Concepts and Examples
Radical Equations in One Variable

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Learning Objectives

1. Memorize the definition of radical equations.
2. Solve radical equations.

1. Definition of Radical Equations

In mathematics we encounter infinitely many different equations, and they all have their own solution method. In this lesson, we will discuss equations containing at least one radical expression with index ***n*** and with a variable in its radicand.

Examples of radical equations:

$$\sqrt{x+3}-3=0$$

$$\sqrt[3]{2x-3}=-2$$

$$\sqrt{2x-1}+2=x$$

2. Solve Radical Equations

Strategy for Solving Radical Equations:

Step 1 - If necessary, arrange the terms so that the radical with index ***n*** is isolated on one side of the equal sign.

Example 1:

Solve $\sqrt{x+3}-3=0$.

We isolate the radical on one side of the equal sign to get $\sqrt{x+3}=3$.

Step 2 - Raise both sides of the equation to the ***nth*** power which eliminates the radical symbol.

Solve Radical Equations (2 of 10)

Example 1 continued:

The radical is of **index 2**. Therefore, we will raise both sides of the equation to the **2^nd^ power**. This eliminates the radical symbol.

$$(\sqrt{x+3})^2 = 3^2$$

and $\boldsymbol{x + 3 = 9}$

Step 3 - Solve the resulting equation. At this point we have “proposed” solutions.

Example 1 continued:

Given $x + 3 = 9$ then $x = 6$.

Solve Radical Equations (3 of 10)

Step 4 - If the index ***n*** in the original equation is EVEN, solutions found in Step 3 are sometimes false. Therefore, it is absolutely necessary to check all proposed solutions in the original equation to find true solutions.

NOTE: The solution check for equations with odd index is optional.

Example 1 continued:

The solution is ***x* = 6**. Since the index of the radical in the original equation is EVEN, we must check the proposed solution ensure that is is a true solution.

Given $\sqrt{x+3} - 3 = 0$ and ***x* = 6**, is the following statement true?

$$\sqrt{6+3} - 3 \stackrel{?}{=} 0$$

Solve Radical Equations (4 of 10)

Example 1 continued:

$$\sqrt{6+3} - 3 \stackrel{?}{=} 0$$

$$\sqrt{9} - 3 \stackrel{?}{=} 0$$

$$3 - 3 \stackrel{?}{=} 0$$

$$0 = 0$$

This is a true statement!

Since ***x* = 6** produces a true statement, we find that ***x* = 6** is a true solution.

Solve Radical Equations (5 of 10)

Example 2:

Solve $\sqrt[3]{2x-3} = -2$.

The radical is already isolated on one side of the equal sign.

The radical is of **index 3**. Therefore, we will raise both sides of the equation to the **3rd power**. This eliminates the radical symbol.

$$\left(\sqrt[3]{2x-3}\right)^3 = (-2)^3$$

and $\mathbf{2x - 3 = -8}$

Solve Radical Equations (6 of 10)

Example 2 continued:

If $\mathbf{2}\boldsymbol{x} - \mathbf{3} = -\mathbf{8}$ then $\mathbf{2}\boldsymbol{x} = -\mathbf{5}$
and $\boldsymbol{x} = -\frac{\mathbf{5}}{\mathbf{2}}$

The solution of the equation is $\boldsymbol{x} = -\frac{\mathbf{5}}{\mathbf{2}}$.

Since the index is ODD, the solution check is optional.

Solve Radical Equations (7 of 10)

Example 3:

Find all real solutions of $\sqrt{2x-1}+2=x$.

The radical must be isolated on one side of the equal sign.

$$\sqrt{2x-1}=x-2$$

The radical is of **index 2** . Therefore, we will raise both sides of the equation to the **2nd power**. This eliminates the radical symbol.

$$\left(\sqrt{2x-1}\right)^2=(x-2)^2$$

and $\mathbf{2x-1=(x-2)^2}$

Solve Radical Equations (8 of 10)

Example 3 continued:

Solve the equation from Step 2.

$\mathbf{2}x - \mathbf{1} = (x - \mathbf{2})(x - \mathbf{2})$ and $\mathbf{2}x - \mathbf{1} = x^2 - \mathbf{4}x + \mathbf{4}$

This is a quadratic equation. Let's move all terms to one side of the equal sign and combine like terms as follows:

$\mathbf{0} = x^2 - \mathbf{4}x + \mathbf{4} - 2x + \mathbf{1}$

and $x^2 - \mathbf{6}x + \mathbf{5} = \mathbf{0}$

This factors nicely into $(x - \mathbf{1})(x - \mathbf{5}) = \mathbf{0}$

We will use the *Zero Product Principle* and set each factor equal to 0.

$x - \mathbf{1} = \mathbf{0}$ and $x - \mathbf{5} = \mathbf{0}$

We find solutions, namely $x = \mathbf{1}$ and $x = \mathbf{5}$.

Solve Radical Equations (9 of 10)

Example 3 continued:

Since the index of the radical in the original equation is EVEN, we must check the proposed solutions in the original equation to ensure that they are true solutions.

Given $\sqrt{2x-1}+2=x$,

1. let ***x* = 1**, then

$$\sqrt{2(1)-1}+2 \stackrel{?}{=} 1$$

$$\sqrt{1}+2 \stackrel{?}{=} 1$$

$$1+2 \stackrel{?}{=} 1$$

This is a false statement!

Solve Radical Equations (10 of 10)

Example 3 continued:

2. Next, we let $\boldsymbol{x}$ **= 5**, then

$$\sqrt{2(5)-1}+2 \stackrel{?}{=} 5$$

$$\sqrt{9}+2 \stackrel{?}{=} 5$$

$$3+2 \stackrel{?}{=} 5$$

This is a true statement!

In summary, since $\boldsymbol{x}$ **= 1** produces a false statement but $\boldsymbol{x}$ **= 5** produces a true statement, we find that the solution of the equation to be only $\boldsymbol{x}$ **= 5**.