Concepts and Examples Exponential Expressions Advanced Order of Operations

Based on power point presentations by Pearson Education, Inc.

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Learning Objectives

1. Evaluate exponential expressions.
2. Use the rules of exponents.
3. Use the advanced Order of Operations.

1. Evaluate Exponential Expressions (1 of 4)

Exponential expressions are of the form $\boldsymbol{b^x}$, where ***b*** is called a **base** and ***x*** **exponent** or **power**. The exponent states how many times to multiply base ***b*** by itself.

For example, given $\mathbf{5^2}$, the base is **5** and the exponent (or power) is **2**.
The exponent states to use **5** two times in a multiplication, so that $\mathbf{5^2}$ equals
$\mathbf{5 \cdot 5 = 25}$.

NOTE: $\mathbf{5^2}$ is read as “five to the second power” or “five raised to the second power” or “five squared”.

Evaluate Exponential Expressions (2 of 4)

Example 1:

Evaluate the following exponential expressions by hand:

a. $\mathbf{4^3}$

The exponent states to use **4** <u>three times</u> in a multiplication, so that $\mathbf{4^3}$ equals $\mathbf{4 \cdot 4 \cdot 4 = 64}$.

NOTE: $\mathbf{4^3}$ is read as "four to the third power" or "four raised to the third power" or "four cubed".

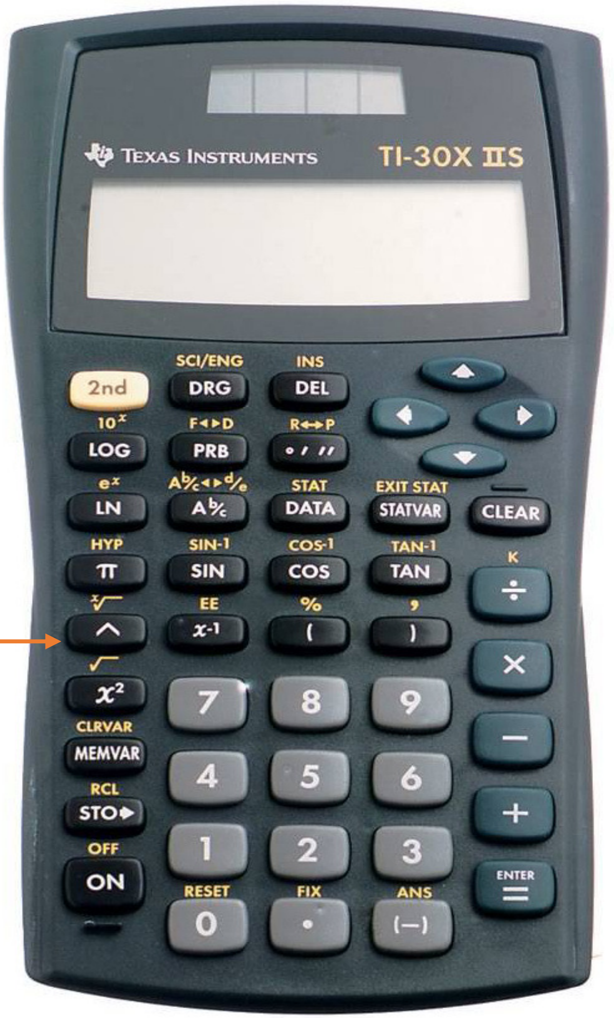
b. $\mathbf{2^4}$

The exponent states to use **2** <u>four times</u> in a multiplication, so that $\mathbf{2^4}$ equals $\mathbf{2 \cdot 2 \cdot 2 \cdot 2 = 16}$.

NOTE: $\mathbf{2^4}$ is read as "two to the fourth power" or "two raised to the fourth power".

