Concepts and Examples
Factoring

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Learning Objectives

1. Memorize the definition of factoring.
2. Factor out common factors.
3. Carry out factoring by grouping.
4. Carry out factory on expressions of the form $ax^2 + bx + c$.
5. Carry out factoring on expressions of the form $ax^2 + bx + c$ where $a = 1$.
6. Carry out factoring using the *Difference of Squares Formula*.

1. Introduction to Factoring

Factors are numbers that divide into other numbers without leaving a remainder.

For example, 2 and 3 divide into 6 without a remainder, but 4 and 5 do not. Therefore, only 2 and 3 are factors of 6.

Factoring is the process of writing a mathematical expression as a product of factors. Be aware that this cannot always be done!

For example, we will soon find that $6x^2 - 11x - 10$ is "factorable". That is, we can write it as a product of factors, namely $(2x - 5)(3x + 2)$.

We usually try several factoring method to find out whether a mathematical expression is *factorable*. In this lesson we will learn how to "factor out" a common factor; factor by grouping; factor expressions of the form $ax^2 + bx + c$; and factor using the *Difference of Squares Formula*.

2. Factor Out Common Factors (1 of 3)

We can write a mathematical expression as a product of factors by simply trying to factor out a **common factor** which is a number that divides into every term of the expression without a reminder. We place this number in front of a set of parentheses containing the "reduced" terms.

For example, given $8x + 12$, we see that the number 2 divides into 8 and 12 without a remainder. Therefore, we can write the expression as the product $2(4x + 6)$. Note the "reduced" terms are $4x$ and 6.

Often, we are asked to factor out **the greatest factor** that every term of an expression has in common.

For example, given $8x + 12$ again, we see that the number 4 also divides into 8 and 12 without a remainder. Therefore, we can write the expression as the product $4(2x + 3)$. We see that 4 is the greatest factor that 8 and 12 have in common because 2 and 3 no longer have a factor in common.

Factor Out Common Factors (2 of 3)

Example 1:

Write $4x^2 - 8x + 12$ as a product of factors by factoring out the greatest common factor.

We notice that 2 and 4 both divide into 4 and – 8 and 12 without a remainder.

Obviously, the greatest common factor is 4. We we divide it out of 4 and – 8 and 12. We place 4 in front of parentheses containing the following:

$4(x^2 - 2x + 3)$

> NOTE: You can check your work by using the *Distributive Property* to convince yourself that you end up with $4x^2 - 8x + 12$ again.

Factor Out Common Factors

Example 2:

Write $6x^3 + 12x^2$ as a product of factors by factoring out the greatest common factor.

We notice that 2, 3, and 6 divide into of 6 and 12 without a remainder. Also, x and x^2 divide into x^3 and x^2 without a remainder.

The largest numeric factor is 6 and the largest variable factor is x^2. Therefore, we can divide $6x^2$ out of of $6x^3$ and $12x^2$. We place $6x^2$ in front of parentheses containing the following:

$6x^2(x + 2)$

> NOTE: You can check your work by using the *Distributive Property* to convince yourself that you end up with $6x^3 + 12x^2$ again.

3. Carry out Factoring by Grouping (1 of 6)

When a mathematical expression contains four terms, we can try to **factor by grouping** to write the expression as a product of factors. Specifically, we can do the following:

Step 1 - Place two term into a ”group” by enclosing them in parentheses and do the same for the other two terms.

Example 3:

Write $2x^3 + 3x^2 + 4x + 6$ as a product of factors.

The easiest grouping is one where we group the first two terms and the last two terms, such as $(2x^2 + 3x^2) + (4x + 6)$.

NOTE: This is a good first try. Sometimes we may have to rearrange the terms before we group them!

Carry out Factoring by Grouping

Step 2 - Factor out the greatest common factor from each group to get the sum or difference of two products.

Example 3 continued:

Given $(2x^3 + 3x^2) + (4x + 6)$, we will factor out the greatest common factor from each group. We end up with a sum of the following two products:

$x^2 (2x + 3) + 2 (2x + 3)$

Carry out Factoring by Grouping

Step 3 - If possible, factor the common factor out of the two products.

Example 3 continued:

Given x^2 $(2x + 3)$ + 2 $(2x + 3)$, we see that the two products have a factor in common, namely the binomial $(2x + 3)$.

This is the goal of factoring by grouping! If we cannot find a common factor, then factoring by grouping will have failed and we will have to use other methods.

Of course, we might rearrange the terms into two different groups before you discard factoring by grouping entirely.

