Concepts and Examples
Products of Mathematical Expressions

Based on power point presentations by Pearson Education, Inc.
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Learning Objectives

1. Use the extension of the *Distributive Property.*
2. Use FOIL.
3. Use the *Difference of Squares Formula*.

1. The Extension of the Distributive Property (1 of 4)

As discussed previously, the *Distributive Property* states the following:

Given any terms *a*, *b*, and *c*, then *a(b + c) = ab + ac.*

For example, $5(3x + 2) = 5(3x) + 5(2) = 15x + 10$.

However, we can also use the *Distributive Property* when none of the factors of a product are a single term.

For example, $(2x^4 + x^2)(x^3 + 5x - 3)$.

The Extension of the Distributive Property (2 of 4)

When none of the factors of a product are monomials, we use an “extension” of the Distributive Property which states the following:

Given any terms *a*, *b*, *c*, and *d* then $(a + b)(c + d) = a(c + d) + b(c + d)$.

Please note that there is an assumed multiplication sign between the two sets of parentheses and again between “a” and “b” and the parentheses.

The Extension of the Distributive Property

Example 1:

Multiply $(2x^4 + x^2)(x^3 + 5x - 3)$.

Here, we use the extension of the *Distributive Property*. Please note that the second factor contains 3 terms. However, this will not be an issue.

According the the extension of the *Distributive Property*, we simply multiply each term of the first expression with the second expression.

$$(2x^4 + x^2)(x^3 + 5x - 3) = 2x^4(x^3 + 5x - 3) + x^2(x^3 + 5x - 3)$$

Now, we carry out “simple” distributions.

$$= 2x^4(x^3) + 2x^4(5x) + 2x^4(-3) + x^2(x^3) + x^2(5x) + x^2(-3)$$

The Extension of the Distributive Property (4 of 4)

Example 1 continued:

We notice that we must multiply several terms that both contain variables. Here we must use the *Product Rule of Exponents*.

That is, $2x^4(x^3) + 2x^4(5x) + 2x^4(-3) + x^2(x^3) + x^2(5x) + x^2(-3) =$

$$= 2x^{4+3} + 10x^{4+1} - 6x^4 \qquad + x^{2+3} + 5x^{2+1} - 3x^2$$

$$= 2x^7 + 10x^5 - 6x^4 \qquad + x^5 + 5x^3 - 3x^2$$

All that's left to do is to combine the like terms $10x^5$ and x^5 to get

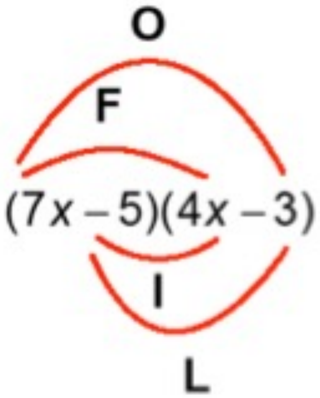
$$(2x^4 + x^2)(x^3 + 5x - 3) = 2x^7 + 11x^5 - 6x^4 + 5x^3 - 3x^2$$

2. Use FOIL (1 of 3)

When we need to multiply two mathematical expressions containing two terms each, we usually use a memory aid for the extension of the *Distributive Property.* It is called **FOIL**.

F represents **the product of the first terms**.
O represents **the product of the outside terms**.
I represents **the product of the inside terms**.
L represents **the product of the last (second) terms**.

For example,



F	O	I	L

$$7x(4x) + 7x(-3) - 5(4x) - 5(-3)$$

Use FOIL (2 of 3)

Example 2:

Multiply $(7x - 5)(4x - 3)$.

Since we are dealing with two mathematical expressions containing two terms each, we will use the memory aid FOIL to multiply.

$$\begin{aligned} (7x - 5)(4x - 3) &= \overset{\textbf{F}}{7x(4x)} + \overset{\textbf{O}}{7x(-3)} - \overset{\textbf{I}}{5(4x)} - \overset{\textbf{L}}{5(-3)} \\ &= 28x^{1+1} - 21x - 20x + 15 && \text{(used the Product Rule of Exponents)} \\ &= 28x^2 - 41x + 15 && \text{(combined like terms)} \end{aligned}$$

Use FOIL (3 of 3)

Example 3:

Multiply $(7x + 8)(7x - 8)$.

Since we are dealing with two mathematical expressions containing two terms each, we will use the memory aid FOIL to multiply.

$$(7x + 8)(7x - 8) = \overset{\textbf{F}}{7x(7x)} + \overset{\textbf{O}}{7x(-8)} + \overset{\textbf{I}}{8(7x)} + \overset{\textbf{L}}{8(-8)}$$

$$= 49x^{1+1} \quad - 56x \quad + 56x \quad - 64 \quad \text{(used the Product Rule of Exponents)}$$

$$= 49x^2 - 64 \quad \text{(combined like terms)}$$

3. Use the Difference of Squares Formula (1 of 2)

The **Difference of Squares Formula** pertains to a special product comprised of a mathematical expression containing two terms and its conjugate. This product occurs so frequently that it's convenient to memorize a short cut.

Given any terms a and b, then $\boldsymbol{(a + b)(a - b) = a^2 - b^2}$

Please note, we can always use FOIL instead of the *Difference of Squares Formula*.

Use the Difference of Squares Formula (2 of 2)

Example 4:

Multiply $(7x + 8)(7x - 8)$ using the *Difference of Squares Formula*.

Here we are multiplying a mathematical expression and its conjugate. Therefore, we can use the *Difference of Squares Formula* $\boldsymbol{(a + b)(a - b) = a^2 - b^2}$ to find its product.

We let $a = 7x$ and $b = 8$.

Then $(7x + 8)(7x - 8) = (7x)^2 - (8)^2$

$= (7)^2 x^2 - 64$ (used the Power-of-the Product Rule of Exponents)

$= 49x^2 - 64$