Concepts and Examples
Common Functions

Based on power point presentations by Pearson Education, Inc.

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Learning Objectives

1. Memorize the domain and range, as well as the characteristics of the graphs of the following common functions:
   a. *Absolute Value Function*
   b. *Square Function*
   c. *Square Root Function*
   d. *Cubic Function*
   e. *Cube Root Function*
2. Graph common functions by hand.

1. Common Functions (1 of 6)

In addition to the *Identity* and *Constant Functions*, the following five common functions are also frequently encountered in algebra:

$f(x) = |x|$ $\qquad$ $f(x) = x^3$

$f(x) = x^2$ $\qquad$ $f(x) = \sqrt[3]{x}$

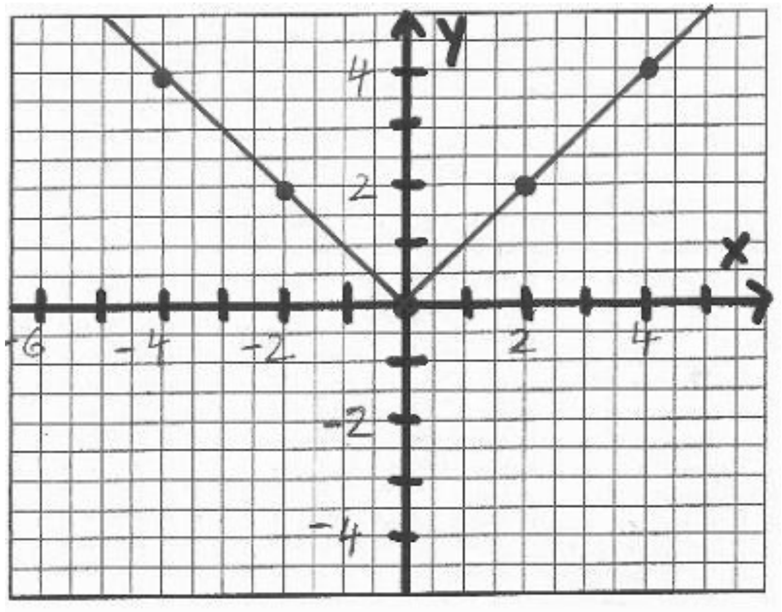
$f(x) = \sqrt{x}$

It is essential to memorize these functions including their domains and ranges and the characteristics of their graphs.

a. The Absolute Value Function (2 of 6)

$$f(x) = |x|$$

Domain: $(-\infty, \infty)$



Basic Characteristics of the Graph:

- The graph touches the axes only at the origin.
- The *x*- and *y*-intercept are both at the origin.
- The graph of this function has a vertex at the origin.
- This graph is V-shaped at its vertex with straight branches. **A V-shaped vertex is also called a "cusp".**

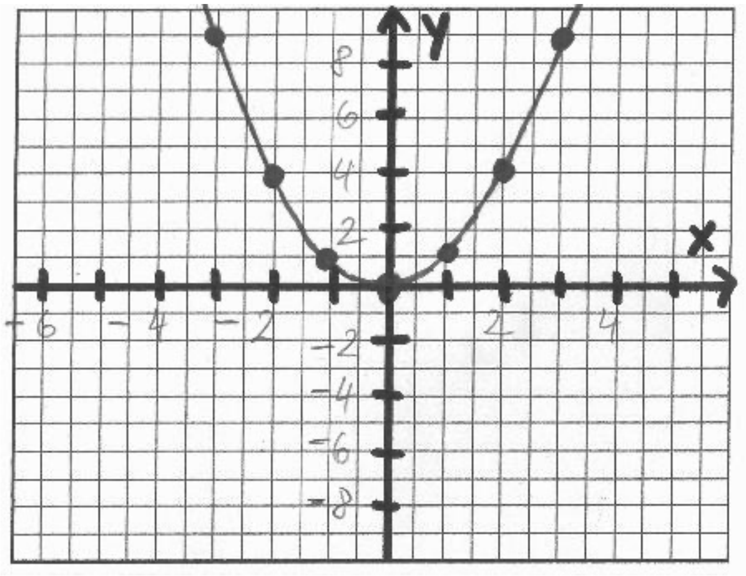
b. The Square Function

$f(x) = x^2$

Domain: $(-\infty, \infty)$

Basic Characteristics of the Graph:

- The graph touches the axes only at the origin.
- The *x*- and *y*-intercepts are both at the origin.
- The graph has a smooth vertex at the origin.
- The graph is U-shaped at its vertex; however, the branches continue to flare away from the *y*-axis.



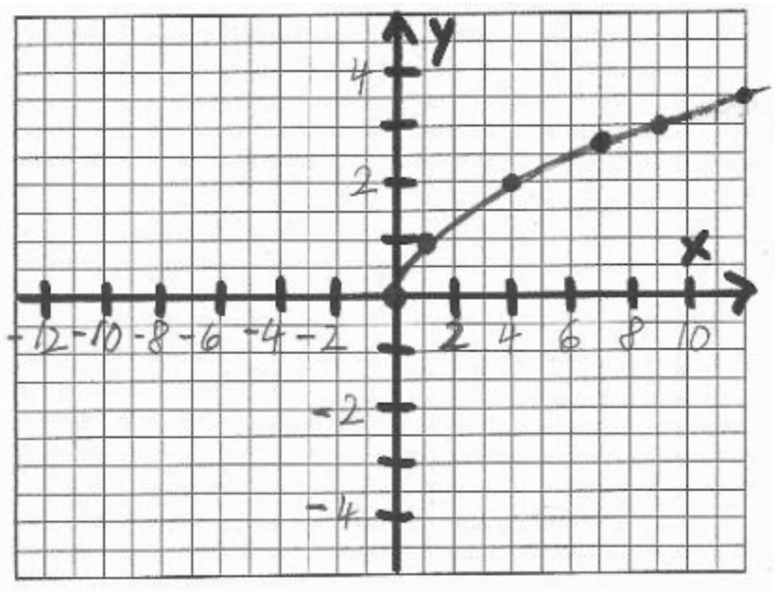
c. The Square Root Function (4 of 6)

