Concepts and Examples
Introduction to Algebra

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Learning Objectives

1. Use the vocabulary of algebra.
2. Use the Order of Operations.
3. Use the Distributive Property.

What is Algebra?

Just like arithmetic, **algebra** is another important branch of mathematics. Arithmetic investigates how numbers are combined and transformed using mathematical operations like addition, subtraction, multiplication, and division.

Algebra relies on the same operations while allowing "unknown" numbers. We call these "unknown" numbers **variables**. They consist of letters of the alphabet that take on different values in different situations. In mathematics, we most often use the letters ***x*** and ***y***. However, we can use any other letter.

For example, in arithmetic we might state $7 + 2 = 9$. In algebra, on the other hand, we might state $x + 2 = 9$. (It should be quite obvious that the variable x equals 7.)

Why Study Algebra?

Algebra is an essential tool that has real-life uses and plays a significant role in different fields such as science, engineering, economics, and statistics. Furthermore, by learning basic algebra, we develop critical thinking and problem-solving skills as well as logical reasoning. All three skills are essential for success in any field and for making informed decisions in everyday life.

1. Some Vocabulary of Algebra

Mathematical Expression: A combination of variables and/or numbers linked by addition, subtraction, multiplication, and division.

For example, $7x + 2y + 8$.

NOTE: There is an implied multiplication sign between the numbers and the variables. $7x$ actually means $7 \cdot x$ and $2y$ means $2 \cdot y$. The multiplication sign is usually not written.

Equation: A mathematical expression that is equal to another mathematical expression. In algebra we use the = (equal) sign.

For example, $7x + 2y + 8 = 5x - 3$.

Some Vocabulary of Algebra (2 of 10)

Term: Those parts of a mathematical expression separated by addition or subtraction.

For example, in $7x + 2y + 8$, the terms are $7x$, $2y$, and 8.

Like Terms: Terms that have the exact same variables.

For example, in $5x – 3y + 2x$, the like terms are $5x$ and $2x$.

Constant: A term that consists of just a number.

For example, in $7x + 2y + 8$, the constant is 8.

Some Vocabulary of Algebra

Coefficient: Any number in front of a variable.

For example, in $3x + y + 9$, the coefficients are 3 and 1.

NOTE: In mathematics, we usually do not write a coefficient of 1. That is, when we don't see a coefficient, it is assumed to be 1!

Grouping Symbols: Grouping symbols are used to indicate that a particular collection of numbers and mathematical operations are to be grouped together and considered as one number.

The most common grouping symbols used in algebra are parentheses ( ), brackets [ ], braces { }, and the fraction bar. Incidentally, the singular of parentheses is one parenthesis.

NOTE: In higher mathematics we usually express multiplication using parentheses ( ) and not the multiplication symbols $\cdot$ or $\times$. That is, instead of $8 \cdot 2$ or 8×2 we write $8(2)$, which is pronounced "8 times 2".

Some Vocabulary of Algebra

Combining Like Terms: The act of simplifying a mathematical expression by adding/subtracting any constants and the coefficients of any like terms.

For example, given $5x - 3y + 2x$ with like terms $5x$ and $2x$, we add their coefficients 5 and 2 to get $7x$. We end up with $7x - 3y$.

Note that we cannot subtract 3 from 7 because they are not coefficients of like terms.

Evaluating a Mathematical Expression: Find the value of the expression when the variables are replaced with numbers.

Some Vocabulary of Algebra

Example 1:

Find the value of the expression $7x + 9y + 8$ when $x = 1$ and $y = 2$.

$7(1) + 9(2) + 8$ then $7 + 18 + 8$ and the value of this is 33.

NOTE: There is implied multiplication between the numbers and the variables. When a variable is replaced by a number, we use parentheses ( ) to indicate multiplication.

Some Vocabulary of Algebra

Conjugate: Given an expression with exactly two terms being added or subtracted, then its conjugate is an expression with the same terms but the operation changes to the opposite one.

For example, given $-5x - 9$, its conjugate is $-5x + 9$.

Simplify: The word "simplify" takes on many meanings in mathematics. The word could mean to add, subtract, multiply, divide, etc. Usually, we must figure out its meaning from the make-up of the given mathematical expression.

For example, “simplify 2 + 3” means the same as “add 2 + 3”.

Some Vocabulary of Algebra

Example 2:

Simplify $xy + 3 + 2x + 6xy - 3x + 9y - 6$.

Here "simplify" means to add and subtract. However, we can only add/subtract like terms. Let's give the individual like terms different colors.

