



Concepts and Examples Radical Expressions

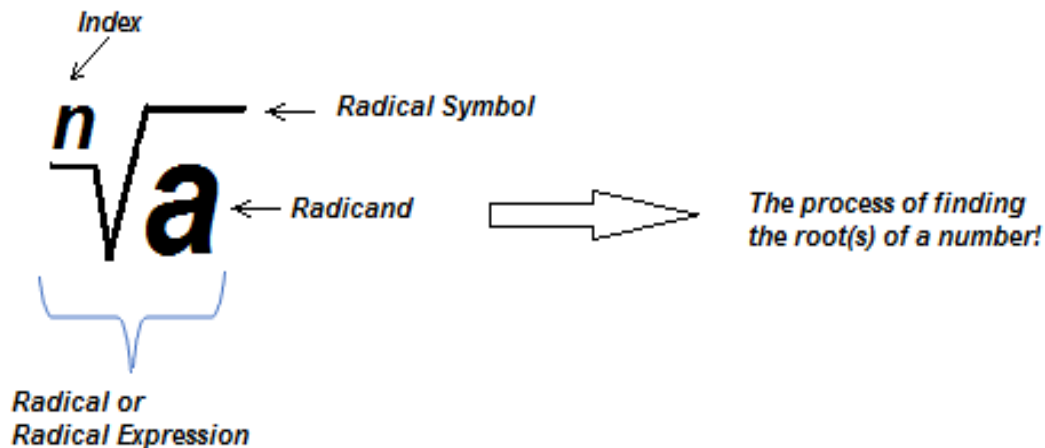
Based on power point presentations by Pearson Education, Inc.
Revised by Ingrid Stewart, Ph.D.

Learning Objectives

1. Evaluate radical expressions.
2. Perform arithmetic operations on radical expressions.
3. Convert radical expressions to exponential expressions and vice versa.

1. Evaluate Radical Expressions (1 of 7)

Radical expressions or simply “radicals” are related to exponential expressions in that they reverse the operation of raising a number to a power. The radical symbol $\sqrt{\quad}$ indicates this process. Evaluating a radical expression is also called “finding a root.” Please examine the general radical expression carefully!



Some radicals can be evaluated by hand. However, we use the calculator most often. They are programmed with the calculus concepts that allow us to easily evaluate all radicals.

Evaluate Radical Expressions (2 of 7)

Radical expressions can have rational, irrational, or imaginary values.

- **Rational values** are of the form $\frac{a}{b}$ where a and b are integers with b not equal to 0. **Integers** are also considered rational values.

Example 1:

- a. Evaluate $\sqrt{9}$ without a calculator. The index is 2 and we often call it a square root!

We know that $3 \cdot 3 = 9$. Therefore, $\sqrt{9} = 3$. Incidentally, $-3 \cdot -3$ also equals 9. However, we also use positive numbers when evaluating a radical.

- b. Evaluate $\sqrt[3]{-8}$ without a calculator. The index is 3 and we often call it a cube root!

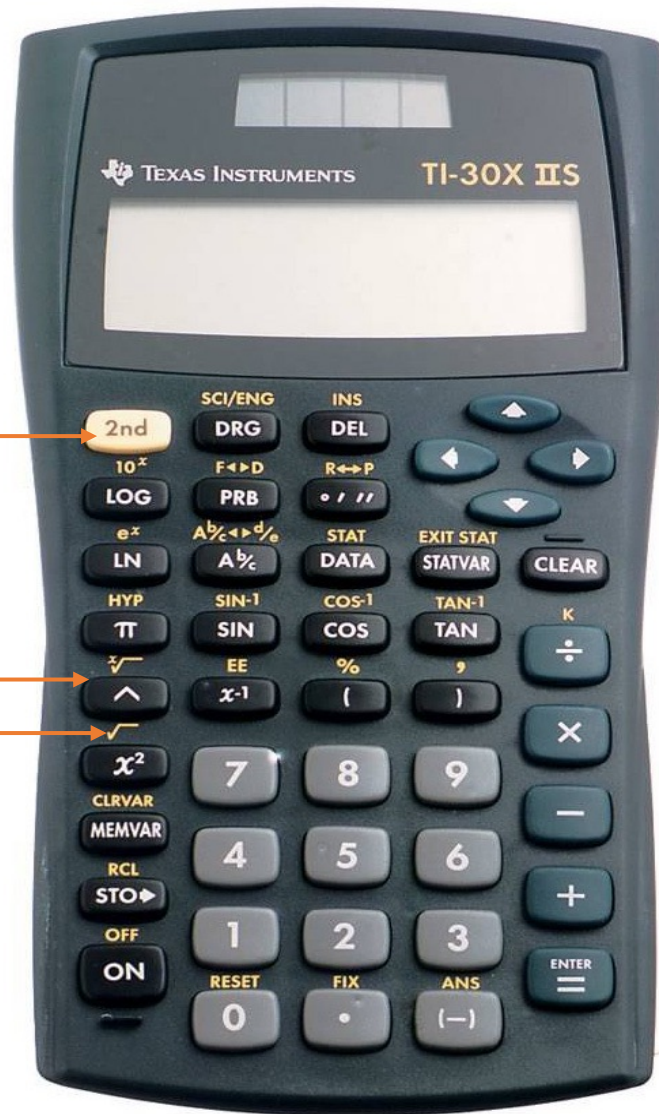
We know that $(-2) \cdot (-2) \cdot (-2) = -8$. Therefore, $\sqrt[3]{-8} = -2$.

