Concepts and Examples
Products of Mathematical Expressions

Based on power point presentations by Pearson Education, Inc.
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Learning Objectives

1. Use an “extension” of the *Distributive Property.*
2. Use the FOIL Method.

1. The "Extension" of the Distributive Property (1 of 4)

As discussed previously, the *Distributive Property* directs how to multiply a sum or difference of two or more numbers within parentheses by a number outside the parentheses. Specifically, it states that we must multiply each number in the parentheses individually by the number outside the parentheses and then add the products.

For example, $5(3x + 2) = 5(3x) + 5(2) = 15x + 10$.

However, we can also use the *Distributive Property* when none of the factors of a product are a single term.

For example, $(2x^4 + x^2)(x^3 + 5x - 3)$.

Here we multiply each term of one factor by each term of the other factor. Then we combine like terms.

The "Extension" of the Distributive Property (2 of 4)

Example 1:

Multiply $(2x^4 + x^2)(x^3 + 5x - 3)$.

Here we will multiply each term of the second factor $(x^3 + 5x - 3)$ by each term of the first factor $(2x^4 + x^2)$. Then we combine like terms.

$$(2x^4 + x^2)(x^3 + 5x - 3) = 2x^4(x^3 + 5x - 3) + x^2(x^3 + 5x - 3)$$

Now, we carry out "simple" distributions.

$$= 2x^4(x^3) + 2x^4(5x) + 2x^4(-3) + x^2(x^3) + x^2(5x) + x^2(-3)$$

The "Extension" of the Distributive Property (3 of 4)

Example 1 continued:

We notice that we must multiply several terms that both contain variables. Here we must use the *Product Rule of Exponents*.

That is, $2x^4(x^3) + 2x^4(5x) + 2x^4(-3) + x^2(x^3) + x^2(5x) + x^2(-3)$

$$= \quad 2x^{4+3} + 10x^{4+1} - 6x^4 \qquad + x^{2+3} + 5x^{2+1} - 3x^2$$

$$= \quad 2x^7 + 10x^5 - 6x^4 \qquad + \ x^5 + 5x^3 - 3x^2$$

All that's left to do is to combine the like terms $10x^5$ and x^5 to get

$$(2x^4 + x^2)(x^3 + 5x - 3) = 2x^7 + 11x^5 - 6x^4 + 5x^3 - 3x^2$$

The "Extension" of the Distributive Property (4 of 4)

Example 2:

Multiply $(7x + 5)(4x - 3)$.

Here we will multiply each term of the second factor $(2x - 3)$ by each term of the first factor $(x + 9)$. Then we combine like terms.

$$(7x + 5)(4x - 3) = 7x(4x) + 7x(-3) + 5(4x) + 5(-3)$$

$$= 28x^{1+1} - 21x + 20x - 15$$ (used the Product Rule of Exponents)

$$= 28x^2 - x - 15$$ (combined like terms)

2. Use the FOIL Method (1 of 3)

When we are asked to find the product of <u>two</u> mathematical expressions containing <u>two</u> terms each, we usually use a memory aid for the "extension" of the *Distributive Property.* It is called **FOIL**.

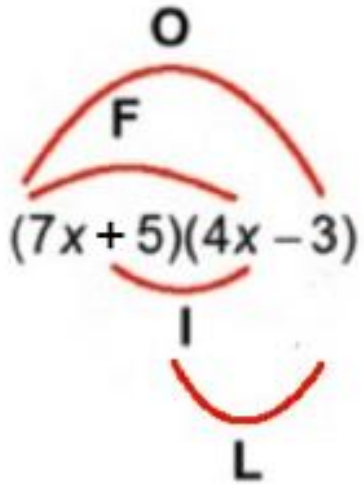
F directs us to multiply the **first terms** of each mathematical expression.

O directs us to multiply the **outside terms** of the product.

I directs us to multiply the **inside terms** of the product.

L directs us to multiply the **last (second) terms** of each mathematical expression.

For example,



F **O** **I** **L**

$$7x(4x) + 7x(-3) + 5(4x) + 5(-3)$$

Use the FOIL Method (2 of 3)

Example 3:

Multiply $(7x + 5)(4x - 3)$ using the FOIL Method.

$$\begin{aligned}
&\qquad\qquad\quad \textbf{F} \qquad\quad \textbf{O} \qquad\quad \textbf{I} \qquad\quad \textbf{L} \\
(7x + 5)(4x - 3) &= 7x(4x) + 7x(-3) + 5(4x) + 5(-3) \\
&= 28x^{1+1} - 21x + 20x - 15 \qquad \text{(used the Product Rule of Exponents)} \\
&= 28x^2 - x - 15 \qquad\qquad\quad \text{(combined like terms)}
\end{aligned}$$

Use the FOIL Method (3 of 3)

Example 4:

Multiply $(x - 1)(4x - 8)$ using the FOIL Method.

$$\begin{aligned} & \qquad\quad \textbf{F} \qquad\quad \textbf{O} \qquad\quad \textbf{I} \qquad\quad \textbf{L} \\ (x - 2)(4x - 8) &= x(4x) + x(-8) - 2(4x) - 2(-8) \\ &= 4x^{1+1} - 8x - 8x + 16 \quad \text{(used the Product Rule of Exponents)} \\ &= 4x^2 - 16x + 16 \quad \text{(combined like terms)} \end{aligned}$$