Carry out Factoring by Grouping

Example 3 continued:

We will now factor $(2x + 3)$ out of each product to get the following:

$(2x + 3)(x^2 + 2)$

This is $2x^3 + 3x^2 + 4x + 6$ written as a product of factors.

Carry out Factoring by Grouping (5 of 6)

Example 4:

Write $6x^3 + 12x^2 + 5x + 10$ as a product of factors using factoring by grouping.

Since we have an expression with four terms, let's place the first and second term into a "group" by enclosing them in parentheses. Do the same for the third and fourth term.

$(6x^3 + 12x^2) + (5x + 10)$ Then we factor out the greatest common factor from each group.

$6x^2\,(x + 2) + 5(x + 2)$

We see that we end up with the same factor in each product. This is the goal of factoring by grouping!

Carry out Factoring by Grouping (6 of 6)

Example 4 continued:

We will now factor the common factor $(x + 2)$ out of each term as follows:

$(x + 2)(6x^2 + 5)$

This is $6x^3 + 12x^2 + 5x + 10$ written as a product of factors.

4. Carry out Factoring on Expressions of the Form $ax^2 + bx + c$

(1 of 10)

Some mathematical expressions of the form $\boldsymbol{ax^2 + bx + c}$ can be written as a product of factors, but not all of them. Following are the steps.

Step 1 - Given $ax^2 + bx + c$, find all pairs of integers whose product is ac.

Example 5:

Write $6x^2 - 11x - 10$ as a product of factors.

We note that $a = 6$, $b = -11$, and $c = -10$. Then $\boldsymbol{ac = 6(-10) = -60}$.
Now we will find all pairs of integers whose product is -60.

$-60 = (-1)(60)$	$-60 = (-2)(30)$	$-60 = (-3)(20)$
$-60 = (-4)(15)$	$-60 = (-5)(12)$	$-60 = (-6)(10)$
$-60 = (1)(-60)$	$-60 = (2)(-30)$	$-60 = (3)(-20)$
$-60 = (4)(-15)$	$-60 = (5)(-12)$	$-60 = (6)(-10)$

Carry out Factoring on Expressions of the Form $ax^2 + bx + c$

Step 2 - If possible, find one pair of integers from Step 1 whose sum equals b. Replace the term bx with a sum or difference of the appropriate pair of integers. NOTE: If this is not possible, factoring will have failed.

Example 5 continued:

We notice that the pair 4 and – 15 has a sum of 4 + (– 15) = – 11, which is the value of $\boldsymbol{b}$. Therefore, we replace the term $\boldsymbol{bx}$ in $6x^2 - 11x - 10$ as follows:

$$6x^2 - 15x + 4x - 10$$

Carry out Factoring on Expressions of the Form $ax^2 + bx + c$

Step 3 - Use factoring by grouping on the final equation in Step 2.

Example 5 continued:

Let's place the first two terms of $6x^2 - 15x + 4x - 10$ into a "group" by enclosing them in parentheses. We do the same for the last two terms.

$$(6x^2 - 15x) + (4x - 10)$$

We will now factor out the greatest common factor from each group. We end up with a sum of the following two products:

$$3x(2x - 5) + 2(2x - 5)$$

Carry out Factoring on Expressions of the Form $ax^2 + bx + c$

Example 5 continued

We see that the products have a factor in common, namely, $(2x - 5)$. We will now factor it out of each product to get the following:

$(2x - 5)(3x + 2)$

In summary, we found that we can write $6x^2 + 11x - 10$ as a product of factors, namely $(2x - 5)(3x + 2)$.

Carry out Factoring on Expressions of the Form $ax^2 + bx + c$

Example 6:

Write $2x^2 + 7x + 6$ as a product of factors.

We are dealing with an expression of the form $\boldsymbol{ax^2 + bx + c}$. We note that $a = 2$, $b = 7$, and $c = 6$. Then $\boldsymbol{ac = 2(6) = 12}$.

Now we will find all pairs of integers whose product is 12.

$12 = (1)(12)$	$12 = (2)(6)$	$12 = (3)(4)$
$12 = (-1)(-12)$	$12 = (-2)(-6)$	$12 = (-3)(-4)$

We notice that 3 and 4 have a sum of $b = 7$.

Carry out Factoring on Expressions of the Form $ax^2 + bx + c$

Example 6 continued:

Now, we replace the middle term $7x$ (in $2x^2 + 7x + 6$) as follows:

$2x^2 + 3x + 4x + 6$

Since we have an expression with four terms, let's place the first and second term into a "group" by enclosing them in parentheses. Do the same for the third and fourth term.

