# Concepts and Examples Exponential Expressions

Based on power point presentations by Pearson Education, Inc.

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# Learning Objectives

1. Evaluate exponential expressions.
2. Use rules of exponents.
3. Use the Order of Operations with exponents.

# 1. Evaluate Exponential Expressions (1 of 4)

Exponential expressions are of the form $\boldsymbol{b^x}$ (expressed as "*b* raised to the *x* power"), where ***b*** is called a **base** and ***x*** the **exponent** or **power**. The exponent states how many times to multiply the base ***b*** by itself.

For example, given $\mathbf{5^2}$ (expressed as "5 raised to the 2nd power"), the base is **5** and the exponent (or power) is **2**.

The exponent states to use **5** <u>two times</u> in a multiplication, so that $\mathbf{5^2}$ equals $\mathbf{5 \cdot 5 = 25}$.

NOTE: When the power is 2, we often use the word "squared." For example, we can say "five squared" when we see $\mathbf{5^2}$.

# Evaluate Exponential Expressions (2 of 4)

Example 1:

Evaluate the following exponential expressions by hand:

a. $\mathbf{4^3}$

The exponent states to use **4** three times in a multiplication, so that $\mathbf{4^3}$ equals $\mathbf{4 \cdot 4 \cdot 4 = 64}$.

NOTE: $\mathbf{4^3}$ is read as “four raised to the third power” or “four cubed”.

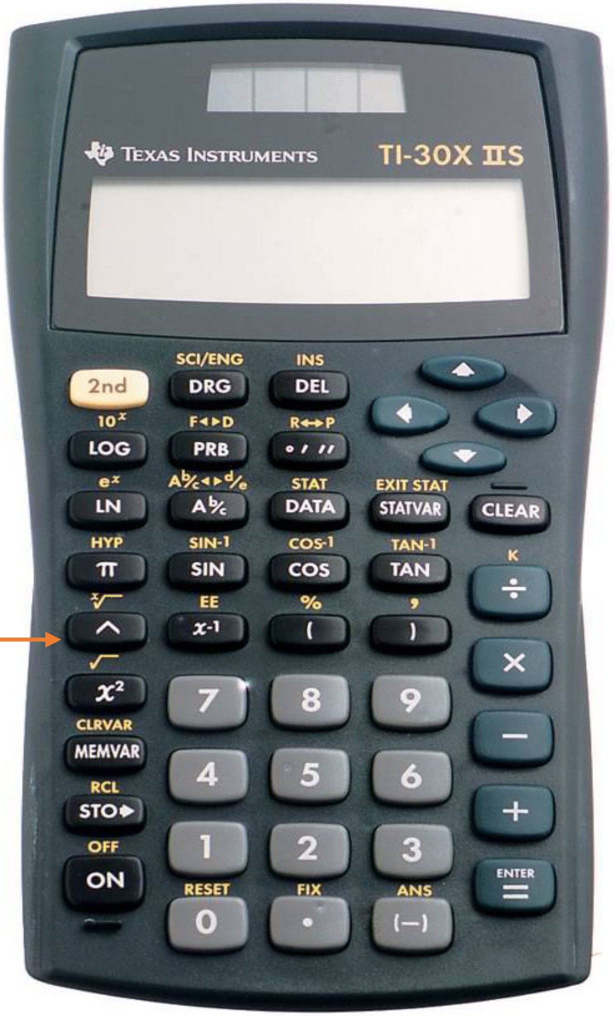
**b.** $\mathbf{2^4}$

The exponent states to use **2** four times in a multiplication, so that $\mathbf{2^4}$ equals $\mathbf{2 \cdot 2 \cdot 2 \cdot 2 = 16}$.

**NOTE:** $\mathbf{2^4}$ is read as “two raised to the fourth power”.

# Evaluate of Exponential Expressions (3 of 4)

power




Example 2:

Evaluate $\mathbf{4^3}$ with a calculator.

