Concepts and Examples
Factoring

Based on power point presentations by Pearson Education, Inc.
Revised by Ingrid Stewart, Ph.D.

Learning Objectives

1. Memorize the definition of factoring.
2. Factor out common factors.
3. Factor by grouping.
4. Factor expressions of the form $ax^2 + bx + c$.
5. Factor expressions of the form $ax^2 + bx + c$ where $a = 1$.
6. Factor a *Difference of Squares*.

1. Introduction to Factoring

Factors are numbers that divide into other numbers without leaving a remainder.

For example, 2 and 3 divide into 6 without a remainder, but 4 and 5 do not. Therefore, only 2 and 3 are factors of 6.

Factoring is the process of writing numbers or mathematical expressions as a product of factors. We usually try several factoring methods to find out whether numbers or mathematical expressions are *factorable*.

For example, 420 can be written as the following product of factors: 2(2)(3)(5)(7)

In this lesson, we are going to concentrate on factoring mathematical expressions, for example $x^2 + 5x – 6$. We will learn how to "factor out" a common factor; factor by grouping; factor expressions of the form $ax^2 + bx + c$; and factor using the *Difference of Squares Formula*.

2. Factor Out Common Factors (1 of 2)

We can write a mathematical expression as a product of factors by simply trying to **factor out a common factor**. This is a term that divides into every term of the expression without a reminder. We place this term in front of a set of parentheses containing the “reduced” terms.

For example, given $8x + 12$, we see that the number 2 divides into 8 and 12 without a remainder. Therefore, we can write the expression as the product $2(4x + 6)$. Note the ”reduced” terms are $4x$ and 6.

Often, we are asked to factor out **the greatest factor** that every term of a mathematical expression has in common.

For example, given $8x + 12$ again, we see that the number 4 also divides into 8 and 12 without a remainder. Therefore, we can write the expression as the product $4(2x + 3)$. We see that 4 is the greatest factor that 8 and 12 have in common because the “reduced” terms have a coefficient of 2 and a constant of 3 which no longer have a factor in common.

Factor Out Common Factors (2 of 2)

Example 1:

Write $6x^3 + 12x^2$ as a product of factors by factoring out the greatest common factor.

We notice that 2, 3, and 6 divide into of 6 and 12 without a remainder. Also, x and x^2 divide into x^3 and x^2 without a remainder.

The largest numeric factor is 6 and the largest variable factor is x^2. Therefore, we can factor out $6x^2$ from $6x^3$ and $12x^2$. We place $6x^2$ in front of parentheses containing the following:

$6x^2(x + 2)$

We see that $6x^2$ is the greatest factor that $6x^3$ and $12x^2$ have in common because the ”reduced” terms have a coefficient 1 and a constant of 2 which no longer have a factor in common.

3. Factor by Grouping (1 of 6)

When a mathematical expression contains an even number of terms, we can try to write it as a product of factors by using **factoring by grouping**. Following are the steps.

Step 1 - Place half of the terms into a ”group” by enclosing them in parentheses and do the same for the other half.

Example 2:

Write $2x^3 + 3x^2 + 4x + 6$ as a product of factors.

This expression has four terms. The easiest grouping is one where we group the first two terms and the last two terms, such as $(2x^2 + 3x^2) + (4x + 6)$.

NOTE: This is a good first try.

Factor by Grouping

Step 2 - Factor out the greatest common factor from each group.

Example 2 continued:

Given $(2x^3 + 3x^2) + (4x + 6)$, we will factor out the greatest common factor from each group. We end up with a sum of the following two products:

$x^2(2x + 3) + 2(2x + 3)$

Factor by Grouping (3 of 6)

Step 3 - If possible, factor out the common factor from the two products created in Step 2.

Example 2 continued:

Given $x^2(2x + 3) + 2(2x + 3)$, we see that the two products created in Step 2 have a factor in common, namely $(2x + 3)$.

This is the goal of factoring by grouping! If we cannot find a common factor, then we might have to rearrange our groups and start over or we must use other methods.