$(2x^2 + 3x) + (4x + 6)$

Then we factor out the greatest common factor from each group.

$x\,(2x + 3) + 2\,(2x + 3)$

Carry out Factoring on Expressions of the Form $ax^2 + bx + c$

Example 6 continued:

We see that we end up with the same factor, namely $(2x + 3)$, in each product. This is the goal of factoring by grouping! We will now factor it out as follows:

$(2x + 3)(x + 2)$

In summary, we found that we can write $2x^2 + 7x + 6$ as a product of factors, namely $(2x + 3)(x + 2)$.

Carry out Factoring on Expressions of the Form $ax^2 + bx + c$

Example 7:

Write $x^2 + 5x - 6$ as a product of factors.

We are dealing with an expression of the form $\boldsymbol{ax^2 + bx + c}$. We note that $a = 1$, $b = 5$, and $c = -6$. Then $\boldsymbol{ac = 1(-6) = -6}$.

Now we will find all pairs of integers whose product is -6.

$-6 = (-1)(6)$ $\quad$ $-6 = (-2)(3)$ $\quad$ $-6 = (1)(-6)$ $\quad$ $-6 = (2)(-3)$

We notice that -1 and 6 have a sum of $b = 5$.

Now, we replace the middle term $5x$ (in $x^2 + 5x - 6$) as follows:

$x^2 - 1x + 6x - 6$

Carry out Factoring on Expressions of the Form $ax^2 + bx + c$

Example 7 continued:

Since we have an expression with four terms, let’s place the first and second term into a ”group” by enclosing them in parentheses. Do the same for the third and fourth term.

$(x^2 - 1x) + (6x - 6)$

Then we factor out the greatest common factor from each group.

$x(x - 1) + 6(x - 1)$

Carry out Factoring on Expressions of the Form $ax^2 + bx + c$

Example 7 continued:

We see that we end up with the same factor, namely $(x - 1)$ in each product. This is the goal of factoring by grouping! We will now factor it out as follows:

$(x - 1)(x + 6)$

In summary, we found that we can write $x^2 + 5x - 6$ as a product of factors, namely $(x - 1)(x + 6)$.

5. Carry out Factoring on Expressions of the Form $ax^2 + bx + c$ where $a = 1$

Given $ax^2 + bx + c$ where $a = 1$, we can try to use a **short-cut method** to write it as a product of factors. Following are the steps.

Step 1 – Given $x^2 + bx + c$, find all pairs of integers whose product is c.

Example 8:

Write $x^2 + 5x - 6$ as a product of factors.

We note that $a = 1$, $b = 5$ and $c = -6$. We will find all pairs of integers whose product is -6.

$-6 = (-1)(+6)$ $\quad$ $-6 = (-2)(+3)$ $\quad$ $-6 = (1)(-6)$ $\quad$ $-6 = (2)(-3)$

Carry out Factoring on Expressions of the Form $ax^2 + bx + c$ where $a = 1$ (2 of 7)

Step 2 – If possible, find one pair from Step 1 whose sum equals *b*. NOTE: If this is not possible, factoring will have failed.

Example 8 continued:

We notice that the pair – 1 and 6 has a sum of $-1 + (+6) = 5$, which is the value of *b*.

Step 3 – Create the template $(x \qquad)(x \qquad)$ and use the numbers from Step 2 as the second terms.

Example 8 continued:

Using – 1 and 6, we can then write $(x - 1)(x + 6)$.

We found that we can write $x^2 + 5x - 6$ as a product of factors, namely $(x - 1)(x + 6)$.

Carry out Factoring on Expressions of the Form $ax^2 + bx + c$ where $a = 1$ (3 of 7)

Example 9:

Write $x^2 - 5x + 6$ as a product of factors.

We are dealing with an expression of the form $\boldsymbol{ax^2 + bx + c}$ where $\boldsymbol{a = 1}$. Therefore, we will use a “short-cut” method.

We note that $a = 1$, $b = 5$, and $c = -6$. Now we will find all pairs of integers whose product is -6.

$-6 = (-1)(6)$ $-6 = (-2)(3)$ $-6 = (1)(-6)$ $-6 = (2)(-3)$

We notice that -2 and -3 have a sum of -5. Using the template $(x \quad)(x \quad)$ and -2 and -3, we can then write $x^2 - 5x + 6$ as a product of factors as follows:

$(x - 2)(x - 3)$

Carry out Factoring on Expressions of the Form $ax^2 + bx + c$ where $a = 1$ (4 of 7)

Example 10:

Write $x^2 - 5x - 6$ as a product of factors.