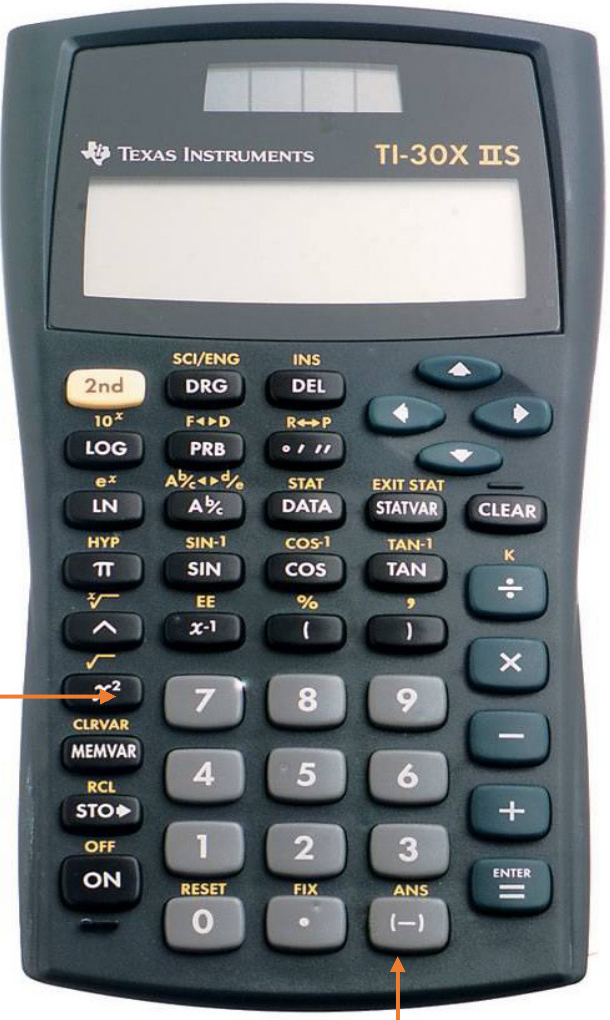
**Using the TI-30X IIS Calculator:**

1. Type 4.
2. Press the caret ^ button. This indicates that the next number will be an exponent.
3. Type **3**.
4. Press the ENTER button.

The answer is **64**.

# Evaluate Exponential Expressions (4 of 4)

power




negative sign – don't confuse with subtraction sign!

Example 3:

Evaluate **$(-5)^2$** by hand and with a calculator.

**By hand: $(-5)^2 = (-5)(-5) = 25$**

**Using the TI-30X IIS Calculator:**

1. Press the left parenthesis button (.
2. Press the negative sign button (–). Do not use the subtraction button!
3. Type **5**.
4. Press the right parenthesis button ).
5. Press the caret **^** button.
6. Type **2** and press the ENTER button.

The answer is **25**.

# 2. The Rules of Exponents (1 of 9)

The *Rules of Exponents*, also called *Laws of Exponents* or *Properties of Exponents*, often make the process of simplifying exponential expressions easier.

In this lesson, we will discuss the following rules:

- Negative-Exponent Rule
- Zero-Exponent Rule
- Product Rule
- Quotient Rule
- Power-of-a-Power Rule
- Power-of-a-Quotient Rule
- Power-of-a-Product Rule

# The Rules of Exponents (2 of 9)

**The Negative-Exponent Rule**

If $b$ is any real non-zero number or mathematical expression and $n$ is a positive integer, then