Evaluate Radical Expressions (3 of 7)

To invoke the radical functions, press the 2nd button!

\sqrt{x} is used for any index

$\sqrt{}$ is used only for index 2



Example 2:

Evaluate $\sqrt{9}$ with a calculator.

We will use the TI-30X IIS.

- Press the 2nd button and then the x^2 button. You will see $\sqrt{($.
- Type **81** and press the right parentheses **)** button to close the set.
- Press the ENTER button.

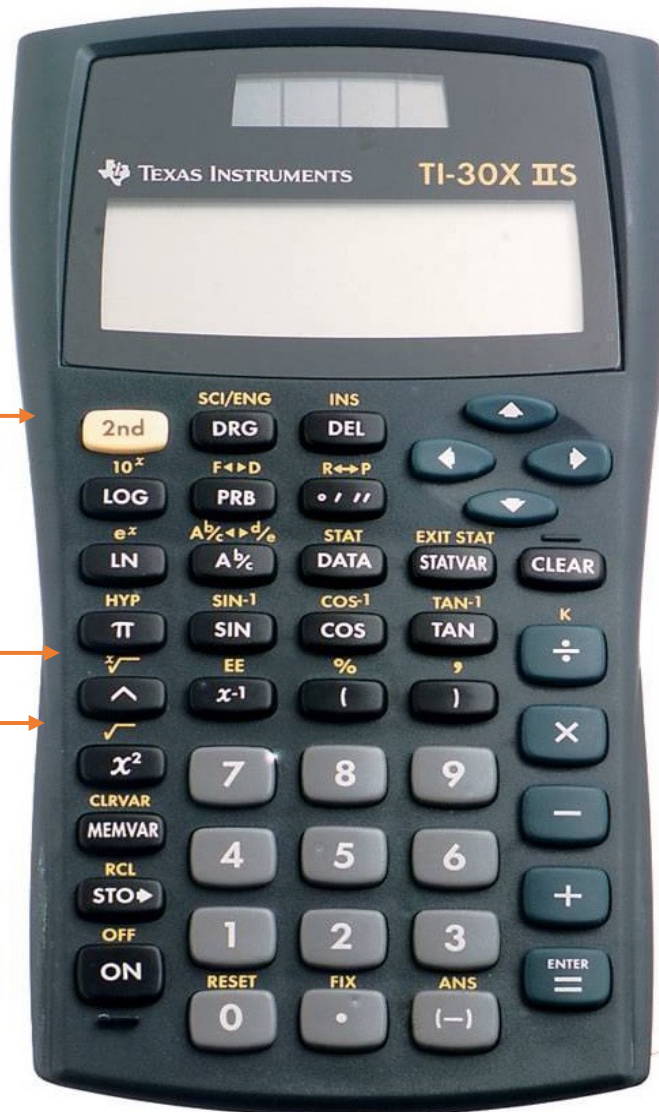
The answer displayed is **3** which is a rational number and more specifically an **integer**.

Evaluate Radical Expressions (4 of 7)

To invoke the radical functions, press the 2nd button!

