Concepts and Examples
Introduction to Logarithms

Based on power point presentations by Pearson Education, Inc.

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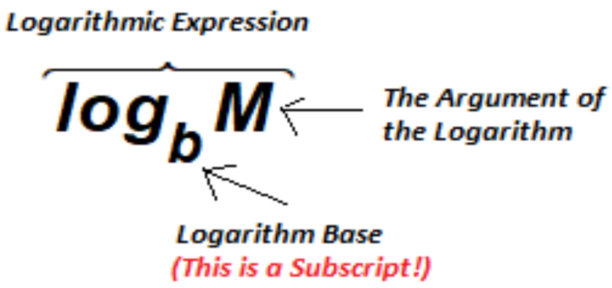
Learning Objectives

1. Change exponential expressions to logarithmic expressions and vice versa.
2. Evaluate logarithms using the calculator.
3. Memorize and apply the *Change-of-Base Property*.
4. Memorize and use basic logarithmic properties.

2. Definition of Logarithmic Expressions (1 of 3)

Just like radical expressions, logarithmic expressions or simply “logarithms” are also related to exponential expressions. Logarithms were developed by ancient mathematicians who were trying to find unknown exponents.

Please examine the general logarithmic expression carefully!



This is expressed as “log base b of M”.

This logarithmic expression asks us to find the power to which *b* (base) must be raised to get *M* (argument)!

Logarithmic expressions can have rational, irrational, or imaginary values.

Exponents Versus Logarithms (2 of 2)

Example 1:

a. Write $5^a = 125$ in logarithmic form.

$log_5\, 125 = a$

b. Write $25 = 5^2$ in logarithmic form.

$log_2\, 25 = 5$

c. Write $log_2\, 8 = 3$ in exponential form.

We get $8 = 2^3$.

Note that $log_2\, 8$ asks *"to what power must we raise 2 to get 8".* Obviously, the answer is 3.

2. Evaluate Logarithms (1 of 5)

The logarithm bases that occur most frequently in applications are **10** and ***e***. Please note that ***e*** is the famous number 2.718281828 …. containing infinitely many decimal places and often rounded to 2.72.

$\log_{10} x$

The 10 is usually left off and we just write **log *x*.**

$\log_e x$

a logarithm of base ***e*** is also referred to as the **natural logarithm**. The ***e*** is usually left off and we write **ln *x***.

Please note that the letter "l" in "ln" is a lower case L. It is neither the upper case letter "i" nor the number 1.

Evaluate Logarithms (2 of 5)

To evaluate most logarithms, we MUST have a calculator. Trying to evaluate logarithms by hand is often extremely cumbersome, but also beyond the scope of this course.

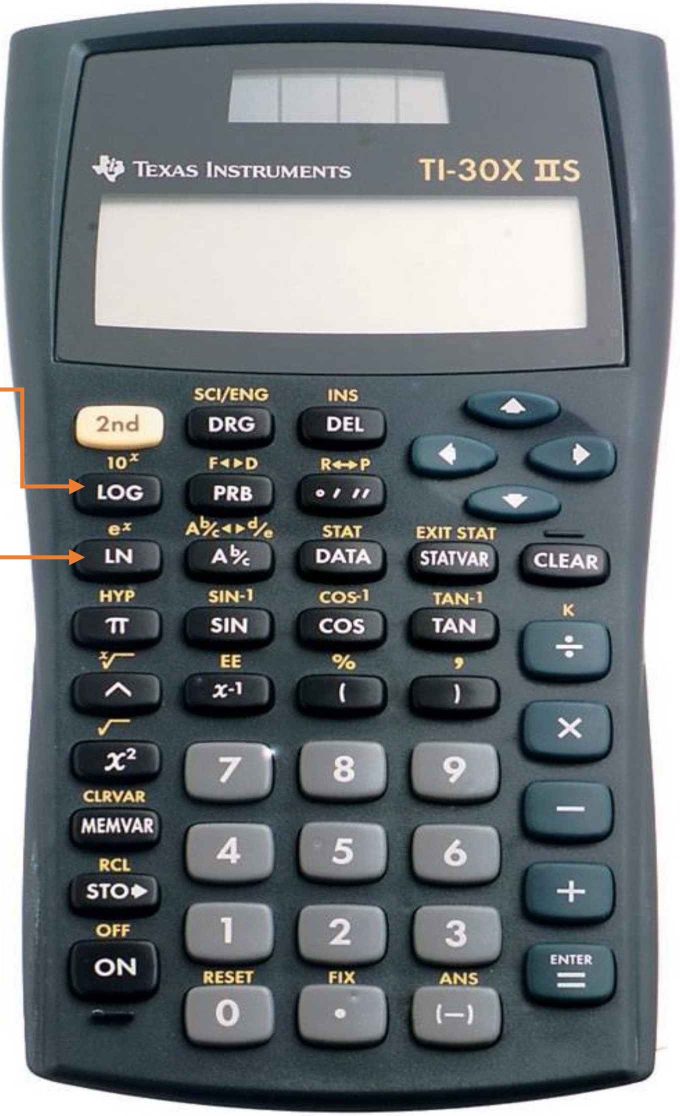
In a calculus course it is shown how to find the values of logarithms by hand and how the calculator is programmed to find values of logarithms.