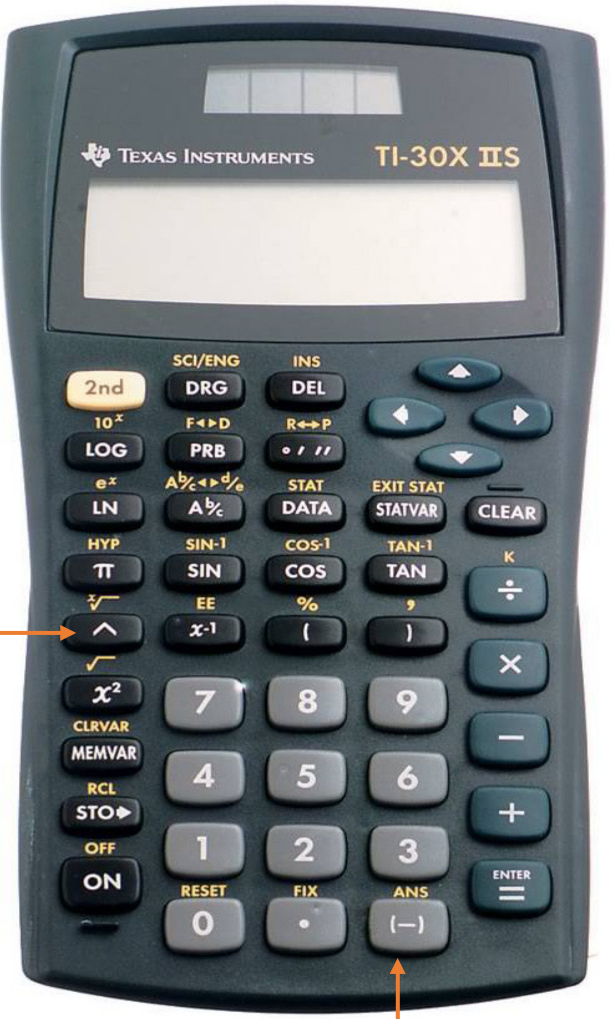
$$b^{-n} = \frac{1}{b^n}$$

This means when we raise a number to a negative power, we write a fraction with a 1 in the numerator and place into the denominator the same number now raised to the positive power.

For example, $4^{-3} = \frac{1}{4^3} = \frac{1}{4 \cdot 4 \cdot 4} = \frac{1}{64}$ or $4^{-1} = \frac{1}{4^1} = \frac{1}{4}$ or $x^{-2} = \frac{1}{x^2}$

Please note that the power of 1 is usually NOT written. For instance, we hardly ever state $4^1$ or $x^1$. It is always just 4 or $x$.

# The Rules of Exponents (3 of 9)




power

negative sign – don’t confuse with subtraction sign!

Example 4:

Evaluate $\mathbf{4^{-3}}$ with a calculator.

**Using the TI-30X IIS calculator:**

1. Type 4.
2. Press the caret ^ button.
3. Press the negative sign button (–). Do not use the subtraction button!
4. Type **3**.
5. Press the ENTER button.

The answer is **0.015625**.

On the previous slide we see that $4^{-3} = \frac{1}{64}$. The TI-30X IIS does not give fractions!

# The Rules of Exponents (4 of 9)

**The Zero-Exponent Rule**

If *b* is any non-zero number or mathematical expression, then

$b^0 = 1$

This means when we raise ANY number (except 0) to the zero power, the result is ALWAYS 1.

For example, $(-4)^0 = 1$

or $7^0 = 1$

or $1000^0 = 1$

or $x^0 = 1$

# The Rules of Exponents (5 of 9)

**The Product Rule**

If $b$ is any real number or mathematical expression and $m$ and $n$ are integers, then

$$b^m \cdot b^n = b^{m+n}$$

This means when we multiply exponential expressions with the same base, we add the exponents.

For example, $2^2 \cdot 2^3 = 2^{2+3} = 2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$

or $x^7 \cdot x^{-3} = x^{7+(-3)} = x^4$

or $x \cdot x \cdot x^4 = x^{1+1+4} = x^6$

# The Rules of Exponents (6 of 9)

**The Quotient Rule**

If *b* is any non-zero real number or mathematical expression and *m* and *n* are integers, then

$$\frac{b^m}{b^n} = b^{m-n}$$

This means when we divide exponential expressions with the same base, we subtract the exponent in the denominator from the exponent in the numerator.

For example, $\frac{2^8}{2^4} = 2^{8-4} = 2^4 = 16$

or $\frac{x^7}{x^3} = x^{7-3} = x^4$ or $\frac{x^2}{x} = x^{2-1} = x^1 = x$

# The Rules of Exponents (7 of 9)

**The Power-of-a-Power Rule**

If $b$ is any real number or mathematical expression and $m$ and $n$ are integers, then

$$\left(b^m\right)^n = b^{m \cdot n}$$

This means when we raise an exponential expression to a power, we multiply the exponents.

For example, $(2^2)^3 = 2^{2(3)} = 2^6 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 64$

or $(x^3)^4 = x^{3(4)} = x^{12}$

Please note the difference between $(x^3)^4$ and $x^3 \cdot x^4$ which equals $x^{3+4} = x^7$!

# The Rules of Exponents (8 of 9)

**The Power-of-a-Quotient Rule**

If *a* and *b* are any real numbers or mathematical expressions where *b* is not 0 and *n* is an integer, then

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

This means when we raise a fraction to a power, we raise the numerator AND the denominator to that power.

