Concepts
Systems of Linear Inequalities in Two Variables

Based on power point presentations by Pearson Education, Inc.
Revised by Ingrid Stewart, Ph.D.

Learning Objectives

1. Graph linear inequalities in two variables.
2. Solve systems of two linear inequalities in two variables.

Example 1: Graph a Linear Inequality in Two Variables (1 of 4)

Graph $2x - y \geq 4$.

Step 1 - Replace the inequality symbol with an equal sign.

$2x - y = 4$

Step 2 – Graph the boundary line.

Using $2x - y = 4$, we will use the intercept method to graph it.

Coordinates of the point associated with the *x*-intercept (when $y = 0$):

$2x - (0) = 4$

$x = 2$

The coordinates of the point associated with the *x*-intercept are (2, 0).

Coordinates of the point associated with the *y*-intercept (when $x = 0$):

$2(0) - y = 4$

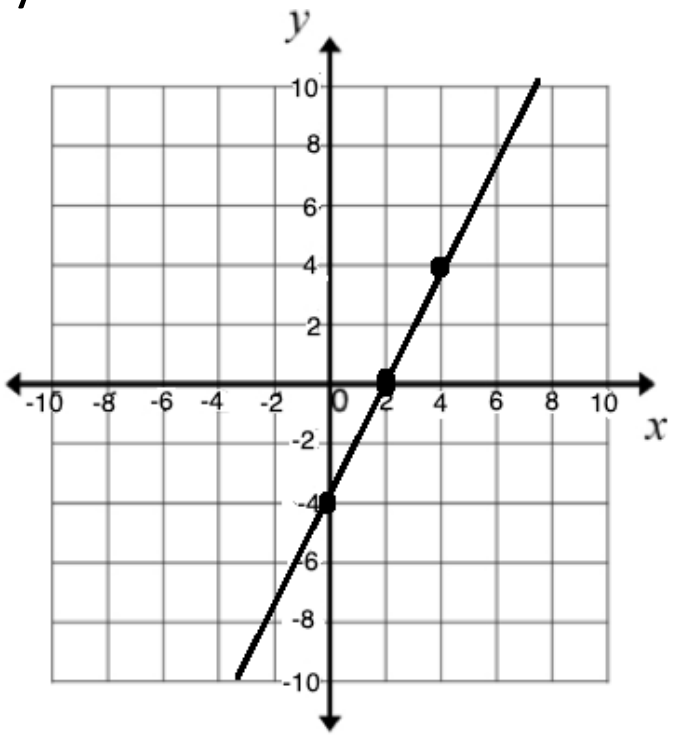
$y = -4$

The coordinates of the point associated with the *y*-intercept are (0, – 4).

Example 1: Graph a Linear Inequality in Two Variables (2 of 4)

Step 2 continued

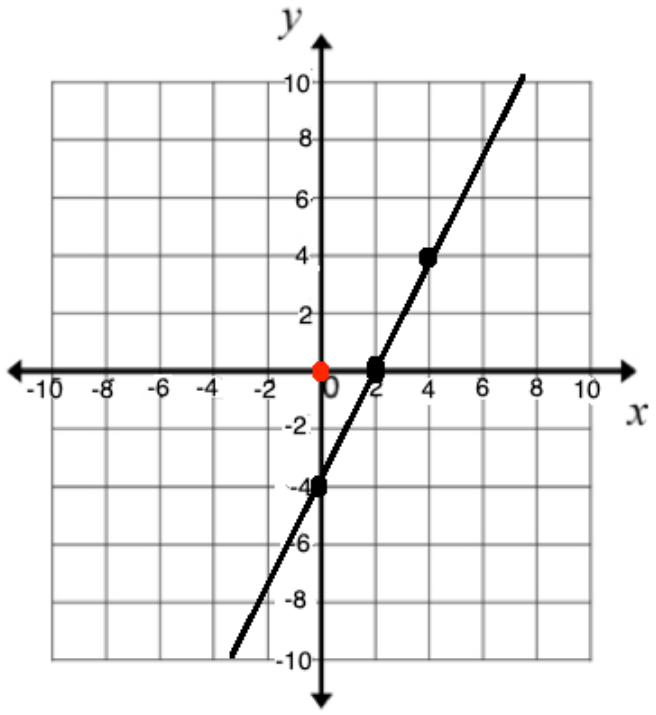
Using the points associated with the intercepts and the additional point (4, 4), we graph a solid line since the inequality contains the "greater than or equal to" (≥) symbol. This is our "boundary" line.



Example 1: Graph a Linear Inequality in Two Variables (3 of 4)

Step 3 - Choose a test point.