All calculators have a LOG button (log base 10) and an LN button (log base ***e***). Some do have features that allow us to evaluate other bases. However, most of the time we must change other bases to base 10 or base ***e*** by using the *Change-of-Base Property*.

Evaluate Logarithms (3 of 5)

$\log_{10}$ button (LOG)

$\log_e$ button (LN)



Example 2:

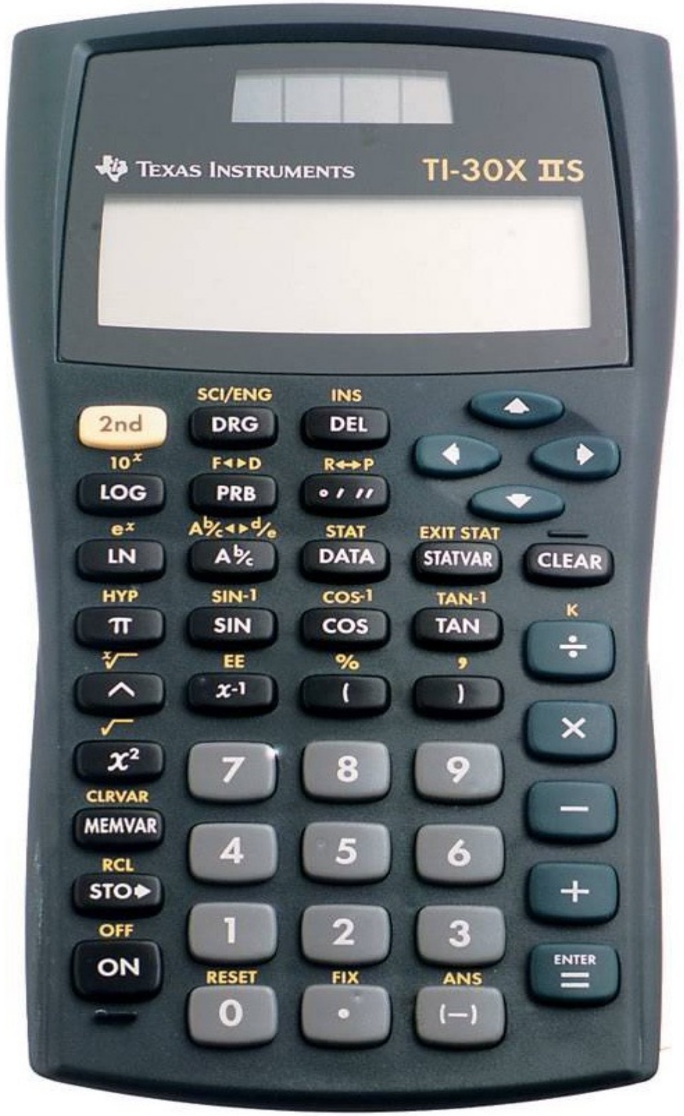
Evaluate $\mathbf{\log_{10} 100}$ with a calculator.

We will use the TI-30X IIS.

- Press the LOG button. You will see **log (**.
- Type **100**.
- Press the right parenthesis button **)** to "close" the set.
- Press the ENTER button.

The answer is **2** which is a rational number and more specifically an **integer**.

Evaluate Logarithms (4 of 5)



Example 3:

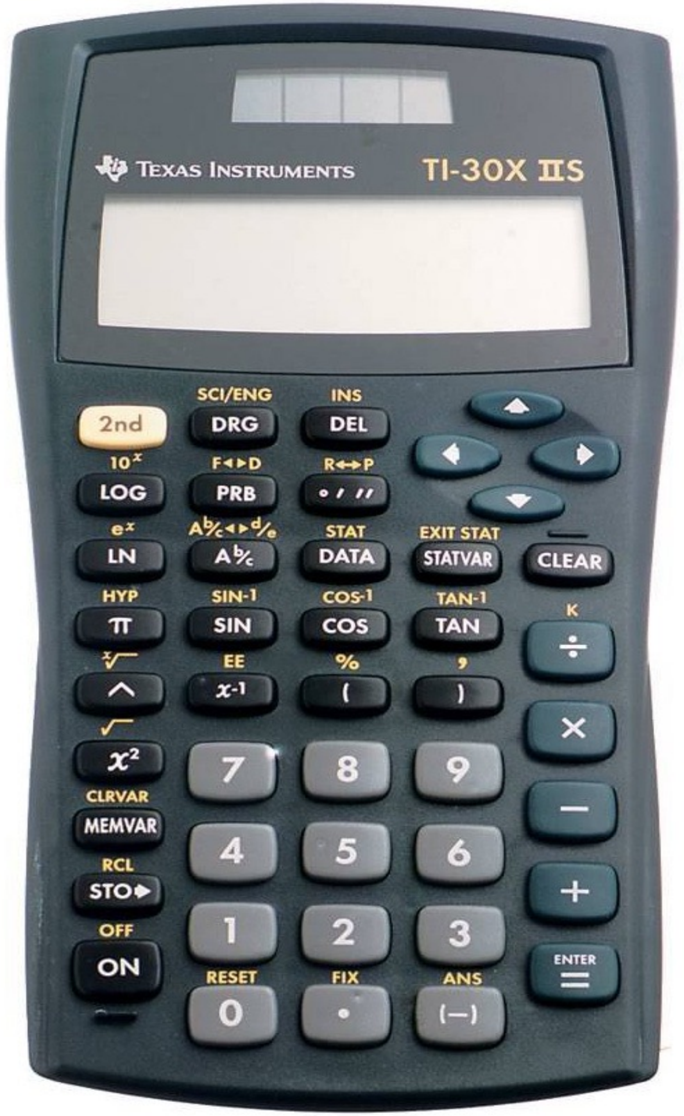
Evaluate **log 99** with a calculator.

We will use the TI-30X IIS.

- Press the LOG button because we are dealing with a log base 10. You will see **log (**.
- Type **99**.
- Press the right parenthesis button **)** to “close” the set.
- Press the ENTER button.

The answer is **1.995635195 …** which has infinitely many decimal places. This makes it an irrational number.

Evaluate Logarithms (5 of 5)



Example 4:

Evaluate **ln 2** with a calculator.

We will use the TI-30X IIS.

- Press the LN button because we are dealing with a log base ***e***. You will see **ln (**.
- Type **2**.
- Press the right parenthesis button **)** to “close” the set.
- Press the ENTER button.

The answer is **0.693147181 …** which has infinitely many decimal places. This makes it an irrational number.

3. The Change-of-Base Property (1 of 2)

The *Change-of-Base Property* gives us a way to change any logarithm to base 10 or base ***e*** so that we can then use the calculator to evaluate it.

It does not matter which one of the following versions of the property we use. Here, we will assume that base **b** is neither equal to 10 nor to ***e***.

Using base 10

$$\log_b M = \frac{\log_{10} M}{\log_{10} b}$$

Using base ***e***

$$\log_b M = \frac{\log_e M}{\log_e b}$$

The Change-of-Base Property (2 of 2)

Example 5:

Evaluate log_7 2506. Round the answer to two decimal places.

Let's use both versions of the *Change-of-Base Property* to illustrate that it does not matter which one we use. In either case, we must use a calculator.

$$log_7 2506 = \frac{log_{10} 2506}{log_{10} 7} \cong 4.02$$

$$log_7 2506 = \frac{log_e 2506}{log_e 7} \cong 4.02$$

4. Basic Logarithm Properties (1 of 2)

Basic logarithm properties allow us to evaluate certain logarithms without having to use a calculator.

a. $log_b\, b = 1$

Note that in exponential form this would be $b^1 = b$, which is always true.

Example: Evaluate $log_9\, 9$ without a calculator.

Since the logarithm base and the argument are equal, we conclude that $log_9\, 9 = 1$.

Basic Logarithm Properties (2 of 2)

b. $log_b\, 1 = 0$

Note that in exponential form this would be $b^0 = 1$, which is always true.

Example: Evaluate $log_8\, 1$ without a calculator

Since the argument is **1**, we conclude that $log_8\, 1 = 0$.

c. $log_b\, b^a = a$

Note that in exponential form this would be $b^a = b^a$, which is always true.

Example: Evaluate $log_5\, 125$ without a calculator.

Remember that $125 = 5^3$. Therefore, $log_5\, 125 = log_5\, 5^3$.

Since the logarithm base and the exponent base in the argument are equal, we conclude that $log_5\, 5^3 = 3$.