$f(x) = \sqrt{x}$

Domain: $[0, \infty)$

Basic Characteristics of the Graph:

- The graph touches the axes only at the origin.
- The *x*- and *y*-intercept are both at the origin.
- The graph of this function starts at the origin.
- This graph consists of a SMOOTH curve which is concave down to the right of the origin.
- The graph is NEVER parallel to the *x*-axis. Instead it moves away from it at a steady pace.



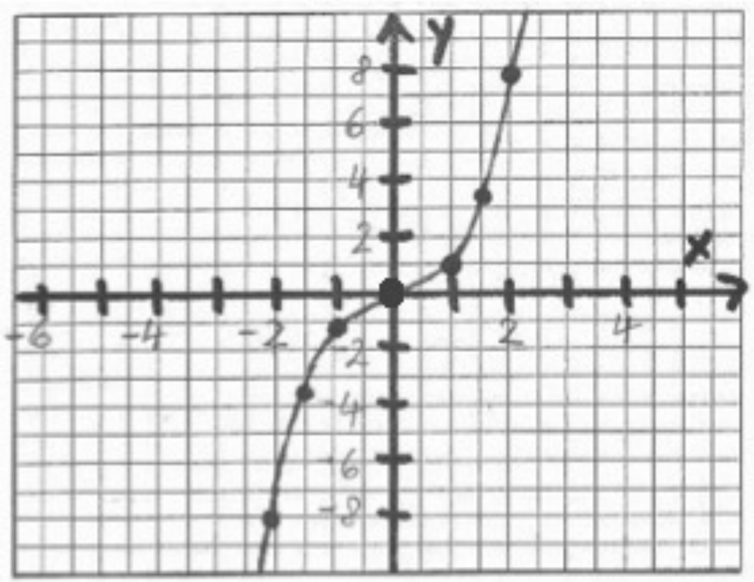
d. The Cubic Function (5 of 6)

$f(x) = x^3$

Domain: $(-\infty, \infty)$

Basic Characteristics of the Graph:

- The graph crosses the axes only at the origin.
- The *x*- and *y*-intercept are both at the origin.
- The graph consists of a SMOOTH curve which is concave up to the right of the origin and concave down to the left.
- The graph of this function changes concavity at the origin.
- The graph is NEVER parallel to the *y*-axis. Instead it moves away from it at a steady pace.



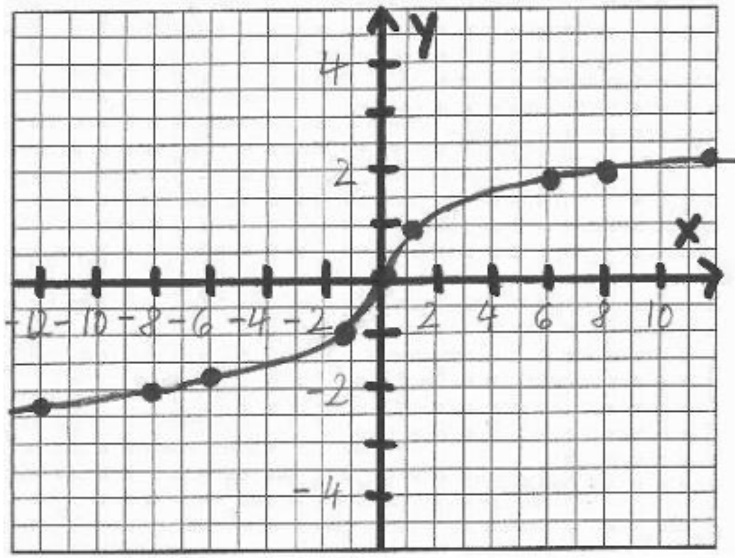
e. The Cube Root Function (6 of 6)

$f(x) = \sqrt[3]{x}$

Domain: $(-\infty, \infty)$

Basic Characteristics of the Graph:

- The graph crosses the axes only at the origin.
- The *x*- and *y*-intercept are both at the origin.
- This graph consists of a SMOOTH curve which is concave down to the right of the origin and concave up to the left.
- The graph of this function changes concavity at the origin.
- The graph is NEVER parallel to the *x*-axis. Instead it moves away from it at a steady pace.



2. Graph Common Functions by Hand

Strategy:

1. Find and plot the coordinates of strategic points, if they exist, such as
   - the coordinates of the vertex of the graph (square and absolute value functions)
   - the point at which concavity changes on the graph (cubic and cube root functions)
   - the point at which the graph of the function starts (square root function)
2. Find and plot two or more other points to help facilitate the shape of the graph (particularly the concavities). If possible, it is best to select points to the right and to the left of the "strategic" point!
3. Connect all points found in the previous steps keeping in mind the shape of the graph.
4. Extend all branches to the appropriate edges of the coordinate system, or we use arrows, if appropriate, to show that the graph continues to infinity!