Examples
Systems of Linear Equations in Three Variables

Based on power point presentations by Pearson Education, Inc.
Revised by Ingrid Stewart, Ph.D.

Learning Objectives

1. Draw a three-dimensional coordinate system.
2. Memorize the characteristics of a system of three linear equations in three variables.
3. Solve systems of three linear equations in three variables.

Example 1: Solve a System (1 of 5)

Solve the following system. Write its solution as an ordered triple.

$$\begin{cases} -2x-4y-2z=-18 & \text{Equation 1} \\ -4x-y+2z=10 & \text{Equation 2} \\ 4x+3y+2z=10 & \text{Equation 3} \end{cases}$$

Let's select Equation 1 and Equation 2 and eliminate the *z*-terms. This will result in a new equation in *x* and *y*.

$$\begin{array}{rll} & -2x-4y-2z=-18 & \text{Equation 1} \\ + & -4x-y+2z=10 & \text{Equation 2} \\ \hline & -6x-5y=-8 & \text{(new) Equation 4} \end{array}$$

Example 1: Solve a System (2 of 5)

Next, let's use Equation 1 and Equation 3 in the system and also eliminate the *z*-terms. This will result in another new equation also in *x* and *y*.

$$
\begin{array}{rl}
 & -2x - 4y - 2z = -18 \quad \text{Equation 1} \\
+ & 4x + 3y + 2z = 10 \quad \text{Equation 3} \\
\hline
 & 2x - y = -8 \quad \text{(new) Equation 5}
\end{array}
$$

Example 1: Solve a System (3 of 5)

Now, we will solve a system containing Equations 4 and 5 by eliminating the *x*-terms. This lets us find the *y*-coordinate of the point of intersection (solution).

$$\begin{cases} -6x - 5y = -8 & \text{Equation 4} \\ 2x - y = -8 & \text{Equation 5} \end{cases}$$

However, here we will first multiply all terms in Equation 5 by the number 3 to get the following:

$$\begin{array}{rll} & -6x - 5y = -8 & \text{Equation 4} \\ + & 6x - 3y = -24 & 3 \cdot \text{Equation 5} \\ \hline & -8y = -32 & \\ & y = 4 & \end{array}$$

This is the *y*-coordinate of the point of intersection!

Example 1: Solve a System (4 of 5)

Next, we will substitute the y in either Equation 4 or 5 with $y = 4$ to find the x-coordinate of the point of intersection (solution).

Let’s use Equation 4 and replace its y with $y = 4$.

$-6x - 5y = -8$ (Equation 4)

$-6x - 5(4) = -8$

$-6x - 20 = -8$

$-6x = 12$

$x = -2$ This is the x-coordinate of the point of intersection!

Example 1: Solve a System

Finally, we will substitute x and y in either Equation 1, 2, or 3 with $y = 4$ and $x = -2$ into to find the z-coordinate of the point of intersection (solution).

Let's use Equation 1 and replace its y with $y = 4$ and its x with $x = -2$.

$-2x - 4y - 2z = -18$ (Equation 1)

$-2(-2) - 4(4) - 2z = -18$

$4 - 16 - 2z = -18$

$-12 - 2z = -18$

$-2z = 6$

$z = 3$ This is the z-coordinate of the point of intersection!

We can now write the solution of the system as the ordered triple $(-2, 4, 3)$.

Example 2: Solve a System (1 of 5)

Solve the following system. Write its solution as an ordered triple.

$$\begin{cases} 2x + 5y + 6z = 19 & \text{Equation 1} \\ \quad\quad y - 2z = 0 & \text{Equation 2} \\ x - y \quad\quad = 2 & \text{Equation 3} \end{cases}$$

Let's select Equation 1 and Equation 2 and eliminate the *y*-terms. This will result in a new equation in *x* and *z*.

However, here we will first multiply all terms of Equation 2 by the number – 5 to get the following:

$$\begin{array}{rrrl} & 2x + 5y + 6z = 19 & & \text{Equation 1} \\ + & -5y + 10z = 0 & & -5 \cdot \text{Equation 2} \\ \hline & 2x \quad\quad + 16z = 19 & & \text{(new) Equation 4} \end{array}$$

Example 2: Solve a System (2 of 5)

Next let's use Equation 1 and Equation 3 and also eliminate the *y*-terms. This will result in another new equation in *x* and *z*.

$$2x + 5y + 6z = 19 \quad \text{Equation 1}$$

$$x - y = 2 \quad \text{Equation 3}$$

However, we will have to multiply all terms of Equation 3 by the number 5 to get the following:

$$\begin{array}{rl} 2x + 5y + 6z = 19 & \text{Equation 2} \\ +\ 5x - 5y \qquad = 10 & 5 \cdot \text{Equation 3} \\ \hline 7x \qquad + 6z = 29 & \text{(new) Equation 5} \end{array}$$

Example 2: Solve a System (3 of 5)

Now, we will solve a system containing Equations 4 and 5 by eliminating the *x*-terms. This lets us find the *z*-coordinate of the point of intersection.

$$\begin{cases} 2x + 16z = 19 & \text{Equation 4} \\ 7x + 6z = 29 & \text{Equation 5} \end{cases}$$

However, here we will first multiply all terms in Equation 4 by the number 7 and all terms in Equation 5 by the number – 2 to get the following:

$$\begin{array}{rrl} & 14x + 112z = 133 & 7 \cdot \text{Equation 4} \\ + & -14x - 12z = -58 & -2 \cdot \text{Equation 5} \\ \hline & 100z = 75 & \end{array}$$

$$z = \frac{75}{100} = \frac{3}{4}$$

This is the *z*-coordinate of the point of intersection!