\sqrt{x} is used for any index

$\sqrt{\quad}$ is used only for index 2



Example 3:

Evaluate $\sqrt[3]{8}$ with a calculator.

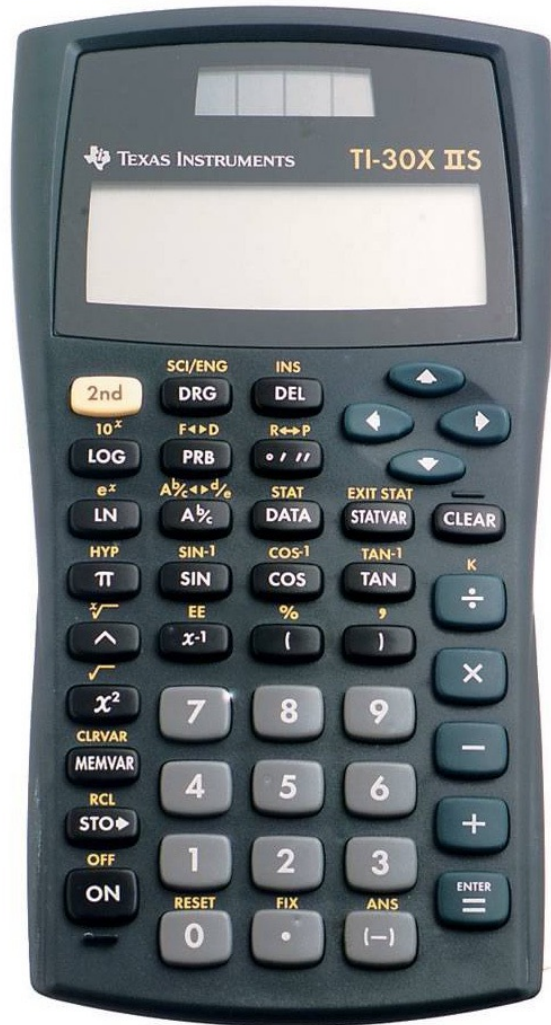
We will use the TI-30X IIS.

- Type the index **3**.
- Press the 2nd button and then the ^ (caret) button. You will see $3\sqrt{\quad}$.
- Type **8**.
- Press the ENTER button.

The answer displayed is **2** which is a rational number and more specifically an integer.

Evaluate Radical Expressions (5 of 7)

- **Irrational values** are those that cannot be converted to the form $\frac{a}{b}$.



Example 4:

Evaluate $\sqrt{10}$ with the TI-30X IIS calculator.

- Press the 2nd button.
- Press the x^2 button. You will see $\sqrt{($.
- Type **10** and press the right parenthesis **)** button to close the set.
- Press the ENTER button.

The answer displayed is **3.16227766**. However, the decimal portion is actually never-ending making the number an **irrational number**.

Evaluate Radical Expressions (6 of 7)

- **Imaginary values** occur when the index is even, and the radicand is negative. We will not discuss imaginary numbers in this course!

Example 5:

Evaluate $\sqrt{-4}$ with a calculator.

Press the 2nd button. Then the x^2 button. This invokes the square root function which resides above the x^2 button. You get $\sqrt{(\ .$ Press the $(-)$ button. Finally type **4** and press the right parenthesis $)$ button to close the parentheses. Press the ENTER button.

Evaluate Radical Expressions (7 of 7)

Example 5 continued:

The calculator states “Domain Error”.

This is how the calculator informs you that $\sqrt{-4}$ is an imaginary number, which is a number that cannot be found on a number line.

Think about it. In $\sqrt{-4}$ the index is 2 which is even. So, what number times itself equals -4 ? We know $2 \cdot 2 = 4$ and $-2 \cdot -2 = 4$. But there is no number that when multiplied by itself gives us -4 ! Therefore, we say that $\sqrt{-4}$ is imaginary!

2. Arithmetic Operations on Radical Expressions (1 of 2)

Addition and Subtraction:

Two or more radical expressions with the same radicand and index can be added and subtracted by adding and subtracting their “non-radical” factors.

For example, $\sqrt{17} - 20\sqrt{17} = (1 - 20)\sqrt{17} = -19\sqrt{17}$.

Note: There is an implied multiplication between the “non-radical” factors and the radical factors.

Also, in the case of $\sqrt{17}$ the “non-radical” factor is 1.

Arithmetic Operations on Radical Expressions (2 of 2)

Multiplication and Division:

Given two nonnegative real numbers a and b .