Factor by Grouping (4 of 6)

Example 2 continued:

We will now factor out the common factor $(2x + 3)$ from $x^2(2x + 3) + 2(2x + 3)$ to get the following:

$(2x + 3)(x^2 + 2)$

This is $2x^3 + 3x^2 + 4x + 6$ written as a product of factors.

Factor by Grouping (5 of 6)

Example 3:

Write $6x^3 + 12x^2 + 5x + 10$ as a product of factors using factoring by grouping.

Since we have an expression with four (4) terms, let's place the first and second term into a "group" by enclosing them in parentheses. Let's do the same for the third and fourth term.

$(6x^3 + 12x^2) + (5x + 10)$ Then we factor out the greatest common factor from each group.

$6x^2(x + 2) + 5(x + 2)$

We see that we end up with the same factor in each product. This is the goal of factoring by grouping!

Factor by Grouping (6 of 6)

Example 3 continued:

We will now factor out the common factor $(x + 2)$ from $6x^2(x + 2) + 5(x + 2)$ to get the following:

$(x + 2)(6x^2 + 5)$

This is $6x^3 + 12x^2 + 5x + 10$ written as a product of factors.

4. Factor Expressions of the Form $ax^2 + bx + c$

Some mathematical expressions of the form $\boldsymbol{ax^2 + bx + c}$ can be written as a product of factors. Following are the steps.

Step 1 - Given $ax^2 + bx + c$, find all pairs of integers whose product is ac.

Example 4:

Try to write $2x^2 + 7x + 6$ as a product of factors.

We are dealing with an expression of the form $\boldsymbol{ax^2 + bx + c}$. We note that $a = 2$, $b = 7$, and $c = 6$. Then $\boldsymbol{ac = 2(6) = 12}$.

Now we will find all pairs of integers whose product is 12.

Factor Expressions of the Form $ax^2 + bx + c$ (2 of 6)

Example 4 continued:

$12 = (+1)(+12)$ $12 = (+2)(+6)$ $12 = (+3)(+4)$

$12 = (-1)(-12)$ $12 = (-2)(-6)$ $12 = (-3)(-4)$

Step 2 - If possible, find one pair of integers from Step 1 whose sum equals $\boldsymbol{b}$ in $\boldsymbol{ax^2 + bx + c}$. Replace the term $\boldsymbol{bx}$ with a sum or difference of the appropriate pair of integers. NOTE: If this is not possible, factoring will have failed.

Example 4 continued:

We notice that the pair $+3$ and $+4$ has a sum of 7, which is the value of $\boldsymbol{b}$. Therefore, we replace the term $7x$ in $2x^2 + 7x + 6$ with $+3x$ and $+4x$ as follows:

$2x^2 + 3x + 4x + 6$

Factor Expressions of the Form $ax^2 + bx + c$ (3 of 6)

Step 3 - Use factoring by grouping on the final equation in Step 2.

Example 4 continued:

Since $2x^2 + 3x + 4x + 6$ is an expression with four terms, let's try factoring by grouping. We'll place the first and second term into a "group" by enclosing them in parentheses. We'll do the same for the third and fourth term.

$(2x^2 + 3x) + (4x + 6)$

Then we factor out the greatest common factor from each group.

$x(2x + 3) + 2(2x + 3)$

Factor Expressions of the Form $ax^2 + bx + c$ (4 of 6)

Example 4 continued:

We see that we end up with the same factor, namely $(2x + 3)$ in $x(2x + 3) + 2(2x + 3)$. We will now factor it out of as follows:

$(2x + 3)(x + 2)$ This is $2x^2 + 7x + 6$ written as a product of factors.

Factor Expressions of the Form $ax^2 + bx + c$ (5 of 6)

Example 5:

Try to write $x^2 + 5x - 6$ as a product of factors.

We are dealing with an expression of the form $\boldsymbol{ax^2 + bx + c}$. We note that $a = 1$, $b = 5$, and $c = -6$. Then $\boldsymbol{ac = 1(-6) = -6}$.

Now we will find all pairs of integers whose product is -6.

$-6 = (-1)(+6)$ $-6 = (-2)(+3)$ $-6 = (+1)(-6)$ $-6 = (+2)(-3)$

We notice that the pair -1 and $+6$ has a sum of 5, which is the value of $\boldsymbol{b}$.