$xy + 3 + 2x + 6xy - 3x + 9y - 6$

Notice that we can combine *xy*-terms and *x*-terms by adding/subtracting their coefficients. We can also add/subtract constants.

Let's write this as $(1 + 6)xy + (2 - 3)x + 9y + 3 - 6$

NOTE: There is an implied coefficient of 1 in front of *xy*!

Some Vocabulary of Algebra

Example 2 continued:

Now we must add some integers. Namely, (2 – 3) for the *x*-terms and + 3 – 6 for the constants. We will use the "gambling gains/losses" method. Here, in both cases, our "gains" (+) are less than our "losses" (–). Therefore, we still end up with losses.

That is, 2 – 3 = – 1 and 3 – 6 = – 3

In summary, we find that $(1 + 6)xy + (2 - 3)x + 9y + 3 - 6$ can be simplified to

$7xy - x + 9y - 3$.

NOTE: In mathematics, we usually do not write a coefficient of 1. This is why the second term is $-x$ and not $-1x$!!!

2. Order of Operations (1 of 2)

Say, you are asked to compute $48 - 20 \div 2 \cdot 5 + 9$. Would you start by subtracting 20 from 48 and then divide this result by 2 and so on? Actually, that's not what you would do.

In mathematics, we have an **Order of Operations**. It consists of rules that state the sequence in which multiple operations should be solved in a mathematical expression.

In this lesson, we will discuss the order given grouping symbols, addition, subtraction, multiplication, and division.

1. Grouping symbols are evaluated first.
2. Multiplication and division are done next, specifically in the order in which they occur, working from left to right.
3. Addition and Subtraction are done last, again in the order in which they occur, working from left to right.

Order of Operations (2 of 2)

Example 3:

Evaluate 7 – 20 ÷ 2 · (2 + 1) + 93.

Given 7 – 20 ÷ 2 · (2 + 1) + 93, we evaluate the parentheses first.

then 7 – 20 ÷ 2 · 3 + 93. Next, we evaluate division:

7 – 10 · 3 + 93. Next, we evaluate multiplication:

7 – 30 + 93. Next, we evaluate subtraction:

– 23 + 93. Next, we evaluate addition:

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3. The Distributive Property (1 of 5)

Say, you are asked to evaluate 3(5 + 2x). How would you go about doing this?

NOTE: There is implied multiplication between 3 and the parentheses ( ).

For this calculation, mathematicians use the **Distributive Property**. It directs how to multiply a sum or difference of two or more numbers within parentheses by a number outside the parentheses.

Specifically, it states that we must multiply each number in the parentheses individually by the number outside the parentheses and then add the products.

For example, $3(5 + 2x) = 3(5) + 3(2x)$.

NOTE: There is an implied multiplication between 3(5) and 3(2x).

The Distributive Property (2 of 5)

Example 4:

Evaluate $3(5 + 2x)$.

Using the *Distributive Property*, we get $3(5) + 3(2x)$.

It is obvious that $3(5)$ equals 15. But what about $3(2x)$?

The multiplication rule is that we multiply numbers with numbers and variables with variables. That is, $3(2x) = (3 \cdot 2) \cdot x = 6 \cdot x = 6x$.

In summary, we find $3(5 + 2x)$ can be simplified to $15 + 6x$.

The Distributive Property (3 of 5)

Example 5:

Simplify $-5(6k + 1)$.

We are going to use the *Distributive Property* to simplify.

$$\begin{aligned} -5(6k+1) &= -5(6k) + (-5)(1) \\ &= -30k + (-5) \\ &= -30k - 5 \end{aligned}$$

Remember, a negative number times a positive number results in a product that is negative!

The Distributive Property (4 of 5)

Example 6:

Simplify $-(3x + 1)$.

We are going to use the *Distributive Property* to simplify.

NOTE: There is actually – 1 in front of the parentheses! Remember, we usually do not write the coefficient of 1.

$$-(3x + 1) = (-1)(3x) + (-1)(1)$$
$$= -3x + (-1)$$
$$= -3x - 1$$

Remember, a negative number times a positive number results in a product that is negative!

The Distributive Property (5 of 5)

Example 7:

Simplify $-2(-4-7x+9y)$.

We are going to use the *Distributive Property* to simplify. Please note that now we have three terms in the parentheses. However, this will not be an issue.

According the the *Distributive Property*, we multiply the number on the outside of the parentheses with EVERY term in the parentheses.

$$\begin{aligned} -2(-4-7x+9y) &= -2(-4)+(-2)(-7x)+(-2)(9y) \\ &= 8+14x+(-18y) \\ &= 8+14x-18y \end{aligned}$$