The easiest test point to use is (0, 0) unless it lies on the boundary line. In our case, we can use it. See graph below. We will then substitute the coordinates of the test point into the inequality.



$2x - y \geq 4$ the inequality

$2(0) - 0 \geq 4$? using (0, 0)

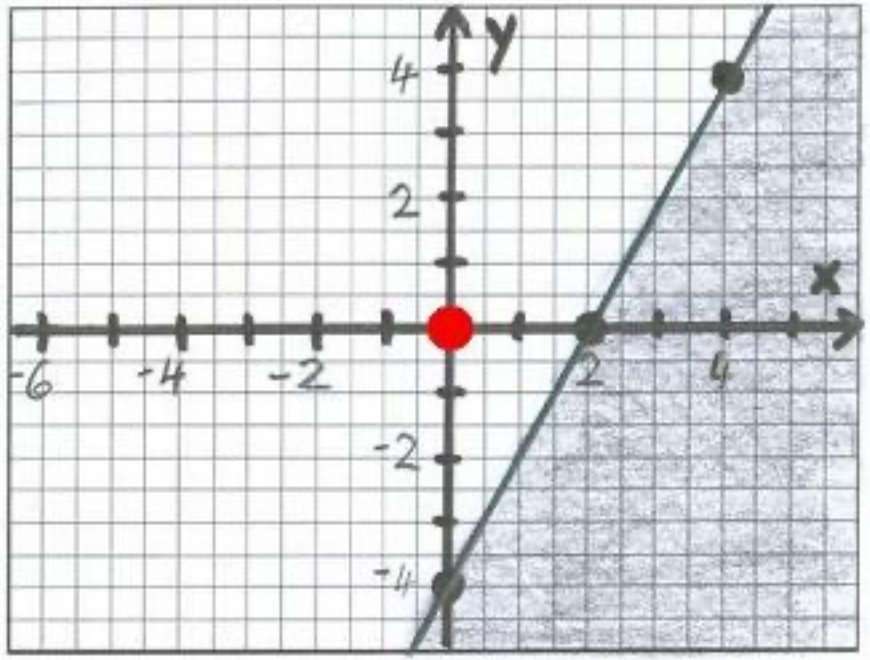
$0 \geq 4$

False (since 0 is not greater than 4 or equal to 4)

Example 1: Graph a Linear Inequality in Two Variables (4 of 4)

Step 4 - Shade the appropriate side of the boundary line.

Since the inequality is false at (0, 0), we shade the side of the boundary line that does not contain (0, 0). This is the graph of $2x - y \geq 4$.



Example 2: Graph a Linear Inequality in Two Variables (1 of 5)

Graph $y > 2x$.

Step 1 - Replace the inequality symbol with an equal sign.

$y = 2x$

Step 2 – Graph the boundary line.

Using $y = 2x$, we will use the intercept method to graph it.

Coordinates of the point associated with the *x*-intercept (when $y = 0$):

$0 = 2x$

$x = 0$

The coordinates of the point associated with the *x*-intercept are (0, 0).

Coordinates of the point associated with the *y*-intercept (when $x = 0$):

$y = 2(0)$

$y = 0$

The coordinates of the point associated with the *y*-intercept are (0, 0).

Example 2: Graph a Linear Inequality in Two Variables (2 of 5)

Step 2 continued

We find that the intercept method did not result in two data points ... and without two data points we cannot graph a line. Let's use the Point-by-Point Plotting Method to find the coordinates of two other points.

