Concepts and Examples
Applications with Logarithmic Equations

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Learning Objectives

1. Calculate acidity and alkalinity.
2. Calculate the magnitude of earthquakes.
3. Calculate sound intensity.

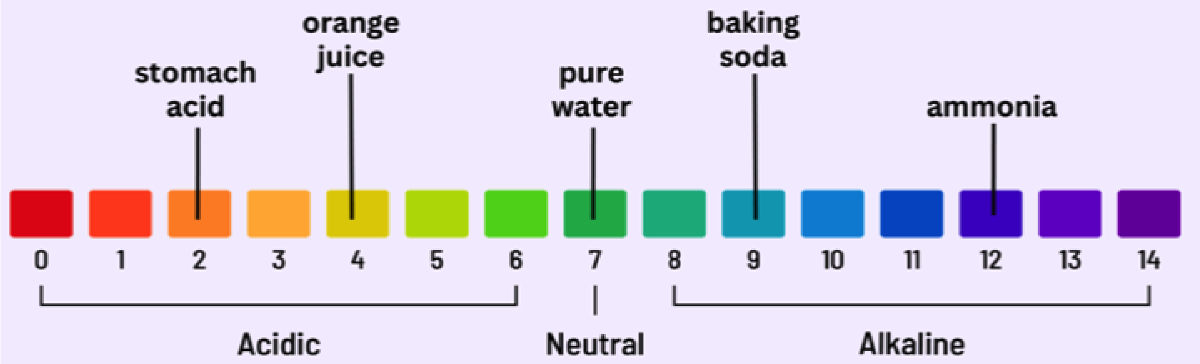
In this lesson, we will use formulas to model some real-life applications using logarithmic functions and equations.

NOTE: The formulas in this lesson were derived using mathematics beyond the scope of this course. They do NOT have to be memorized, but it is expected that you are able to use them.

1. Calculate Acidity and Alkalinity (1 of 3)

In chemistry, *pH* (the potential or power of Hydrogen) is a number that indicates acidity or alkalinity of a chemical solution in which the solvent is water.

The *pH* scale normally runs from 0 to 14. A *pH* value of 7 is neutral. This is the *pH* of pure water. Values less than 7 are considered acidic, while those greater than 7 are considered alkaline.



The *pH* scale scores are calculated using the following formula:

pH* = – log [*H*$^+$]**, where the logarithm base is 10 and ***H$^+$ is the hydrogen ion concentration of a substance in *moles per liter*. The **mole** is a unit of measurement for an amount of substance in the International System of Units (SI).

Calculate Acidity and Alkalinity (2 of 3)

Example 1:

A medical technologist creates a substance with a *pH* of 7.48. Find the concentration of hydrogen ions $[H^+]$ in it. Round the answer keeping two nonzero decimal places.

Using the given value *pH* = 7.48 in the formula $\boldsymbol{pH = -\log [H^+]}$, we get

$$\mathbf{7.48 = -\log [H^+]}$$

We now solve a logarithm equation by isolating the logarithm. That is, we divide both sides by – 1 to get the following:

$$\mathbf{-7.48 = \log [H^+]}$$

Calculate Acidity and Alkalinity (3 of 3)

Example 1 continued:

Now, we can solve for $\boldsymbol{H^+}$ by changing $\mathbf{-7.48 = \log [\boldsymbol{H^+}]}$ from logarithmic form to exponential form.

$$\boldsymbol{H^+} = \mathbf{10^{-7.4}}$$

Using a calculator, we get the following:

$$\boldsymbol{H^+} = \mathbf{10^{-7.48}} \cong \mathbf{0.\ 00\ 00\ 00\ 033}$$

We found that the hydrogen ion concentration in a substance with a *pH* of 7.48 is approximately 0. 00 00 00 033 moles per liter.

2. Calculate the Magnitude of Earthquakes (1 of 4)

The **Richter Scale** is used to assign a magnitude score to earthquakes by comparing the amplitude (height) of the seismic waves of the earthquake to those of a "magnitude 0 event", which was chosen to be a reading of 0.001 mm on a seismometer 100 km from the earthquake's epicenter.

The Richter Scale was developed by seismologist Charles Francis Richter in collaboration with the seismologist Beno Gutenberg.

The Richter Scale was developed using the following formula:

$R = \log \frac{I}{I_0}$, where the logarithm base is 10 and

R is the earthquake magnitude score
I is the seismometer reading in millimeters
I_0 is a reading of 0.001 = 10^{-3} millimeters (considered a "magnitude 0 event")

Calculate the Magnitude of Earthquakes

Example 2:

Find the magnitude score of an earthquake given a seismometer reading of 50,000 mm. Round the answer to one decimal place.