Product Rule: $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$

For example, $5\sqrt{2} \cdot 4\sqrt{3} = (5 \cdot 4)\sqrt{2 \cdot 3} = 20\sqrt{6}$.

Note: We separately multiplied the “non-radical” factors and then the radical factors using the Product Rule.

Quotient Rule $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$, ($b \neq 0$)

For example, $\frac{\sqrt{6}}{\sqrt{2}} = \sqrt{\frac{6}{2}} = \sqrt{3}$.

3. Convert Radical Expressions to Exponential Expressions and Vice Versa (1 of 3)

If a is a real number and $\frac{m}{n}$ is a positive rational number with $n \geq 2$ then

$$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m \quad \text{or} \quad a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

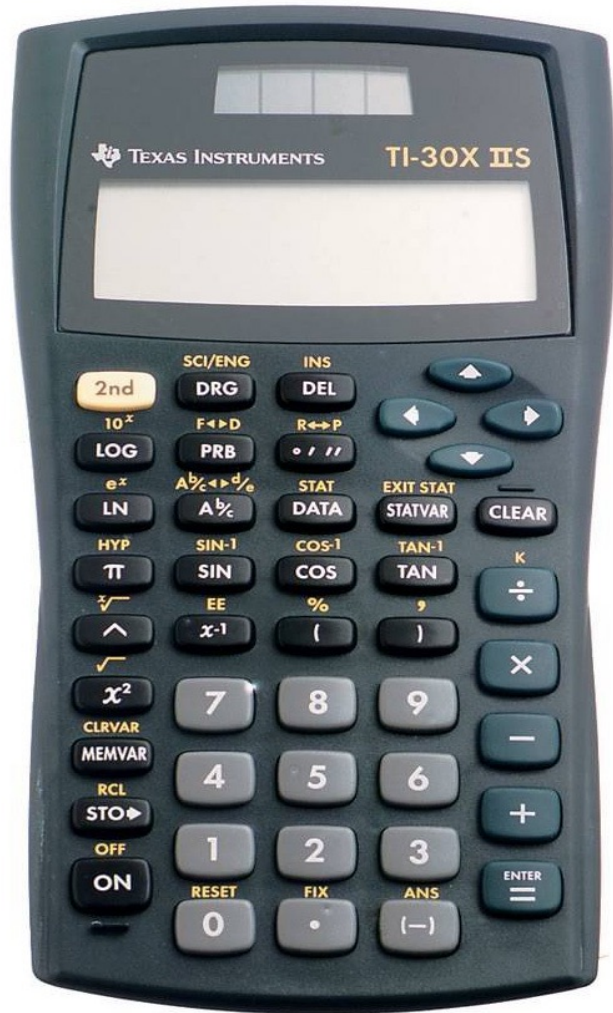
For example, $\sqrt[7]{x^3} = x^{\frac{3}{7}}$

or $\sqrt{x} = x^{\frac{1}{2}}$

Note, here the index n is 7 and the power m on x is 3!

Note, here the index n is 2 and the power m on x is 1!

Convert Radical Expressions to Exponential Expressions and Vice Versa (2 of 3)



Example 11:

Evaluate $27^{\frac{1}{3}}$ by hand and using a calculator.

By hand: $27^{\frac{1}{3}} = \sqrt[3]{27} = 3$ because $3(3)(3) = 3^3 = 27$.

TI-30X IIS Calculator:

- Type 27.
- Press the caret ^ button.
- Press the left parentheses button (.
- Type $1 \div 3$.
- Press the right parenthesis button).
- Press the ENTER button.

Note: You must place $\frac{1}{3}$ in parentheses otherwise the calculator will use *Order of Operations!*

Convert Radical Expressions to Exponential Expressions and Vice Versa (3 of 3)

Example 12:

Change $27^{\frac{4}{3}}$ to a radical expression and then evaluate without a calculator.

We know that $27^{\frac{4}{3}}$ can either be written as $(\sqrt[3]{27})^4$ or $\sqrt[3]{27^4}$.

$\sqrt[3]{27^4}$ is difficult to evaluate without a calculator because we have to find 27^4 .

However, $(\sqrt[3]{27})^4$ is manageable since we know that $\sqrt[3]{27} = 3$, then $3^4 = 81$.
See Example 21.