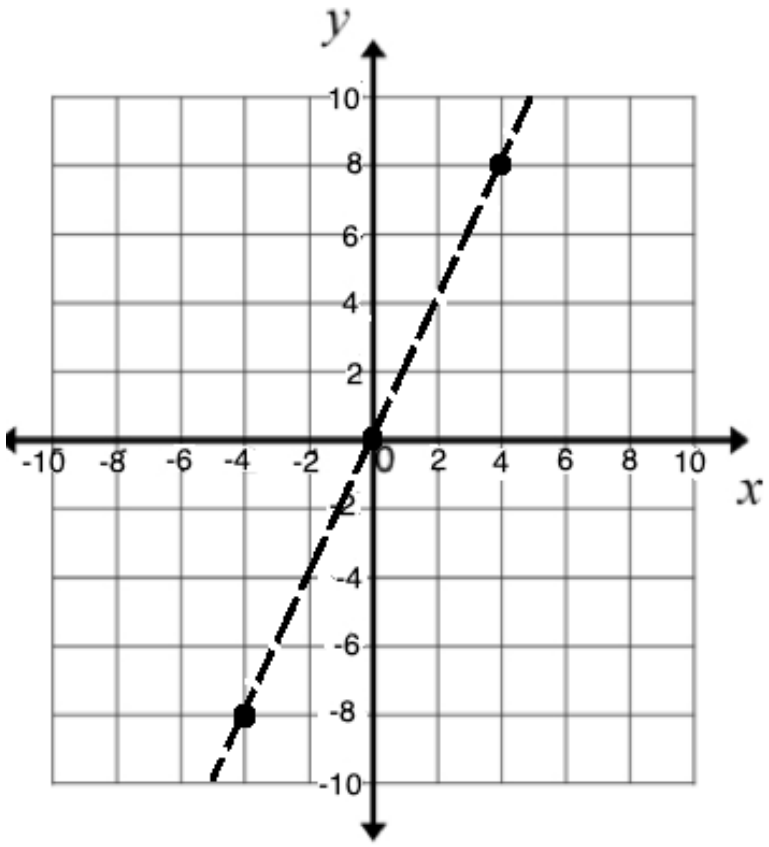
Let's use $x = 4$, then $y = 2(4)$ and $y = 8$. The coordinates of the additional point are $(4, 8)$.

Let's also use $x = -4$, then $y = 2(-4)$ and $y = -8$. The coordinates of the second point are $(-4, -8)$.

Example 2: Graph a Linear Inequality in Two Variables (3 of 5)

Step 2 continued

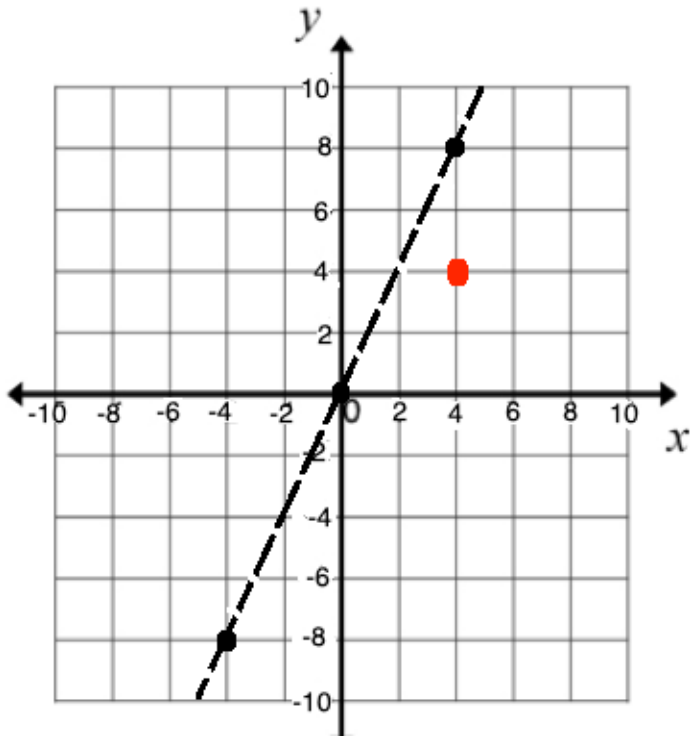
Using the three points we just found, we graph a dashed line since the inequality contains the “greater than” (>) symbol. This is our “boundary” line.



Example 2: Graph a Linear Inequality in Two Variables (4 of 5)

Step 3 - Choose a test point.

We will choose any test point not lying on the boundary line. How about (4, 4)? See graph below. We will then substitute the coordinates of the test point into the inequality.



$y > 2x$ the inequality

$4 > 2(4)$? using (4, 4)