Evaluate of Exponential Expressions (3 of 4)

power



Example 2:

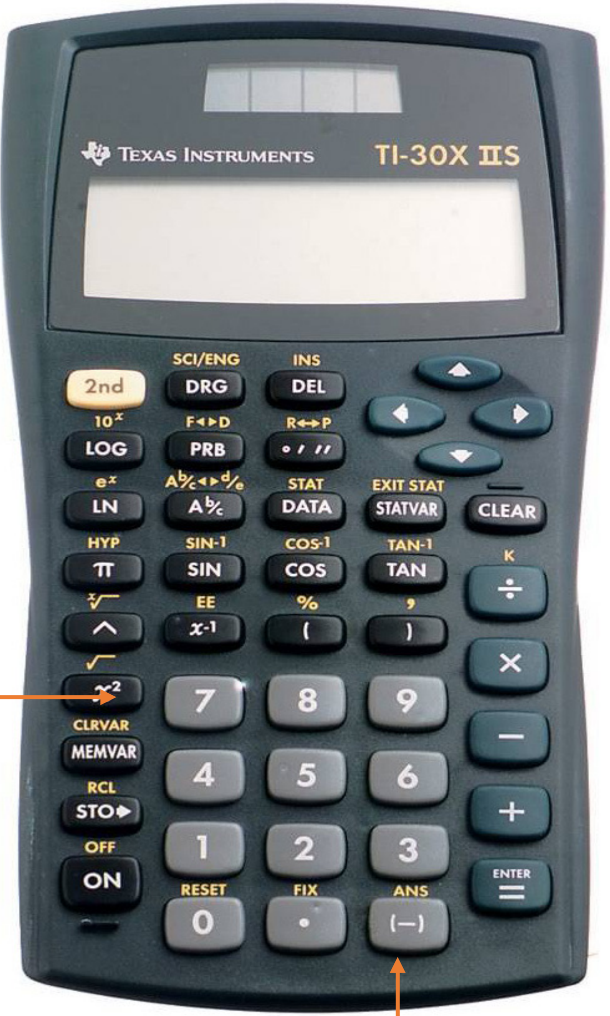
Evaluate $\mathbf{4^3}$ on the calculator.

1. Type 4.
2. Press the caret ^ button. This indicates “raising to a power.
3. Type **3**.
4. Press the ENTER button.

The answer is **64**.

Evaluate Exponential Expressions (4 of 4)

power



negative sign – don’t confuse with subtraction sign!

Example 3:

Evaluate $(-5)^2$ by hand and using a calculator.

$(-5)^2 = (-5)(-5) = 25$

Using the TI-30X IIS Calculator:

Press the left parenthesis button (.
Press the negative sign button (–). Do not use the subtraction button!
Type **3**.
Press the right parenthesis button).
Press the caret **^** button on the calculator.
Type **2** and press the ENTER button.

The answer is **25**.

2. The Rules of Exponents (1 of 9)

The *Rules of Exponents*, also called *Laws of Exponents* or *Properties of Exponents*, make the process of simplifying expressions involving exponents easier. Many arithmetic operations like addition, subtraction, multiplication, and division can be conveniently performed in quick steps using the Rules of Exponents.

Specifically, we will discuss the Negative-Exponent Rule, the Zero-Exponent Rule, the Product Rule, the Quotient Rule, the Power-of-a-Power Rule, the Power-of-a-Quotient Rule, and the Power-of-a-Product Rule.

The Rules of Exponents (2 of 9)

