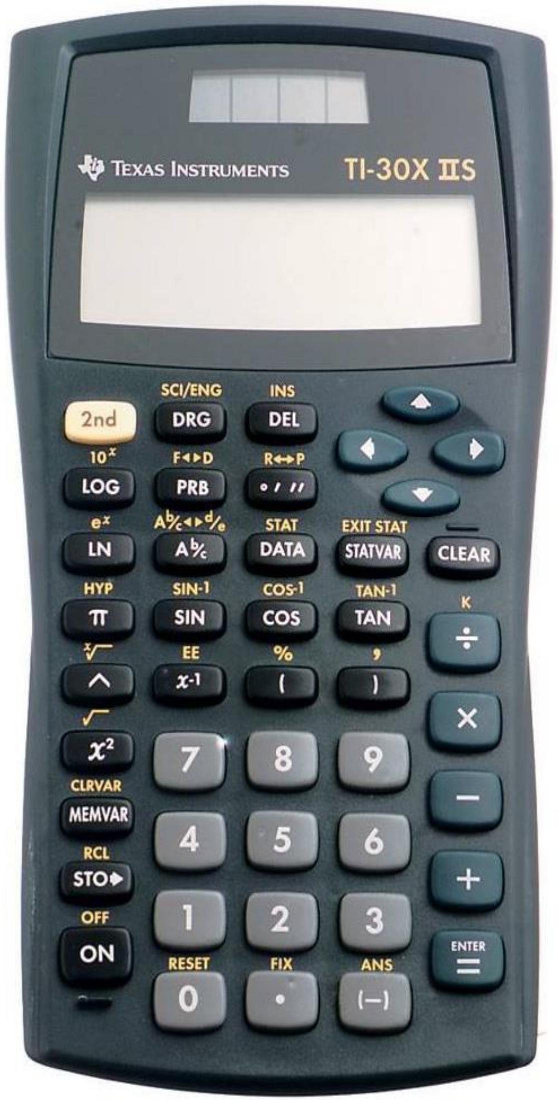
Example 9 continued:

We will use the TI-30X IIS Calculator:

- Type 27.
- Press the caret ^ button.
- Press the left parentheses button and type $1 \div 3$.
- Press the right parenthesis button).
- Press the ENTER button.

The answer displayed is also **3.**

Note: You must place $\frac{1}{3}$ in parentheses otherwise the calculator will use the *Order of Operations*!



Convert Radical Expressions to Exponential Expressions and Vice Versa (4 of 4)

Example 10:

Change $27^{\frac{4}{3}}$ to a radical expression and then evaluate <u>without</u> a calculator.

We know that $27^{\frac{4}{3}}$ can either be written as $\left(\sqrt[3]{27}\right)^4$ or $\sqrt[3]{27^4}$.

$\sqrt[3]{27^4}$ is difficult to evaluate without a calculator because we have to find 27^4.

However, $\left(\sqrt[3]{27}\right)^4$ is manageable since we know that 27 = 3(3)(3) following that $\sqrt[3]{27}$ = 3.

Then $(3)^4$ which equals 81.