We will use the formula $R = \log \frac{I}{I_0}$ and replace I with 50,000 as follows:

$$R = \log \frac{50000}{10^{-3}}$$

Using a calculator, we get $\boldsymbol{R \cong 7.7}$

We find that a seismometer reading of 50,000 mm indicates a 7.7 magnitude earthquake.

Calculate the Magnitude of Earthquakes (3 of 4)

Example 3:

How many times more intense is an earthquake with a magnitude score of 7.7 as compared to one with a magnitude score of 6.7?

We must use the formula $R = \log \frac{I}{I_0}$ for both given magnitudes.

Let's call one intensity $I_{7.7}$ and the other one $I_{6.7}$.

$7.7 = \log \frac{I_{7.7}}{I_0}$ and $6.7 = \log \frac{I_{6.7}}{I_0}$

We now solve the logarithm equations by isolating their logarithms. That is, we solve for $I_{7.7}$ and $I_{6.7}$ by changing from logarithmic form to exponential form.

$10^{7.7} = \frac{I_{7.7}}{I_0}$ and $10^{6.7} = \frac{I_{6.7}}{I_0}$

Calculate the Magnitude of Earthquakes

Example 3 continued:

Now, let's write the equation $\mathbf{10^{6.7}}\ \boldsymbol{x} = \mathbf{10^{7.7}}$ where x is the intensity multiplier we are seeking. We will solve for x by dividing both sides by $10^{6.7}$.

$$x = \frac{10^{7.7}}{10^{6.7}}$$

$$x = 10^{7.7-6.7}$$

$$x = 10^1 = 10$$

In summary, we find that the earthquake that registered 7.7 on the Richter scale is 10 times as intense as the one that measures 6.7.

3. Calculate Sound Intensity (1 of 6)

We use an audiometer to measure sound intensity scores in units called **decibels**. Decibels (*dB*) are named in honor of Alexander Graham Bell, the inventor of both the telephone and the audiometer.

The sound intensity score is calculated using the following formula:

$L = 10 \log \frac{I}{I_0}$, where the logarithm base is 10 and

L is the sound intensity score measured in decibels
I is the intensity of the sound in watts per square meter
$I_0 = 10^{-12}$ is the intensity of sound that is barely audible to the human ear

Calculate Sound Intensity (2 of 6)

Example 4:

Human ears should not be exposed to more than 85 decibels for extended periods without protection. What sound intensity in watts per square meter does this represent? Round the answer to 6 decimal places.

We will use the model $L = 10 \log \frac{I}{I_0}$.

Specifically, in the given problem, we must solve $85 = 10 \log \frac{I}{10^{-12}}$.

We now solve the logarithmic equation by isolating the logarithm. That is, we divide both sides by 10 to get the following:

$$8.5 = \log \frac{I}{10^{-12}}$$

Calculate Sound Intensity (3 of 6)

Example 4 continued:

Now, we can solve for I by changing $8.5 = \log \frac{I}{10^{-12}}$ from logarithmic form to exponential form.

$$10^{8.5} = \frac{I}{10^{-12}}$$

Multiplying both sides by $\boldsymbol{10^{-12}}$ and then using the calculator, we find the following:

$$\boldsymbol{I = 10^{8.5 + (-12)} = 10^{-3.5} \cong 0.\ 00\ 03\ 16}$$

In summary, a sound intensity score of 85 decibels represents a sound intensity of 0. 00 03 16 watts per square meter.

Calculate Sound Intensity (4 of 6)

Example 5:

How many more times intense is the sound of normal conversation with score 60 dB as compared to whispered conversation with score 30 dB?

We will use the model $L = 10 \log \frac{I}{I_0}$ both for normal and whispered conversation. Let's use I_N for normal conversation and I_W for whispered conversation.

$60 = 10 \log \frac{I_N}{I_0}$ and $30 = 10 \log \frac{I_W}{I_0}$

We now solve both logarithmic equations by isolating their logarithms. That is, we divide both sides by 10.

Calculate Sound Intensity (5 of 6)

Example 5 continued:

$6 = log \frac{I_N}{I_0}$ and $3 = log \frac{I_W}{I_0}$

Now, we can solve for I_N and I_W by changing from logarithmic form to exponential form.

$10^6 = \frac{I_N}{I_0}$ and $10^3 = \frac{I_W}{I_0}$

We are supposed to find how many more times intense the sound of normal conversation (60 *dB*) is than whispered conversation (30 *dB*).

Let's write the equation $\mathbf{10^3 \, x = 10^6}$ where *x* is the intensity multiplier we are seeking. We will solve for *x* by dividing both sides by 10^3.

Calculate Sound Intensity (6 of 6)

Example 5 continued:

$$x = \frac{10^6}{10^3}$$

$$x = 10^{6-3}$$

$$x = 10^3 = 1000$$

In summary, we find that normal conversation at 60 *dB* is 1000 times as intense as whispered conversation at 30 *dB*.