Concepts and Examples
Introduction to Complex Numbers

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Learning Objectives

1. Define imaginary and complex numbers.
2. Find the conjugate of a complex number.
3. Perform arithmetic operations on complex numbers.
4. Write the square root of a negative number in terms of *i*.

1. Definition of Imaginary and Complex Numbers (1 of 2)

Up to this point, we pretty much only discussed the set of real numbers. Actually, we have been mentioning "real" numbers a lot. They consist of integers, rational numbers, and irrational numbers. We will now discuss **imaginary numbers** and **complex numbers**.

The basic **imaginary number** is called *i* and is defined as

$i = \sqrt{-1}$ where $i^2 = -1$

Given imaginary numbers, we can now define another set of numbers, namely the **complex numbers**.

Complex numbers consist of all numbers in the standard form ***a + bi*** where ***a*** and ***b*** are real numbers, and ***i*** is the basic imaginary number.

Definition of Imaginary and Complex Numbers (1 of 2)

Example 1:

a. Write 19 as a complex number $a + bi$.

We only have a real number a and $b = 0$. Therefore, we can write 19 as

$19 + 0i$

Please note that ALL real numbers are also complex numbers where $b = 0$.

b. Write $-2i$ as a complex number $a + bi$.

We only have an imaginary number b and $a = 0$. Therefore, we can write $-2i$ as

$0 - 2i$

2. The Conjugate of a Complex Number (1 of 2)

We discussed conjugates already. We stated that given an expression with exactly two terms, its conjugate is an expression with the same terms but the arithmetic operator in the middle of these terms is changed to the opposite one.

For example, the conjugate of $-2x - 3$ is $-2x + 3$.

Conjugates of complex numbers are similar. That is, for the complex number ***a + bi***, we define its **conjugate** to be ***a – bi***. Only the sign between the real and imaginary part changes!

For example, the conjugate of $-2 + 3i$ is $-2x - 3i$. Similarly, the conjugate of $5i$ is $-5i$.

The Conjugate of a Complex Number (2 of 2)

Example 2:

a. Find the conjugate of $3i$.

Here we don't have a real part. We assume it to be 0. We can then write $3i$ as $0 + 3i$. Changing the sign between $0 + 3i$, we get

$0 - 3i$ or simply $-3i$

b. Find the conjugate of $-3i$.

Here we don't have a real part. We assume it to be 0. We can then write $-3i$ as $0 - 3i$. Changing the sign between $0 - 3i$, we get

$0 + 3i$ or simply $3i$

3. Arithmetic Operations on Complex Number (1 of 2)

To add and subtract complex numbers, we combine the real numbers and the coefficients of the number i.

Example 3:

Find the sum of $(5 – 2i) + (3 + 3i)$.

Removing parentheses, we get $5 – 2i + 3 + 3i$.

Next, we will group real numbers and imaginary numbers as follows:

$5 + 3 – 2i + 3i$

Now we will add the real numbers and the coefficients of the number i to get

$8 + i$

Note, that the coefficient of i is 1, but we usually do not write it.

Arithmetic Operations on Complex Number (2 of 2)

To multiply and divide complex numbers, we combine the real numbers and the coefficients of the number *i*.

Example 4:

Find the product of $(5 + 4i)(6 - 7i)$.

We will use FOIL!

That is $5(6) + 5(-7i) + 4i(6) + 4i(-7i) = 30 - 35i + 24i - 28i^2$

$= 30 - 11i - 28i^2$

Here we need to do some extra steps. We know that $i^2 = -1$. Therefore, we can continue as follows:

$30 - 11i - 28(-1) = 30 - 11i + 28$

$= 58 - 11i$

4. The Square Root of a Negative Number

The square root of the number **– *b***, where ***b*** itself is positive is an imaginary number.

We usually rewrite this as $\sqrt{-b}$ = $i\sqrt{b}$.

For example, $\sqrt{-3} = i\sqrt{3}$

or $\sqrt{-4} = i\sqrt{4}$ = 2i

Note that the imaginary number *i* is always written to the RIGHT of integers and fractions. It is usually written to the left of radicals!

Incidentally, if you were to evaluate $\sqrt{-3}$ or $\sqrt{-4}$ on the calculator, it would state "Domain Error."