Concepts and Examples
Applications of Quadratic Functions

Based on power point presentations by Pearson Education, Inc.

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Learning Objectives

1. Find the minima and maxima of quadratic functions.
2. Find the minima and maxima given business formulas.

Quadratic functions are widely used in many fields. For example, in business we often use quadratic functions to represent revenue, profit, and cost of products sold or produced.

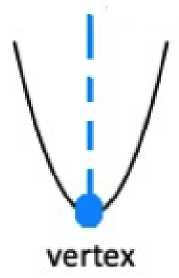
In this lesson, we will study minimum and maximum values of quadratic functions in general, and then we will look at some common quadratic functions used in business.

1. Minima and Maxima of Quadratic Functions (1 of 5)

Please note that the plural of minimum and maximum is minima and maxima!

Let's consider the quadratic function $f(x) = ax^2 + bx + c$.

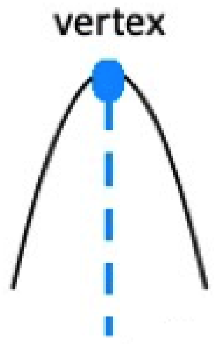
1. If $a > 0$, the graph of the quadratic function is a parabola open upward.



We say that the function has a **minimum** value at $f\left(-\frac{b}{2a}\right)$ which is the y-coordinate of the vertex.

We further state that the location of this minimum value is at $x = -\frac{b}{2a}$ which is the x-coordinate of the vertex.

Minima and Maxima of Quadratic Functions (2 of 5)



2. If a < 0, the graph of the quadratic function is a parabola open downward.

We say that the function has a **maximum** value at $f\left(-\frac{b}{2a}\right)$ which is the *y*-coordinate of the vertex.

We further state that the location of this maximum value is at $x = -\frac{b}{2a}$ which is the *x*-coordinate of the vertex.

Minima and Maxima of Quadratic Functions

Example 1:

The number of walleye (a type of fish) living in a pond on a fish farm depends on the temperature of the water.

The function modeling this dependency is given by $\boldsymbol{w = -3t^2 + 372t - 10639}$ where $\boldsymbol{w}$ is the number of walleye and $\boldsymbol{t}$ is the water temperature in Fahrenheit.

a. What must the temperature of the water be to maximize the number of walleye?

Note that the model is a quadratic function, and its graph is a parabola open down because the leading coefficient is negative. This means that this function has a maximum value.

Minima and Maxima of Quadratic Functions (4 of 5)

Example 1 continued:

We will find the value of ***t*** (temperature) at which the maximum number of walleye ***w*** occurs. That is, we use the formula for the vertex point.

Specifically, we will use $t = -\frac{b}{2a}$.

Given $\boldsymbol{w = -3t^2 + 372t - 10639}$ where $a = -3$ and $b = 372$, we find the following using a calculator as necessary:

$$t = -\frac{372}{2(-3)} = 62$$

This means that the temperature ***t*** of the water must be 62° F to maximize the walleye population.

Minima and Maxima of Quadratic Functions

Example 1 continued:

b. What is the maximum walleye population?

We just found that the maximum walleye population occurs when the temperature of the water is 62^{o} Fahrenheit.

Given the fish population function $\boldsymbol{w = -3t^2 + 372t - 10639}$, we will now find $\boldsymbol{w}$ when $\boldsymbol{t}$ equals 62. Do use a calculator!

$w = -3(62)^2 + 372(62) - 10639 = 839$

The maximum fish population is 839, and it is achieved when the temperature of the water is 62^{o} Fahrenheit.

2. Quadratic Functions in Business Applications (1 of 4)

A common use of quadratic functions in business is to minimize cost and to maximize profits. Following are some frequently used business functions:

The **Cost Function *C(x)*** with *x* being a quantity sold or produced.

It includes fixed costs (e.g., rent) and variable costs (e.g., utilities).

The **Price Function *p(x)***.

It represents the selling price with *x* being a quantity sold or produced.

The **Revenue Function** $R(x) = x \cdot p(x)$ with *x* being a quantity sold or produced. (Quantity sold/produced times the Price!)

The **Profit Function** $P(x) = R(x) - C(x)$ with *x* being a quantity sold or produced. (Revenue minus Cost!)

Quadratic Functions in Business Applications (2 of 4)

Example 2:

The MEGA company sells *mPhones*. They found that the cost function to sell x number of *mPhones* is $\mathbf{C(x) = -21x^2 + 540x + 20603}$.

The company further determined that if they sell x number of *mPhones* for the maximum price they can fetch, the revenue function will be $\mathbf{R(x) = -25x^2 + 1900x}$.

a. How many *mPhones* should the MEGA company sell to maximize profit? Use the profit function $P(x) = R(x) - C(x)$. Utilize a calculator as necessary!

$P(x) = -25x^2 + 1900x - (-21x^2 + 540x + 20603)$ (Revenue minus Cost!)

$P(x) = -25x^2 + 1900x + 21x^2 - 540x - 20603$

$\mathbf{P(x) = -4x^2 + 1360x - 20603}$

Note that the resulting profit function is quadratic. Therefore, its graph is a parabola which is open down because the leading coefficient is negative. This means that this profit function does have a maximum value.

Quadratic Functions in Business Applications (3 of 4)

Example 2 continued:

Now, we will find the number of *mPhones* at which the maximum profit P occurs. Specifically, we will use the formula for the vertex point of the quadratic function $P(x)$.

Given the $\mathbf{P(x) = -4x^2 + 1360x - 20603}$ where where $a = -4$ and $b = 1360$ and using $x = -\frac{b}{2a}$, we find the following using a calculator:

$$x = -\frac{1360}{2(-4)} = 170$$

This means that the profit is at a maximum when 170 *mPhones* are sold.

Please notice that selling more than 170 *mPhones* would actually result in a profit decrease!

Quadratic Functions in Business Applications

Example 2 continued:

b. What is the maximum profit?

The maximum profit occurs when 170 *mPhones* are sold. Given the profit function $\boldsymbol{P(x) = -4x^2 + 1360x - 20603}$, we must find $P(170)$. Do use a calculator!

$$\begin{aligned} P(170) &= -4(170)^2 + 1360(170) - 20603 \\ &= 94997 \end{aligned}$$

The MEGA company's maximum profit is \$94,997.