Example 2: Solve a System (4 of 5)

Next, we will substitute z in either Equation 4 or 5 with $z = \frac{3}{4}$ to find the x-coordinate of the point of intersection (solution).

Let’s use Equation 4 and replace its z with $z = \frac{3}{4}$.

$2x + 16z = 19$ (Equation 4)

$2x + 16\left(\frac{3}{4}\right) = 19$

$2x + 12 = 419$

$2x = 7$

$x = \frac{7}{2}$ This is the x-coordinate of the point of intersection!

Example 2: Solve a System (5 of 5)

Finally, we will substitute *x* and *z* in either Equation 1, 2, or 3 with $x = \frac{7}{2}$ and $z = \frac{3}{4}$ to find the *y*-coordinate of the point of intersection (solution).

Let's use Equation 1 which is $2x + 5y + 6z = 19$ and replace its *z* with $z = \frac{3}{4}$ and its *x* with $x = \frac{7}{2}$.

$$2\left(\frac{7}{2}\right) + 5y + 6\left(\frac{3}{4}\right) = 19$$

$$7 + 5y + \frac{18}{4} = 19$$

$$\frac{46}{4} + 5y = 19$$

$$5y = \frac{30}{4}$$

$y = \frac{3}{2}$ This is the *y*-coordinate of the point of intersection!

We can now write the solution of the system as the ordered triple $\left(\frac{7}{2}, \frac{3}{2}, \frac{3}{4}\right)$.

Example 3: Solve a System (1 of 5)

Solve the following system. Write its solution as an ordered triple.

$$\begin{cases} x+4y-z=20 & \text{Equation 1} \\ 3x+2y+z=8 & \text{Equation 2} \\ 2x-3y+2z=-16 & \text{Equation 3} \end{cases}$$

Let's select Equation 1 and Equation 2 and eliminate the *z*-terms. This will result in a new equation in *x* and *y*.

$$\begin{array}{rl} x+4y-z=20 & \text{Equation 1} \\ +\ \underline{3x+2y+z=8} & \text{Equation 2} \\ 4x+6y=28 & \text{(new) Equation 4} \end{array}$$

Example 3: Solve a System (2 of 5)

Next, let's use Equation 1 and Equation 3 in the system and also eliminate the *z*-terms. This will result in another new equation also in *x* and *y*.

$x + 4y - z = 20$	Equation 1
$2x - 3y + 2z = -16$	Equation 3

However, here we will first multiply all terms of Equation 1 by the number 2 to get the following:

$2x + 8y - 2z = 40$	2 · Equation 1
$+ \; 2x - 3y + 2z = -16$	Equation 3
$4x + 5y = 24$	(new) Equation 5

Example 3: Solve a System (3 of 5)

Now, we will solve a system containing Equations 4 and 5 by eliminating the *x*-terms. This lets us find the *y*-coordinate of the point of intersection (solution).

$$\begin{cases} 4x + 6y = 28 & \text{Equation 4} \\ 4x + 5y = 24 & \text{Equation 5} \end{cases}$$

However, here we will first multiply all terms in Equation 5 by the number – 1 to get the following:

$$\begin{array}{rl} 4x + 6y = 28 & \text{Equation 4} \\ + \quad \underline{-4x - 5y = -24} & -1 \cdot \text{Equation 5} \\ y = 4 & \text{This is the } y\text{-coordinate of the point of intersection!} \end{array}$$

Example 3: Solve a System (4 of 5)

Next, we will substitute y in either Equation 4 or 5 with $y = 4$ to find the x-coordinate of the point of intersection (solution).

Let's use Equation 5 and replace its y with $y = 4$.

$4x + 5y = 24$ (Equation 5)

$4x + 5(4) = 24$

$4x + 20 = 24$

$4x = 4$

$x = 1$ This is the x-coordinate of the point of intersection!

Example 3: Solve a System (5 of 5)

Finally, we will substitute x and y in either in either Equation 1, 2, or 3 with $x = 1$ and $y = 4$ to find the z-coordinate of the point of intersection (solution).

Let's use Equation 2 and replace its y with $y = 4$ and its x with $x = 1$.

$3x + 2y + z = 8$ (Equation 2)

$3(1) + 2(4) + z = 8$

$3 + 8 + z = 8$

$11 + z = 8$

$z = -3$ This is the z-coordinate of the point of intersection!

We can now write the solution of the system as the ordered triple $(1, 4, -3)$.