Concepts and Examples
Applications of Quadratic Functions

Based on power point presentations by Pearson Education, Inc.
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Learning Objectives

1. Find the minima and maxima of quadratic functions
2. Find the minima and maxima given business formulas.
3. Find the minima and maxima involving gravity.

Quadratic equations are widely used in science, business, and engineering. For example, we often use a quadratic equation to represent revenue, profit, and cost of products sold or produced. Quadratic equations are also used when gravity is involved, such as the path of a projectile or the shape of cables in a suspension bridge.

In this lesson, we will study minimum and maximum values of quadratic functions in general, and in business and science applications.

1. Minima and Maxima of Quadratic Functions (1 of 5)

Please note that the plural of minimum and maximum is minima and maxima!

Now, consider the quadratic function $f(x) = ax^2 + bx + c$.

1. If a > 0, the graph of the quadratic function is a parabola open upward.

   We say that the function has a **minimum** value at $f\left(-\frac{b}{2a}\right)$ which is the *y*-coordinate of the vertex.

   We further state that the location of this minimum value is at $x = -\frac{b}{2a}$ which is the *x*-coordinate of the vertex.

Minima and Maxima of Quadratic Functions

2. If a < 0, the graph of the quadratic function is a parabola open downward.

We say that the function has a **maximum** value at $f\left(-\frac{b}{2a}\right)$ which is the *y*-coordinate of the vertex.

We further state that the location of this maximum value is at $x=-\frac{b}{2a}$ which is the *x*-coordinate of the vertex.

Minima and Maxima of Quadratic Functions

Example 1:

The number of walleye (a type of fish) living in a small pond depends on the temperature of the water.

Let W be the number of walleye and t (in Fahrenheit) the water temperature. The function modeling this dependency is given by $\boldsymbol{W(t) = -3t^2 + 372t - 10639}$.

a. What is the temperature of the water that will maximize the number of walleye?

Note that the model is a quadratic function, and its graph is a parabola open down because the leading coefficient is – 3. This means that this function has a maximum value.

Minima and Maxima of Quadratic Functions (3 of 5)

Example 1 continued:

We will find the value of *t* (temperature) at which the maximum number of walleye *W* occurs. That is, we use the formula for the vertex point.

Specifically, we will use .

Given $W(t) = -3t^2 + 372t - 10639$ where a = – 3 and $b = 372$ and using $t = -\frac{b}{2a}$, we find the following:

$$t = -\frac{372}{2(-3)} = 62$$

This means that the temperature of the water must be 62° F to maximize the walleye population.

Minima and Maxima of Quadratic Functions

Example 1 continued:

b. What is the maximum walleye population?

To find the maximum walleye population, we will again use the formula for the vertex.
But this time we will use $W\left(-\frac{b}{2a}\right)$.

We just found that the maximum walleye population occurs when the temperature of the water is 62° Fahrenheit.

Given the fish population function $W(t) = -3t^2 + 372t - 10639$, we must find $W(62)$.

$W(62) = -3(62)^2 + 372(62) - 10639 = 839$

The maximum fish population is 839.

2. Quadratic Functions in Business Applications (1 of 4)

A common use of quadratic functions in business is to maximize profit which is the difference between the total revenue (money taken in) and the production costs (money spent). Following are some frequently used business functions:

The **Cost Function** ***C(x)*** with *x* being a quantity sold or produced.

It includes fixed costs (e.g. rent) and variable cost (e.g., utilities)

The **Price-Demand Function** ***p(x)***.

It represents the selling price with *x* being a quantity sold or produced.

The **Revenue Function** ***R(x) = x · p(x)*** with *x* being a quantity sold or produced.

The **Profit Function** ***P(x) = R(x) – C(x)*** with *x* being a quantity sold or produced.

Quadratic Functions in Business Applications

Example 2:

The Mega company sells mPhones. The cost function to manufacture x number of mPhones is $\boldsymbol{C(x) = -21x^2 + 54000x + 20603}$.

If the company sells x number of mPhones for the maximum price they can fetch, the revenue function will be $\boldsymbol{R(x) = -25x^2 + 190000x}$.

a. How many mPhones should the Mega company produce and sell to maximize profit where profit $P(x) = R(x) - C(x)$.

$$P(x) = -25x^2 + 190000x - (-21x^2 + 54000x + 20603)$$
$$P(x) = -25x^2 + 190000x + 21x^2 - 54000x - 20603$$
$$P(x) = -4x^2 + 136000x - 20603$$

Note that the model is a quadratic function, and its graph is a parabola open down because the leading coefficient is – 4. This means that this function has a maximum value.

Quadratic Functions in Business Applications (3 of 4)

Example 2 continued:

Now, we will find the value of *x* at which the maximum profit *P* occurs. That is, we use the formula for the vertex point of a quadratic function.