We are dealing with an expression of the form $\boldsymbol{ax^2 + bx + c}$ where $\boldsymbol{a = 1}$. Therefore, we will use a "short-cut" method.

We note that $a = 1$, $b = -5$, and $c = -6$. Now we will find all pairs of integers whose product is -6.

$-6 = (-1)(6)$ $\quad$ $-6 = (-2)(3)$ $\quad$ $-6 = (1)(-6)$ $\quad$ $-6 = (2)(-3)$

We notice that $+1$ and -6 have a sum of -5. Using the template $(x \quad)(x \quad)$ and $+1$ and -6, we can then write $x^2 - 5x - 6$ as a product of factors as follows:

$(x + 1)(x - 6)$

Carry out Factoring on Expressions of the Form $ax^2 + bx + c$ where $a = 1$

Example 11:

$x^2 + 5x + 6$ as a product of factors.

We are dealing with an expression of the form $\boldsymbol{ax^2 + bx + c}$ where $\boldsymbol{a = 1}$. Therefore, we will use a "short-cut" method.

We note that $a = 1$, $b = 5$, and $c = 6$. Now we will find all pairs of integers whose product is 6.

$6 = (1)(6)$ $\quad$ $6 = (2)(3)$ $\quad$ $6 = (-1)(-6)$ $\quad$ $6 = (-2)(-3)$

We notice that $+ 2$ and $+ 3$ have a sum of 5. Using the template $(x \quad)(x \quad)$ and $+ 2$ and $+ 3$, we can then write $x^2 + 5x + 6$ as a product of factors as follows:

$(x + 2)(x + 3)$

Carry out Factoring on Expressions of the Form $ax^2 + bx + c$ where $a = 1$

Example 12:

Write $x^2 + 14x + 49$ as a product of factors.

We are dealing with an expression of the form $\boldsymbol{ax^2 + bx + c}$ where $\boldsymbol{a = 1}$. Therefore, we will use a “short-cut” method.

We note that $a = 1$, $b = 14$, and $c = 49$. Now we will find all pairs of integers whose product is 49.

$49 = (1)(49)$ $\quad$ $49 = (7)(7)$ $\quad$ $49 = (-1)(-49)$ $\quad$ $49 = (-7)(-7)$

We notice that $+7$ and $+7$ have a sum of 14. Using the template $(x \quad)(x \quad)$ and $+7$ and $+7$, we can then write $x^2 + 14x + 49$ as a product of factors as follows:

$(x + 7)(x + 7)$

Carry out Factoring on Expressions of the Form $ax^2 + bx + c$ where $a = 1$ (7 of 7)

Example 13:

Write $x^2 - 6x + 7$ as a product of factors.

We are dealing with an expression of the form $\boldsymbol{ax^2 + bx + c}$ where $\boldsymbol{a = 1}$. Therefore, we will use a "short-cut" method.

We note that $a = 1$, $b = -6$, and $c = 7$. Now we will find all pairs of integers whose product is 7.

$7 = (1)(7)$ $\quad$ $7 = (-1)(-7)$

We notice that NO pair gives us a sum of 6.

Therefore, this trinomial CANNOT be written as a product of factors.

6. Carry out Factoring Using the Difference of Squares Formula (1 of 2)

Given mathematical expressions of the form $a^2 - b^2$, we can use the *Difference of Squares Formula* $a^2 - b^2 = (a + b)(a - b)$ to write them as a product of factors.

Example 14:

Write $x^2 - 49$ as a product of factors.

We notice that the given expression is of the form $a^2 - b^2$, where $a = x$ and $b = 7$.

Therefore, according to the *Difference of Squares Formula* $a^2 - b^2 = (a + b)(a - b)$, we can write $x^2 - 49$ as a product of factors as follows:

$(x + 7)(x - 7)$

Please note that the product consists of two expressions that are conjugates!

Carry Out Factoring Using the Difference of Squares Formula (2 of 2)

Example 15:

Write $36x^2 - 25$ as a product of factors.

We notice that the given expression is of the form $a^2 - b^2$, where $a = 6x$ and $b = 5$.

Therefore, according to the *Difference of Squares Formula* $a^2 - b^2 = (a + b)(a - b)$ *,* we can write $36x^2 - 25$ as a product of factors as follows:

$(6x + 5)(6x - 5)$