Concepts and Examples
Applications with Logarithmic Equations

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Learning Objectives

1. Calculate acidity and alkalinity.
2. Calculate sound intensity.
3. Calculate the magnitude of earthquakes.

Logarithms have been used for years and have reinforced various scientific discoveries over time. From early astronomical experiments to developing the first data storage device, concepts of logarithms are widely applied.

Considering the significance of this concept, in this lesson, we will cover some real-life applications of logarithms. Specifically, we will calculate acidity and alkalinity, sound intensity, and the magnitude of earthquakes.

NOTE: The formulas in this lesson were derived using mathematics beyond the scope of this course. They do NOT have to be memorized, but it is expected that you are able to use them.

1. Calculate Acidity and Alkalinity (1 of 3)

In chemistry, *pH* (potential or power of hydrogen) is a number that indicates acidity or alkalinity of a chemical solution in which the solvent is water. The *pH* scale normally runs from 0 to 14. A *pH* value of 7 is neutral. This is the *pH* of pure water. Values less than 7 are acidic, while those greater than 7 are alkaline.

The *pH* scale is calculated using the following formula:

> ***pH = – log [H⁺]***, where $[H^+]$ is the hydrogen ion concentration of a substance in *moles per liter*. The **mole** is a unit of measurement for an amount of substance in the International System of Units (SI).

Calculate Acidity and Alkalinity (2 of 3)

Example 1:

A medical technologist creates a substance with a *pH* of 7.48. Find the concentration of hydrogen ions $[H^+]$ in it. Round the answer keeping two nonzero decimal places.

Using the given value pH = 7.48 in the formula $\boldsymbol{pH = -\log [H^+]}$, we get

$7.48 = -\log [H^+]$

We now isolate the logarithm by dividing both sides by – 1.

$-7.48 = \log [H^+]$

Calculate Acidity and Alkalinity

Example 1 continued:

Now, we can solve for H^+ by changing from logarithmic form to exponential form.

$H^+ = 10^{-7.48} \cong 0.000000033$

We found that the hydrogen ion concentration is approximately 0.000000033 moles per liter.

2. Calculate Sound Intensity (1 of 6)

We measure sound intensity in units called decibels. Decibels (dB) are named in honor of Alexander Graham Bell, the inventor of both the telephone and the audiometer.

The sound intensity level is calculated using the following formula:

$L = 10 \log \frac{I}{I_0}$, where

I is the intensity of the sound in watts per square meter
$I_0 = 10^{-12}$ is the intensity of sound that is barely audible to the human ear
L is the sound intensity level measured in decibels.

Calculate Sound Intensity (2 of 6)

Example 2:

Human ears should not be exposed to more than 85 decibels for extended periods without protection. What sound intensity in watts per square meter does this represent? Round the answer to 6 decimal places.

We will use the formula for sound intensity $L = 10 \log \frac{I}{I_0}$.

Specifically, in the given problem, we must solve $85 = 10 \log \frac{I}{10^{-12}}$.

We now isolate the logarithm on one side of the equal sign by dividing both sides by 10 to get

$$8.5 = \log \frac{I}{10^{-12}}$$

Calculate Sound Intensity (3 of 6)

Example 2 continued:

Now, we can solve for I by changing from logarithmic form to exponential form.

$$10^{8.5} = \frac{I}{10^{-12}}$$

We find the following:

$I = 10^{-3.5} \cong 0.000316$

In summary, 85 decibels represents a sound intensity of 0.000316 watts per square meter.

Calculate Sound Intensity (4 of 6)

Example 3:

How many more times intense is the sound of normal conversation (60 dB) than whispered conversation (30 dB)?

We will use the model for sound intensity $L = 10 \log \frac{I}{I_0}$.

In the given problem, we must use the model for sound intensity both for normal conversation, lets call its intensity I_N, and for whispered conversation, let's call its intensity I_W.

$60 = 10 \log \frac{I_N}{I_0}$ and $30 = 10 \log \frac{I_W}{I_0}$

We now isolate both logarithms on one side of the equal sign by dividing both sides by 10.