Given $P(x) = -4x^2 + 136000x - 20603$ where where $a = -4$ and $b = 136000$ and using $x = -\frac{b}{2a}$, we find the following:

$$x = -\frac{136000}{2(-4)} = 17000$$

This means that the profit is at a maximum when 17,000 mPhones are produced and sold.

Quadratic Functions in Business Applications

Example 2 continued:

b. What is the maximum profit?

The maximum profit occurs when 17000 mPhones are produced and sold. Given the profit function $P(x) = -4x^2 + 136000x - 20603$, we must find $P(17000)$.

$$\begin{aligned} P(17000) &= -4(17000)^2 + 136000(17000) - 20603 \\ &= 1155979397 \end{aligned}$$

The maximum profit is $1,155,979,397.

3. Quadratic Functions in Gravity Applications (1 of 7)

A major applications of quadratic function relates to what is called projectile motion. For our purposes, a projectile is any object that is thrown, shot, or dropped. Almost always, in this context, the object is initially moving directly up or straight down. If it starts by going up then, naturally, it will later be coming back down! This initial movement speed is the **velocity**.

In projectile motion, the coefficient on the squared term is $-\frac{1}{2}g$. The g stands for the constant of gravity (on Earth), which is −32 feet per second squared. The "minus" signs reflect the fact that Earth's gravity pulls us, and the object in question, downward.

The projectile-motion equation is $s(t) = -\frac{1}{2}gt^2 + v_0t + h_0$ where

g is the constant of gravity

v_0 is the initial velocity (that is, the velocity at time $t = 0$)

h_0 is the initial height of the object at the time of release

Quadratic Functions in Gravity Applications

Example 3:

A ball is thrown upward from the top of a building at a speed of 80 feet per second. The ball's height above ground can be modeled by the equation $h(t) = -16t^2 + 80t + 40$ (where t is in seconds).

a. What is the initial height of the ball?

We are going to take advantage of the fact that the ball has not yet been thrown. That is, we will let t equal 0 and solve for h.

$h(0) = -16(0)^2 + 80(0) + 40 = 40$

We find that the initial height of the ball is 40 feet. Incidentally, this means that building is 40 feet tall!

Quadratic Functions in Gravity Applications

Example 3 continued:

b. When does the ball reach maximum height?

Note that the model is a quadratic function, and its graph is a parabola open down because the leading coefficient is – 16. This means that this function has a maximum value.

We will find the value of *t* at which the ball reaches maximum height. That is, we use the formula for the vertex point. Specifically, we will use

$$t = -\frac{b}{2a}.$$

Quadratic Functions in Gravity Applications

Exampled 3 continued:

Given $h(t) = -16t^2 + 80t + 40$ where a = – 16 and $b = 80$ and using $t = -\frac{b}{2a}$, we find the following:

$$t = -\frac{80}{2(-16)} = \frac{80}{32}$$

$$= \frac{5}{2} = 2.5$$

This means that the ball reaches maximum height 2.5 seconds after it was thrown in the air.

Quadratic Functions in Gravity Applications (5 of 7)

Example 3 continued:

c. What is the maximum height of the ball? If necessary, round your answer to one decimal place.

To find the maximum hei $h\left(-\frac{b}{2a}\right)$ ball, we will again use the formula for the vertex.

But this time we will use .

We just found that the ball reaches maximum height after 2.5 seconds.

Given the height function $h(t) = -16t^2 + 80t + 40$, we must find $h(2.5)$.

$h(2.5) = -16(2.5)^2 + 80(2.5) + 40 = 140$

The maximum height of the ball is 140 feet.

Quadratic Functions in Gravity Applications

Example 3 continued:

d. When does the ball hit the ground? If necessary, round your answer to one decimal place.

We are going to take advantage of the fact that once the ball is back on the ground its height is 0. That is, we will let *h* equal 0 and solve for *t*.

Specifically, we are going to solve the quadratic equation $0 = -16t^2 + 80t + 40$. The best solution method given the rather large numbers is to use the Quadratic Formula.

Given $a = -16$ and $b = 80$ and $c = 40$ we get the following. We will be making liberal use of the calculator.

$$t = \frac{-80 \pm \sqrt{80^2 - 4(-16)(40)}}{2(-16)} = \frac{-80 \pm \sqrt{8960}}{-32}$$

Quadratic Functions in Gravity Applications (7 of 7)

Example 3 continued:

Again using the calculator and inputting the entire fraction using both the + and – in the formula, we get $t \cong 5.5$ and $t \cong -0.5$.

Since we are dealing with seconds, which are never negative, the only useful answer is $t \cong 5.5$.

In summary, we found that the ball hits the ground after approximately 5.5 seconds.