Examples
Transformations of Common Functions

Based on power point presentations by Pearson Education, Inc.

Revised by Ingrid Stewart, Ph.D.

Learning Objectives

1. Recognize vertical shifts of graphs of common functions.
2. Recognize horizontal shifts of graphs of common functions.
3. Recognize reflections of graphs of common functions.
4. Recognize vertical stretches and compressions of graphs of common functions.

Example 1: Transformations of Functions

Starting with the graph of the common function $f(x) = x^2$, write the equation of the graph that results from the following:

(1) a vertical shift up 3 units

$y = x^2 + 3$

(2) a vertical shift down 3 units

$y = x^2 - 3$

(3) a horizontal shift to the right 3 units

$y = (x - 3)^2$

(4) a horizontal shift to the left 3 units

$y = (x + 3)^2$

Example 2: Transformations of Functions

Starting with the graph of the common function $k(x) = x^3$, write the equation of the graph that results from the following:

(1) a vertical shift up 1 unit and a horizontal shift to the right 2 units

$y = (x - 2)^3 + 1$

(2) a vertical shift up 3 units and a horizontal shift to the left 4 units

$y = (x + 4)^3 + 3$

(3) a vertical shift down 5 units and a horizontal shift to the right 1 unit

$y = (x - 1)^3 - 5$

(4) a vertical shift down 2 units and a horizontal shift to the left 3 unit

$y = (x + 3)^3 - 2$

Example 3: Transformations of Functions

Starting with the graph of the common function $f(x) = |x|$, write the equation of the graph that results from the following:

(1) a reflection across the *x*-axis

$y = -|x|$

(2) a reflection across the *y*-axis

$y = |-x|$

(3) a reflection across the *x*- and *y*-axis

$y = -|-x|$

Example 4: Transformations of Functions

Given $g(x) = \sqrt{x-2} + 1$, do the following:

a. Determine the common function from which the function g can be obtained.

$y = \sqrt{x}$

b. Next, identify each transformation that can be applied to the common function in order to obtain the function *g*.

vertical shift up one unit

horizontal shift to the right 2 units

Example 5: Transformations of Functions

Given $k(x) = (x + 5)^3 - 9$, do the following:

a. Determine the common function from which the function *k* can be obtained.

$y = x^3$

b. Next, identify each transformation that can be applied to the common function in order to obtain the function *k*.

vertical shift down 9 units

horizontal shift to the left 5 units

Example 6: Transformations of Functions

Given $f(x) = \sqrt[3]{x+2} + 10$ do the following:

a. Determine the common function from which the function f can be obtained.

$y = \sqrt[3]{x}$

b. Next, identify each transformation that can be applied to the common function in order to obtain the function f .

vertical shift up 10 units

horizontal shift to the left 2 units

Example 7: Transformations of Functions

Given $h(x) = (x - 7)^2 - 2$, do the following:

a. Determine the common function from which the function *h* can be obtained.

$y = x^2$

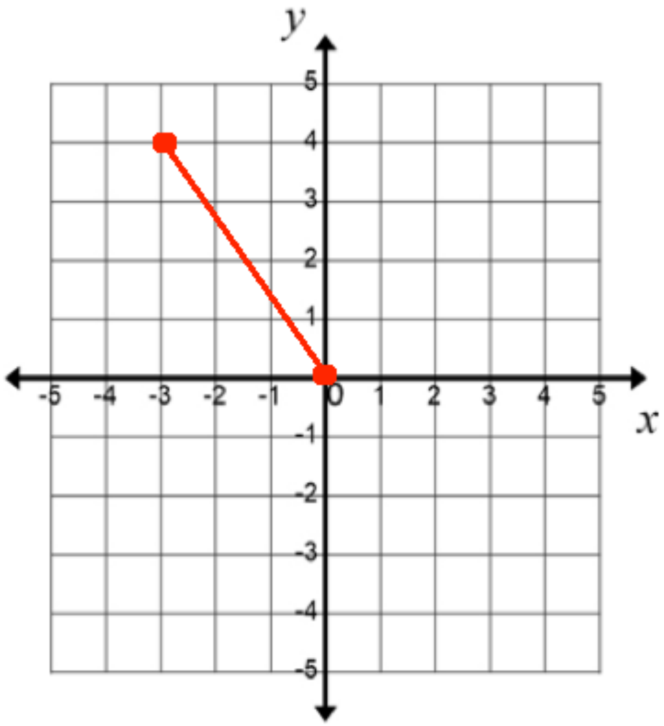
b. Next, identify each transformation that can be applied to the common function in order to obtain the function *h*.

vertical shift down 2 units

horizontal shift to the right 7 units

Example 8: Transformations of Functions (1 of 2)

The graph of the function *f* is shown below. Graph its transformation which is the function $g(x) = f(x - 2) - 1$.



$g(x) = f(x - 2) - 1$ indicates a vertical shift of the graph of *f* of 1 unit down and a horizontal shift of 2 units to the right. We see that the graph of the function *f* contains the points $(-3, 4)$ and $(0, 0)$.

We know that a vertical shift affects *y*-coordinates, and a horizontal shift affects *x*-coordinates.

Subsequently, the graph of the function *g* contains the points $(-3 + 2, 4 - 1) = (-1, 3)$ and $(0 + 2, 0 - 1) = (2, -1)$.

Please note during a horizontal shift right as it pertains to points, we ADD the appropriate number of units.

Example 7: Transformations of Functions (2 of 2)

Below is the graph of the function *f* in red and the graph of its transformation *g* in blue.