Now, we replace the term $5x$ in $x^2 + 5x - 6$ with $-1x$ and $+6x$ as follows:

$x^2 - 1x + 6x - 6$

Factor Expressions of the Form $ax^2 + bx + c$ (6 of 6)

Example 5 continued:

Since we have an expression with four terms, let's try factoring by grouping. We'll place the first and second term into a "group" by enclosing them in parentheses. We will do the same for the third and fourth term.

$(x^2 - 1x) + (6x - 6)$

Then we factor out the greatest common factor from each group.

$x(x - 1) + 6(x - 1)$

We see that we end up with the same factor, namely $(x - 1)$. We will now factor it out as follows:

$(x - 1)(x + 6)$ This is $x^2 + 5x - 6$ written as a product of factors.

5. Factor Expressions of the Form $ax^2 + bx + c$ where $a = 1$ (1 of 7)

Given $\boldsymbol{ax^2 + bx + c}$ where $\boldsymbol{a = 1}$, we can try to use a **short-cut method** to write it as a product of factors. Following are the steps.

Step 1 – Given $\boldsymbol{ax^2 + bx + c}$ where $\boldsymbol{a = 1}$, find all pairs of integers whose product is $\boldsymbol{c}$.

Example 6:

Try to write $x^2 + 5x - 6$ as a product of factors.

We note that $a = 1$, $b = 5$ and $c = -6$. We will find all pairs of integers whose product is -6, which is the value of $\boldsymbol{c}$.

$-6 = (-1)(+6)$ $\quad$ $-6 = (-2)(+3)$ $\quad$ $-6 = (+1)(-6)$ $\quad$ $-6 = (+2)(-3)$

Factor Expressions of the Form $ax^2 + bx + c$ where $a = 1$

Step 2 – If possible, find one pair from Step 1 whose sum equals *b*. NOTE: If this is not possible, factoring will have failed.

Example 6 continued:

We notice that the pair -1 and $+6$ has a sum of $-1 + (+6) = 5$, which is the value of ***b***.

Step 3 – Create the template $(x \quad)(x \quad)$ and use the numbers from Step 2 as the second terms.

Example 6 continued:

Using the template $(x \quad)(x \quad)$ and -1 and $+6$, we can then write $x^2 + 5x - 6$ as a product of factors as follows:

$(x - 1)(x + 6)$

Factor Expressions of the Form $ax^2 + bx + c$ where $a = 1$

Example 7:

Try to write $x^2 - 5x + 6$ as a product of factors.

We are dealing with an expression of the form $\boldsymbol{ax^2 + bx + c}$ where $\boldsymbol{a = 1}$. Therefore, we will use a "short-cut" method.

We note that $a = 1$, $b = -5$, and $c = 6$. Now we will find all pairs of integers whose product is 6, which is the value of $\boldsymbol{c}$.

$6 = (+1)(+6)$ $6 = (+2)(+3)$ $6 = (-1)(-6)$ $6 = (-2)(-3)$

We notice that the pair -2 and -3 has a sum of -5, which is the value of $\boldsymbol{b}$. Using the template $(x \quad)(x \quad)$ and -2 and -3, we can then write $x^2 - 5x + 6$ as a product of factors as follows:

$(x - 2)(x - 3)$

Example 8:

Try to write $x^2 - 5x - 6$ as a product of factors.

We are dealing with an expression of the form $\boldsymbol{ax^2 + bx + c}$ where $\boldsymbol{a = 1}$. Therefore, we will use a "short-cut" method.

We note that $a = 1$, $b = -5$, and $c = -6$. Now we will find all pairs of integers whose product is -6, which is the value of $\boldsymbol{c}$.