Calculate Sound Intensity

Example 3 continued:

$$6 = \log \frac{I_N}{I_0} \quad \text{and} \quad 3 = \log \frac{I_W}{I_0}$$

Now, we can solve for I_N and I_W by changing from logarithmic form to exponential form.

$$10^6 = \frac{I_N}{I_0} \quad \text{and} \quad 10^3 = \frac{I_W}{I_0}$$

We are supposed to find how many more times intense the sound of normal conversation (60 dB) is than whispered conversation (30 dB).

Let’s write the equation $10^3\,x = 10^6$ where x is the intensity multiplier we are seeking. We will solve for x by dividing both sides by 10^3.

Calculate Sound Intensity (6 of 6)

Example 3 continued:

$$x = \frac{10^6}{10^3}$$

$$x = 10^{6-3}$$

$$x = 10^3 = 1000$$

In summary, we find that normal conversation at 60 dB is 1000 times as intense as whispered conversation at 30 dB.

3. Calculate the Magnitude of Earthquakes (1 of 6)

The Richter Scale indicates the magnitude of an earthquake by comparing the amplitude (height) of the seismic waves of the earthquake to those of a "magnitude 0 event", which was chosen to be a reading of 0.001 millimeters on a seismometer 100 kilometers from the earthquake's epicenter.

The Richter scale was developed by the seismologist Charles Francis Richter in collaboration with the seismologist Beno Gutenberg.

The Richter Scale was developed using the following formula:

$R = \log \frac{I}{I_0}$, where I is the seismometer reading in millimeters and $I_0 = 10^{-3}$

Calculate the Magnitude of Earthquakes (2 of 6)

Example 4:

Find the magnitude of an earthquake given a seismometer readying of 50,000 mm. Round the answer to one decimal place.

We will use the formula $R = \log \frac{I}{I_0}$ and replace I with 50,000 as follows:

$$R = \log \frac{50000}{10^{-3}} \cong 7.7$$

We find that a seismometer reading of 50,000 mm indicates a 7.7 magnitude earthquake.

Calculate the Magnitude of Earthquakes (3 of 6)

Example 5:

How many times more intense is an earthquake with magnitude 7.7 than one with magnitude 6.7?

We must use the formula $R = \log \frac{I}{I_0}$ for both given magnitudes.

Let's call one intensity $I_{7.7}$ and the other one $I_{6.7}$.

$7.7 = \log \frac{I_{7.7}}{I_0}$ and $6.7 = \log \frac{I_{6.7}}{I_0}$

Now, we can solve for $I_{7.7}$ and $I_{6.7}$ by changing from logarithmic form to exponential form.

$10^{7.7} = \frac{I_{7.7}}{I_0}$ and $10^{6.7} = \frac{I_{6.7}}{I_0}$

Calculate the Magnitude of Earthquakes

Example 5 continued:

Now, let's write the equation $10^{6.7}\,x = 10^{7.7}$ where x is the intensity multiplier we are seeking. We will solve for x by dividing both sides by $10^{6.7}$.

$$x = \frac{10^{7.7}}{10^{6.7}}$$

$$x = 10^{7.7-6.7}$$

$$x = 10^1 = 10$$

In summary, we find that the earthquake that registered 7.7 on the Richter scale is 10 times as intense as the one that measures 6.7.

Calculate the Magnitude of Earthquakes

Example 6:

An earthquake is measured with a wave amplitude 584 times as great as that of a "magnitude 0 event". What is the magnitude of this earthquake using the Richter Scale? Round the answer to the nearest tenth.

Given the information in the problem description, we can write $I = 584\ I_0$.

We will substitute I into the Richter Scale formula $R = \log \frac{I}{I_0}$ and then evaluate the logarithm.

$$R = \log \frac{584\ I_0}{I_0} = \log \frac{584\ \cancel{I_0}}{\cancel{I_0}}$$

Calculate the Magnitude of Earthquakes

Example 6 continued:

We find that $R = \log 584$ which is approximately equal to 2.8

In summary, we find that the earthquake with wave amplitude 584 times greater than that of a "magnitude 0 event" has a magnitude of 2.8 on the Richter Scale.