The Negative-Exponent Rule

If b is any real number other than 0 and n is a natural number, then

$$b^{-n} = \frac{1}{b^n}$$

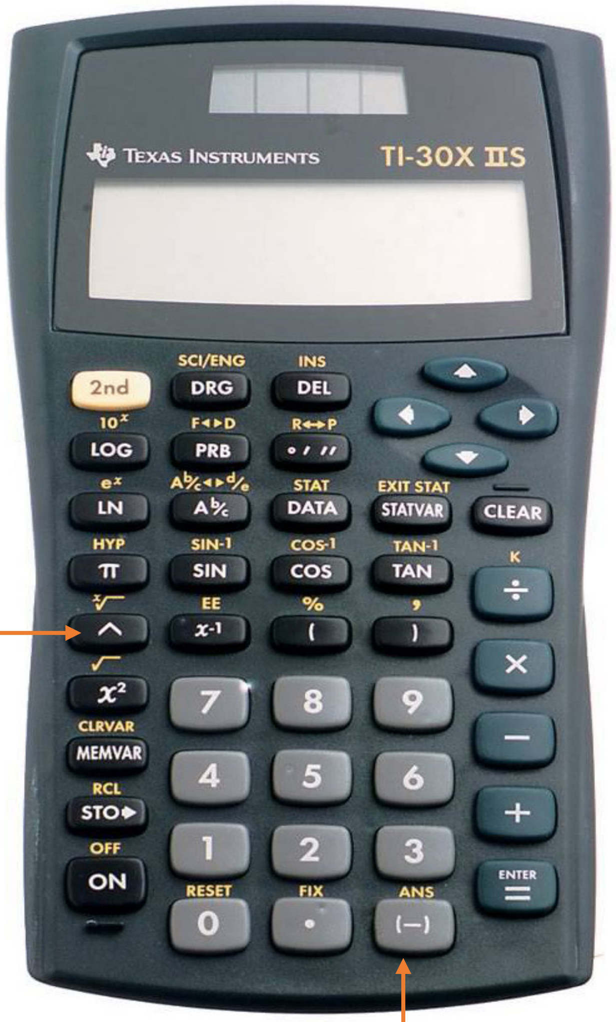
This means when a number is raised to a negative power, we write a fraction with a 1 in the numerator and place into the denominator the same number raised to the positive power.

For example, $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$ or $2^{-1} = \frac{1}{2^1} = \frac{1}{2}$ or $x^{-2} = \frac{1}{x^2}$

Please note that the power of 1 is usually NOT written. For instance, we hardly ever state 2^1 or x^1. It is always just 2 or x.

The Rules of Exponents (3 of 9)

power



negative sign – don't confuse with subtraction sign!

Example 4:

Evaluate $\mathbf{4^{-3}}$ on the TI-30X IIS calculator.

1. Type 4.
2. Press the caret ^ button.
3. Press the negative sign button (–). Do not use the subtraction button!
4. Type **3**.
5. Press the ENTER button.

The answer is $\mathbf{0.015625 = \frac{1}{64}}$.

Note: The TI-30X IIS does not give you a fraction!

The Rules of Exponents (4 of 9)

The Zero-Exponent Rule

If b is any real number other than 0, then $b^0 = 1$.

This means when a number is raised to the zero power, the result is ALWAYS 1.

For example, $(-4)^0 = 1$

or $7^0 = 1$

or $1000^0 = 1$

or $x^0 = 1$

The Rules of Exponents (5 of 9)

The Product Rule

If b is a real number or algebraic expression, and m and n are integers, then

$$b^m \cdot b^n = b^{m+n}$$

This means when multiplying exponential expressions with the same base, add the exponents.

For example, $2^2 \cdot 2^3 = 2^{2+3} = 2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$

or $x^7 \cdot x^3 = x^{7+3} = x^{10}$

or $x \cdot x \cdot x^4 = x^{1+1+4} = x^6$

The Rules of Exponents (6 of 9)

The Quotient Rule

If b is a nonzero real number or algebraic expression, and m and n are integers, then

$$\frac{b^m}{b^n} = b^{m-n}$$

This means when dividing exponential expressions with the same nonzero base, subtract the exponent in the denominator from the exponent in the numerator.

For example, $\frac{2^8}{2^4} = 2^{8-4} = 2^4 = 16$

or $\frac{x^7}{x^3} = x^{7-3} = x^4$ or $\frac{x^2}{x} = x^{2-1} = x^1 = x$

The Rules of Exponents (7 of 9)

The Power-of-a-Power Rule

If *b* is a real number or algebraic expression, and *m* and *n* are integers, then

$$\left(b^m\right)^n = b^{m \cdot n}$$

This means when an exponential expression is raised to a power, multiply the exponents.

For example, $(2^2)^3 = 2^{2(3)} = 2^6 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 64$

or $(x^3)^4 = x^{3(4)} = x^{12}$

Please note the difference between $x^3 \cdot x^4$ and $(x^3)^4$. Hint: $x^3 \cdot x^4 = x^{3+4} = x^7$.

