Concepts and Examples
Imaginary and Complex Numbers

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Learning Objectives

1. Define imaginary numbers.
2. Define complex numbers.
3. Find the conjugate of a complex number.
4. Perform arithmetic operations on complex numbers.

1. Definition of Imaginary Numbers (1 of 2)

Up to this point, we pretty much only discussed the set of real numbers. Actually, we have been mentioning “real” numbers a lot. They consist of integers, rational numbers, and irrational numbers. We will now discuss **imaginary numbers** and **complex numbers**.

Of note is that imaginary numbers most often occur given radicals with even index and negative radicands.

We briefly mentioned imaginary numbers earlier when we tried to evaluate $\sqrt{-4}$ with a calculator and we were told “Domain Error.” We learned that this is the calculator’s way of telling us that there is no real number that, when multiplied twice, results in – 4.

Definition of Imaginary Numbers (2 of 2)

We will now define the basic **imaginary number**. It is universally assigned to the lower-case letter ***i*** and equals $\sqrt{-1}$.

Another definition that comes in handy is the fact that $\boldsymbol{i}^{\mathbf{2}}$ equals – 1 .

And the third definition we will use a lot is the fact that any imaginary number of the form $\sqrt{-b}$ can be written as $\boldsymbol{i}\sqrt{b}$. Note that there is an implied multiplication between ***i*** and $\sqrt{b}$.

For example, $\sqrt{-4}$ can be written as $\boldsymbol{i}\sqrt{4}$, and, since $\sqrt{4}$ = 2, we can further state that $\sqrt{-4}$ = 2***i***. Note that there is an implied multiplication between the ***i*** and 2.

Further note that the imaginary number ***i*** is always written to the RIGHT of integers and fractions. However, it is usually written to the left of radicals!

2. Definition of Complex Numbers (1 of 2)

Given imaginary numbers, we can now define another set of numbers, namely the **complex numbers**.

Complex numbers consist of all numbers in the standard form ***a + bi*** where ***a*** and ***b*** are real numbers, and ***i*** is the basic imaginary number. Note that there is an implied multiplication between ***i*** and *b*.

Examples of complex numbers:

$2 + 3\boldsymbol{i}$ $(a = 2,\ b = 3)$

$-8 - \boldsymbol{i}$ $(a = -8,\ b = -1)$

$1 - \frac{2}{3}\boldsymbol{i}$ $\left(a = 1, b = -\frac{2}{3}\right)$

Definition of Imaginary and Complex Numbers (2 of 2)

Example 1:

a. Write 19 as a complex number $a + b\boldsymbol{i}$.

We only have a real numbers. So, $a = 19$. Since there is no imaginary part, we say $b = 0$ and write 19 as

$19 + 0\boldsymbol{i}$

Please note that ALL real numbers are also complex numbers where $b = 0$.

b. Write $-2\boldsymbol{i}$ as a complex number $a + b\boldsymbol{i}$.

We only have an imaginary number. So, $b = -2$. Since there is no real part, we say $a = 0$ and write $-2\boldsymbol{i}$ as

$0 - 2\boldsymbol{i}$

2. The Conjugate of a Complex Number (1 of 2)

We discussed conjugates already. We stated that given a mathematical expression with exactly two terms being added or subtracted, then the conjugate retains the same terms but the operational sign between the terms changes to the opposite one.

For example, the conjugate of $-2-3x$ is $-2+3x$. Note that the operational sign between the terms changed!

Conjugates of complex numbers are similar. The operational sign between the real and imaginary part changes!

For example, the conjugate of $-2+3\boldsymbol{i}$ is $-2-3\boldsymbol{i}$.

The Conjugate of a Complex Number (2 of 2)

Example 2:

a. Find the conjugate of $3\boldsymbol{i}$.

Here we don't have a real part. We assume it to be 0. We can then write $3\boldsymbol{i}$ as $0 + 3\boldsymbol{i}$. Changing the operational sign between $0 + 3\boldsymbol{i}$, we get

$0 - 3\boldsymbol{i}$ or simply $-3\boldsymbol{i}$

b. Find the conjugate of $-3\boldsymbol{i}$.

Here we don't have a real part. We assume it to be 0. We can then write $-3\boldsymbol{i}$ as $0 - 3\boldsymbol{i}$. Changing the operational sign between $0 - 3\boldsymbol{i}$, we get

$0 + 3\boldsymbol{i}$ or simply $3\boldsymbol{i}$

4. Arithmetic Operations on Complex Numbers (1 of 2)

To add and subtract complex numbers, we combine the real numbers and the coefficients of the number $\boldsymbol{i}$.

Example 3:

Find the sum of the two complex numbers $5 - 2\boldsymbol{i}$ and $3 + 3\boldsymbol{i}$.

We write $(5 - 2\boldsymbol{i}) + (3 + 3\boldsymbol{i})$ or $5 - 2\boldsymbol{i} + 3 + 3\boldsymbol{i}$.

Let's group the real numbers and imaginary numbers as follows:

$5 + 3 - 2\boldsymbol{i} + 3\boldsymbol{i}$

Then, we'll add the real numbers and the coefficients of the number $\boldsymbol{i}$ to get

$8 + \boldsymbol{i}$

Note, that the coefficient of $\boldsymbol{i}$ is 1, but we usually do not write it.

Arithmetic Operations on Complex Numbers (2 of 2)

To multiply complex numbers, we use the FOIL method.

Example 4:

Find the product of $(5 + 4\boldsymbol{i})(6 - 7\boldsymbol{i})$.

Using ***F*** ***O*** ***I*** ***L***

we get $5(6) + 5(-7\boldsymbol{i}) + 4\boldsymbol{i}(6) + 4\boldsymbol{i}(-7\boldsymbol{i}) = 30 - 35\boldsymbol{i} + 24\boldsymbol{i} - 28\boldsymbol{i}^2$

$= 30 - 11\boldsymbol{i} - 28\boldsymbol{i}^2$

Here we need to do some extra steps. We know that $\boldsymbol{i}^2$ is defined to be -1. Therefore, we can continue as follows:

$30 - 11\boldsymbol{i} - 28(-1) = 30 - 11\boldsymbol{i} + 28$

$= 58 - 11\boldsymbol{i}$ (combined the real numbers)