For example, $\left(\frac{2}{5}\right)^3 = \frac{2^3}{5^3} = \frac{2\cdot2\cdot2}{5\cdot5\cdot5} = \frac{8}{125}$

or $\left(\frac{x^7}{x^3}\right)^4 = \frac{x^{7(4)}}{x^{3(4)}} = \frac{x^{28}}{x^{12}} = x^{28-12} = x^{16}$

# The Rules of Exponents (9 of 9)

**The Power-of-a-Product Rule**

If *a* and *b* are any real numbers or mathematical expressions and *n* is an integer, then

$$(ab)^n = a^n b^n$$

This means when we raise a product to a power, we raise each factor to that power.

For example, $(-2y)^4 = (-2)^4 y^4 = 16y^4$

or $(-2xy)^3 = (-2)^3 x^3 y^3 = -8x^3y^3$.

# 3. The Order of Operations with Exponents (1 of 6)

In an earlier lesson, we discussed the *Order of Operations* given grouping symbols, addition, subtraction, multiplication, and division.

In this lesson, we will add exponents. Following is the *Order of Operations* given grouping symbols, exponents, addition, subtraction, multiplication, and division.

1. Grouping Symbols are evaluated first.
2. Exponents are evaluated next.
3. Multiplications and Divisions are done next, in the order in which they occur, working from left to right.
4. Addition and Subtraction are done last, in the order in which they occur, working from left to right.

# The Order of Operations with Exponents (2 of 6)

Example 5:

Evaluate $7^2 - 20 \div 2^2 \cdot (2 + 3)^3$.

Given $7^2 - 20 \div 2^2 \cdot (2 + 3)^3$, we evaluate the parentheses first.

Then $7^2 - 20 \div 2^2 \cdot 5^3$. Next, we evaluate all exponents:

$49 - 20 \div 4 \cdot 125$. Next, we evaluate division:

$49 - 5 \cdot 125$. Next, we evaluate multiplication:

$49 - 625$. Next, we evaluate subtraction:

$-576$

# The Order of Operations with Exponents (3 of 6)

Example 6:

Evaluate $6^2 - 24 \div 2^2 \cdot 3 + 1$.

Given $6^2 - 24 \div 2^2 \cdot 3 + 1$, we evaluate all exponent first.

Then $36 - 24 \div 4 \cdot 3 + 1$. Next, we evaluate division:

$36 - 6 \cdot 3 + 1$. Next, we evaluate multiplication:

$36 - 18 + 1$. Next, we evaluate subtraction:

$18 + 1$. Next, we evaluate addition:

$19$

# The Order of Operations with Exponents (4 of 6)

Example 7:

Evaluate $(-7)^0$ and $-7^0$.

In $(-7)^0$ the negative number is raised to the 0 power, therefore, $(-7)^0$ equals 1.

However, $-7^0$ actually equals $-1(7^0)$. Therefore, we must use the *Order of Operations* rule that states that exponents are evaluated before multiplication.

Given that $7^0$ equals 1, we find that $-7^0$ must equal $-1(1)$ or $-1$.

In summary, $(-7)^0 = 1$ and $-7^0 = -1$.

# The Order of Operations with Exponents 

Example 8:

Evaluate $(-6)^2$ and $-6^2$.

In $(-6)^2$, the negative number is raised to the 2nd power, therefore, $(-6)^2$ equals $(-6)(-6) = 36$.

However, $-6^2$ actually equals $-1(6^2)$. Therefore, we must use the *Order of Operations* rule that states that exponents are evaluated before multiplication.

Given that $6^2$ equals 36, we find that $-6^2$ must equal $-1(36)$ or $-36$.

In summary, $(-6)^2 = 36$ and $-6^2 = -36$.

# The Order of Operations with Exponents (6 of 6)

Example 9:

Evaluate $(-4)^3$ and $-4^3$.

In $(-4)^3$, the negative number is raised to the 3rd power, therefore, $(-4)^3$ equals $(-4)(-4)(-4) = -64$

However, $-4^3$ actually equals $-1(4^3)$. Therefore, we must use the *Order of Operations* rule that states that exponents are evaluated before multiplication.

Given that $4^3$ equals 64, we find that $-4^3$ must equal $-1(64)$ or $-64$.

In summary, $(-4)^3 = -64$ and $-4^3 = -64$.

Please note when a negative number is raised to an odd power the parentheses make no difference!