The Rules of Exponents (8 of 9)

The Power-of-a-Quotient Rule

If *a* and *b* are real numbers, $b \neq 0$, or algebraic expressions, and *n* is an integer, then

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

This means, when a fraction is raised to a power, raise the numerator AND the denominator to that power.

For example, $\left(\frac{2}{5}\right)^3 = \frac{2^3}{5^3} = \frac{2 \cdot 2 \cdot 2}{5 \cdot 5 \cdot 5} = \frac{8}{125}$

or $\left(\frac{x^7}{x^3}\right)^4 = \frac{x^{7(4)}}{x^{3(4)}} = \frac{x^{28}}{x^{12}} = x^{28-12} = x^{16}$

The Rules of Exponents (9 of 9)

The Power-of-a-Product Rule

If *a* and *b* are real numbers or algebraic expressions, and *n* is an integer, then

$$(ab)^n = a^n b^n$$

This means when a product is raised to a power, raise each factor to that power.

For example, $(-2y)^4 = (-2)^4 y^4 = 16y^4$

or $(-2xy)^3 = (-2)^3 x^3 y^3 = -8x^3 y^3$.

3. Advanced Order of Operations (1 of 5)

In an earlier lesson, we discussed the simple Order of Operations given only addition, subtraction, multiplication, and division.

In this lesson, we will add grouping symbols and exponents to mathematical expressions.

1. Grouping Symbols are evaluated first.
   Grouping symbols are parentheses ( ), brackets [ ], braces { }, and fraction bars.
2. Exponents are evaluated next.
3. Multiplications and Divisions are done next, in the order in which they occur, working from left to right.
4. Addition and Subtraction are done last, in the order in which they occur, working from left to right.

Advanced Order of Operations (2 of 5)

Example 5:

Evaluate $6^2 - 24 \div 2^2 \cdot 3 + 1$.

There are no grouping symbols. Therefore, we begin by evaluating the exponent.

$6^2 - 24 \div 2^2 \cdot 3 + 1 =$

$= 36 - 24 \div 4 \cdot 3 + 1$ (evaluated the exponent)

$= 36 - 6 \cdot 3 + 1$ (evaluated the division)

$= 36 - 18 + 1$ (evaluated the multiplication)

$= 18 + 1$ (evaluated the subtraction)

$= 19$ (evaluated the addition)

Advanced Order of Operations (3 of 2)

Example 6:

Evaluate $(-5)^0$ and -5^0.

$(-5)^0 = 1$

In -5^0 only 5 is raised to the 0 power. Therefore, you must use the *Order of Operations* rule that states that exponents are evaluated before multiplication.

$$\begin{aligned} -5^0 &= -1(5^0) \\ &= -1(1) \\ &= -1 \end{aligned}$$

We can conclude that a negative number raised to the 0 power **has a different value** than its positive counterpart raised to the 0 power and multiplied by – 1!

Advanced Order of Operations (4 of 2)

Example 7:

Evaluate $(-4)^2$ and -4^2.

$(-4)^2 = (-4) \cdot (-4) = 16$

In -4^2 only 4 is raised to the 2nd power. Therefore, you must use the *Order of Operations* rule that states that exponents are evaluated before multiplication.

$$\begin{aligned} -4^2 &= -1(4^2) \\ &= -1(16) \\ &= -16 \end{aligned}$$

We can conclude that a negative number raised to an EVEN power **has a different value** than its positive counterpart raised to the same even power and multiplied by – 1!

Advanced Order of Operations (5 of 5)

Example 8:

Evaluate $7 - 20 \div 2 \cdot (2 + 1)^3 + 93$.

There are grouping symbols. Therefore, we begin by evaluating the parentheses.

$7 - 20 \div 2 \cdot (2 + 1)^3 + 93$

$= 7 - 20 \div 2 \cdot 3^3 + 93$ (evaluated parentheses)

$= 7 - 20 \div 2 \cdot 27 + 93$ (evaluated the exponent)

$= 7 - 10 \cdot 27 + 93$ (evaluated the division)

$= 7 - 270 + 93$ (evaluated the multiplication)

$= -263 + 93$ (evaluated the subtraction)

$= -170$ (evaluated the addition)