$-6 = (-1)(+6)$ $\quad$ $-6 = (-2)(+3)$ $\quad$ $-6 = (+1)(-6)$ $\quad$ $-6 = (+2)(-3)$

We notice that the pair $+1$ and -6 has a sum of -5, which is the value of $\boldsymbol{b}$. Using the template $(x \quad)(x \quad)$ and $+1$ and -6, we can then write $x^2 - 5x - 6$ as a product of factors as follows:

$(x + 1)(x - 6)$

Factor Expressions of the Form $ax^2 + bx + c$ where $a = 1$

Example 9:

Try to write $x^2 + 5x + 6$ as a product of factors.

We are dealing with an expression of the form $\boldsymbol{ax^2 + bx + c}$ where $\boldsymbol{a = 1}$. Therefore, we will use a "short-cut" method.

We note that $a = 1$, $b = 5$, and $c = 6$. Now we will find all pairs of integers whose product is 6, which is the value of $\boldsymbol{c}$.

$6 = (+1)(+6)$ $6 = (+2)(+3)$ $6 = (-1)(-6)$ $6 = (-2)(-3)$

We notice that the pair $+2$ and $+3$ has a sum of 5, which is the value of $\boldsymbol{b}$. Using the template $(x \quad)(x \quad)$ and $+2$ and $+3$, we can then write $x^2 + 5x + 6$ as a product of factors as follows:

$(x + 2)(x + 3)$

Factor Expressions of the Form $ax^2 + bx + c$ where $a = 1$

Example 10:

Try to write $x^2 + 14x + 49$ as a product of factors.

We are dealing with an expression of the form $\boldsymbol{ax^2 + bx + c}$ where $\boldsymbol{a = 1}$. Therefore, we will use a "short-cut" method.

We note that $a = 1$, $b = 14$, and $c = 49$. Now we will find all pairs of integers whose product is 49, which is the value of $\boldsymbol{c}$.

$49 = (+1)(+49)$ $49 = (+7)(+7)$ $49 = (-1)(-49)$ $49 = (-7)(-7)$

We notice that the pair $+7$ and $+7$ has a sum of 14, which is the value of $\boldsymbol{b}$. Using the template $(x \quad)(x \quad)$ and $+7$ and $+7$, we can then write $x^2 + 14x + 49$ as a product of factors as follows:

$(x + 7)(x + 7)$

Factor Expressions of the Form $ax^2 + bx + c$ where $a = 1$

Example 11:

Try to write $x^2 - 6x + 7$ as a product of factors.

We are dealing with an expression of the form $\boldsymbol{ax^2 + bx + c}$ where $\boldsymbol{a = 1}$. Therefore, we will use a “short-cut” method.

We note that $a = 1$, $b = -6$, and $c = 7$. Now we will find all pairs of integers whose product is 7, which is the value of $\boldsymbol{c}$.

$7 = (+1)(+7)$ $7 = (-1)(-7)$

We notice that NO pair has a sum of -6, which is the value of $\boldsymbol{b}$.

Therefore, this mathmati al expression CANNOT be written as a product of factors.

6. Factor a Difference of Squares (1 of 3)

In mathematics, we call two terms consisting of squares of numbers, variables, or mathematical expressions, which are then subtracted from each other a **difference of squares.**

For example, $64 - 9x^2$. Please note that the two values that are squared are 8 (from $8 \cdot 8 = 64$) and $3x$ (from $3x \cdot 3x = 9x^2$).

The difference of the squares of the two terms can then be written as the product of a sum and difference of those terms.

For example, $64 - 9x^2$ can be written as $(8 + 3x)(8 - 3x)$ or $(8 - 3x)(8 + 3x)$. It does not matter if the sum/difference is written first or second.

Factor a Difference of Squares (2 of 3)

Example 12:

Try to write $49 - x^2$ as a product of factors.

We notice that the given expression is the difference of two terms that are squares, namely 7 and x.

Therefore, we can write $49 - x^2$ as a product of factors as follows:

$(7 + x)(7 - x)$ or $(7 - x)(7 + x)$

Factor a Difference of Squares (3 of 3)

Example 13:

Try to write $36x^2 - 25$ as a product of factors.

We notice that the given expression is the difference of two terms that are squares, namely $6x$ and 5.

Therefore, we can write $36x^2 - 25$ as a product of factors as follows:

$(6x + 5)(6x - 5)$ or $(6x - 5)(6x + 5)$