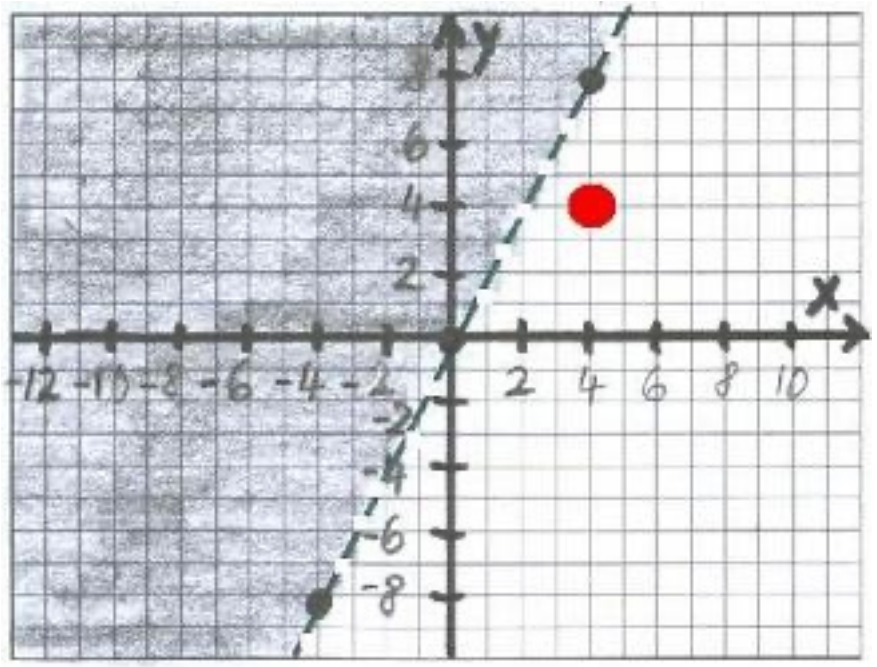
$4 > 8$

False (since 4 is not greater than 8)

Example 2: Graph a Linear Inequality in Two Variables (5 of 5)

Step 4 - Shade the appropriate side of the boundary line.

Since the inequality is false at (4, 4), we shade the side of the boundary line that does not contain (4, 4). This is the graph of $y > 2x$.



Example 3: Graph a Linear Inequality in Two Variables (1 of 5)

Graph $3y > -9x$.

Step 1 - Replace the inequality symbol with an equal sign and graph the linear equation.

$3y = -9x$

Step 2 – Graph the boundary line.

Using $3y = -9x$, we will use the intercept method to graph it.

Coordinates of the point associated with the *x*-intercept (when $y = 0$):

$3(0) = -9x$

$x = 0$

The coordinates of the point associated with the *x*-intercept are (0, 0).

Coordinates of the point associated with the *y*-intercept (when $x = 0$):

$3y = -9(0)$

$y = 0$

The coordinates of the point associated with the *y*-intercept are (0, 0).

Example 3: Graph a Linear Inequality in Two Variables (2 of 5)

Step 2 continued

We find that the intercept method did not result in two data points ... and without two data points we cannot graph a line. Let's use the Point-by-Point Plotting Method to find the coordinates of two other points.

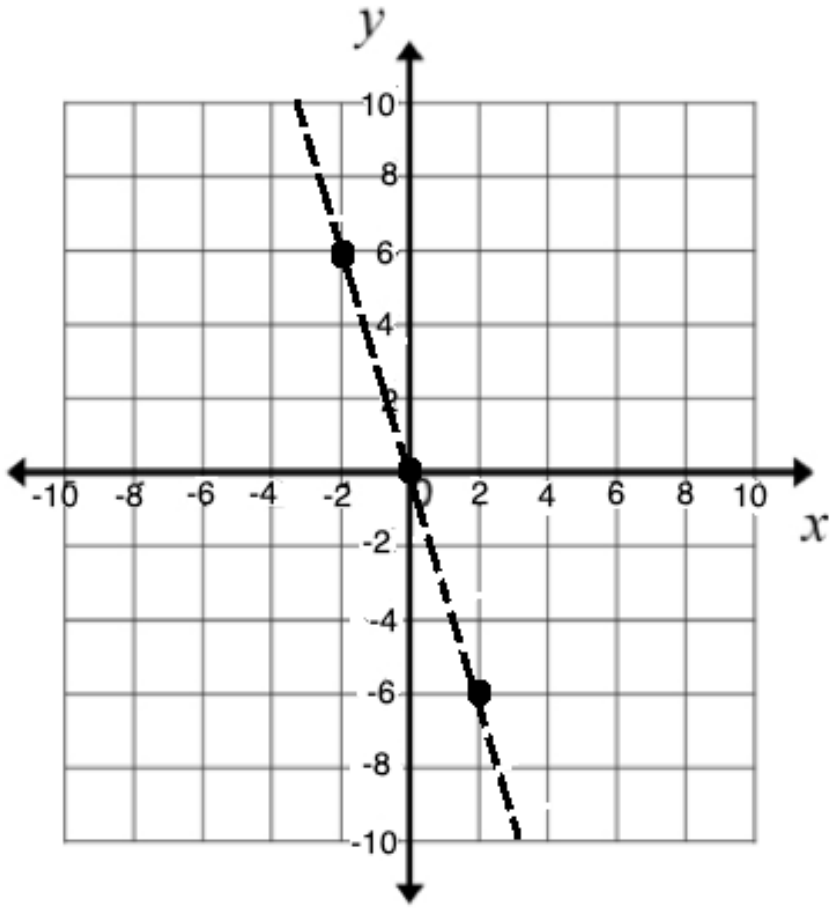
Let's use $x = 2$, then $3y = -9(2)$ and $y = -6$. The coordinates of the additional point are $(2, -6)$.

