Concepts and Examples
Introduction to Functions

Based on power point presentations by Pearson Education, Inc.

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Learning Objectives

1. Define a function and use the Vertical Line Test.
2. Write function notation.
3. Use function notation.
4. Identify the domain and range of a function.

1. Definition of a Function (1 of 2)

In mathematics, some equations in two variables can be called **functions**. However, there are many that are not functions.

One easy way to determine if an equation in two variable is a function is to utilize the **Vertical Line Test**. It is used on the graphs of equations in two variables to prove or disprove that they are functions.

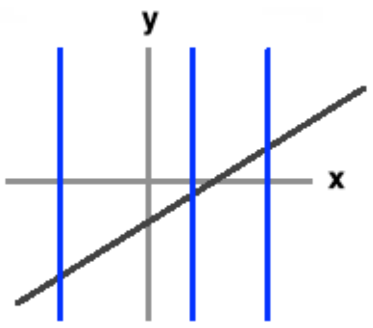
The *Vertical Line Test* states to following:

> If ALL vertical lines intersect with a graph of an equation in two variables exactly once, then the equation is a function.
>
> If just one vertical line intersects with a graph of an equation in two variables more than once, then the equation is NOT a function.

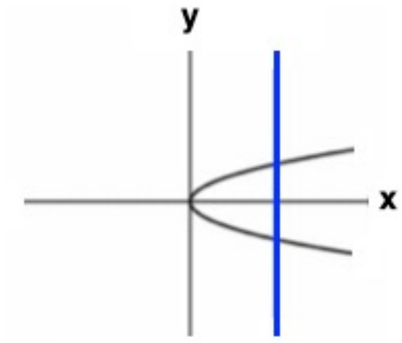
Definition of a Function (2 of 2)

Let's use the *Vertical Line Test* on the following two graphs of equations in two variables to determine if they are functions.

(a)



(b)



The graph in (a) is an increasing line. It should be apparent that all vertical lines drawn though it intersect with it only once. Therefore, its equation is a function.

NOTE: Since increasing (and decreasing) lines are strictly associated with linear equations, it should be obvious that linear equations are functions!

The graph in (b) is "bowl-shaped". We can immediately state that its equation is NOT a function because we readily found a vertical line that intersects with the graph twice.

2. Function Notation (1 of 2)

To indicate that an equation in two variables is a function, we can assign special notation to the dependent variable. Following are the steps how to go about this.

Step 1 – If necessary, we must isolate the dependent variable on one side of the equal sign giving it a coefficient of 1.

Example 1:

Given the function $4x - 2y + 6 = 0$, re-write it using function notation. <u>Assume *x* is the independent variable</u>. Therefore, we must "isolate" the variable *y*.

$4x - 2y + 6 = 0$
then $2y = 4x + 6$
and $y = 2x + 3$

Function Notation (2 of 2)

Step 2 - Pick a function name. In mathematics it is often a lower-case **f**. However, we might also see g, h, k, F, G or any other upper- or lower-case letter of the alphabet.

Example 1 continued:

Let's pick **f** as our function name.

Step 3 - Replace the dependent variable with the function name together with the independent variable in parentheses.

Example 1 continued:

$y = 2x + 3$ can be written as $f(x) = 2x + 3$.

IMPORTANT: The notation $f(x)$ is pronounced "f of x".

3. Use Function Notation (1 of 2)

Function notation can be used to efficiently ask for certain information pertaining to the function. For instance, we might want to find the value of the dependent variable given a value for the independent variable.

Example 2:

Given is a function **f** where x is the independent variable. Write the following sentence using function notation:

"Find the value of the dependent variable when $x = 4$."

We write: Find f(4).

Use Function Notation (2 of 2)

Example 3:

Given $f(x) = 3 - x$, find f(4).

f(4) is asking us to find the value of the dependent variable when $x = 4$.

All we have to do is replace the *x*-variables in the function with 4.

That is, $f(4) = 3 - (4)$ Note that 4 is now in the place of *x*.

$f(4) = -1$

We found that $f(4) = -1$, which means when $x = 4$ then the value of the dependent variable is -1 .

4. The Domain and Range of a Function (1 of 2)

Whenever we discuss functions, usually the names **domain** and **range** come up. The domain is especially important because it tells us what numbers we are allowed to pick for the independent variable.

Domain:

The numbers we assign to the independent variable. They CANNOT make the dependent variable imaginary or undefined.

NOTE: So far, we only discussed "real" numbers. Soon we will talk more about "imaginary" numbers. We already discussed "undefined" numbers in the "slope" lesson. Those are numbers that usually have a 0 in the denominator.

Range:

The numbers produced for the dependent variable when using domain values.

The Domain and Range of a Function (2 of 2)

Example 4:

Based on the table below, find the domain and range of the function **$g(x) = 3 - x$.**

x	**– 1**	**0**	**1**	**2**	**3**
$g(x) = 3 - x$	**$3-(-1)=4$**	**$3-(0)=3$**	**$3-(1)=2$**	**$3-(2)=1$**	**$3-(3)=0$**

Domain values: – 1, 0, 1, 2, 3

Range values: 4, 3, 2, 1, 0