Let's also use $x = -2$, then $3y = -9(-2)$ and $y = 6$. The coordinates of the second point are $(-2, 6)$.

Example 3: Graph a Linear Inequality in Two Variables (3 of 5)

Step 2 continued

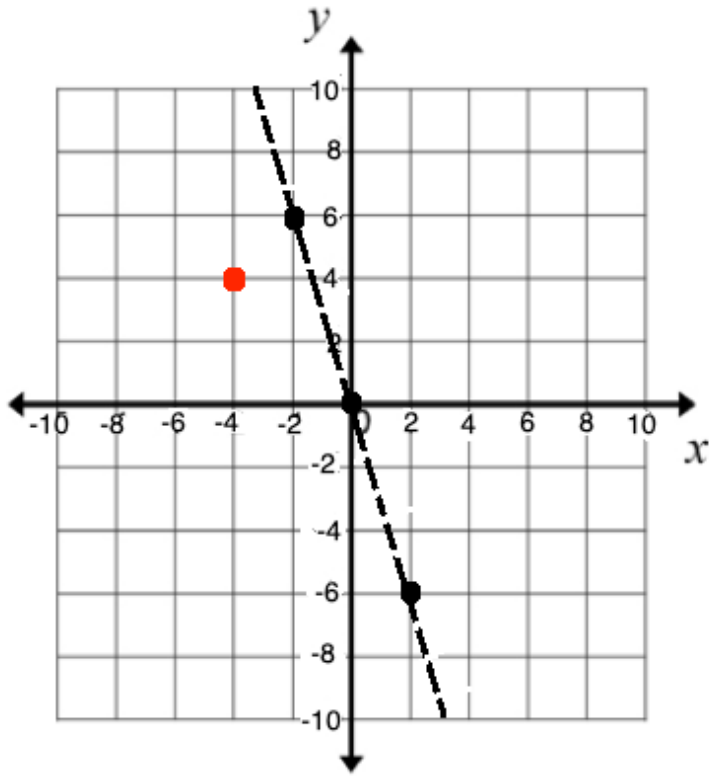
Using the points we just found, we graph a dashed line since the inequality contains the “greater than” (>) symbol. This is our “boundary” line.



Example 3: Graph a Linear Inequality in Two Variables (4 of 5)

Step 3 - Choose a test point.

We will choose any test point not lying on the boundary line. How about (– 4, 4)? See graph below. We will then substitute the coordinates of the test point into the inequality.



$3y > -9x$ the inequality

$3(4) > -9(-4)$? using (– 4, 4)

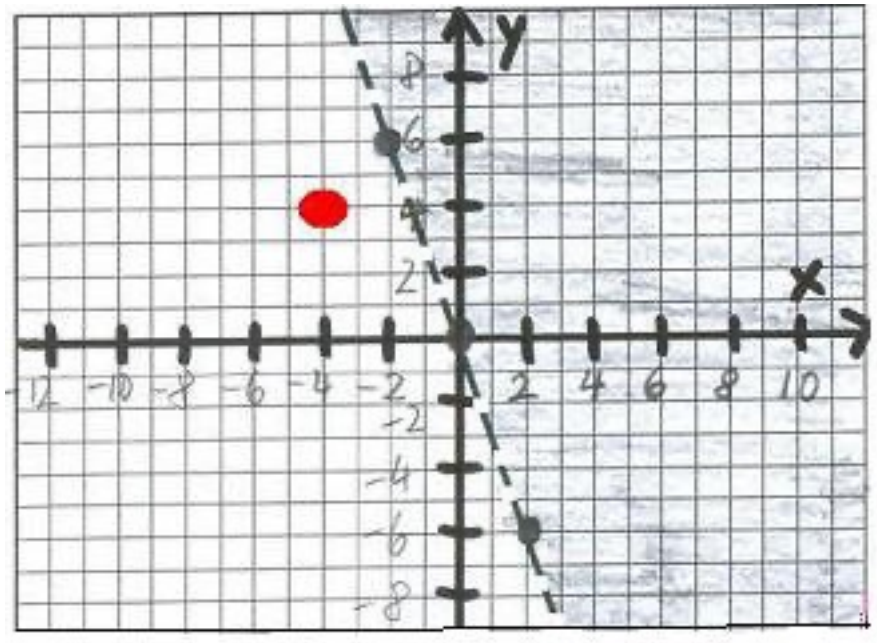
$12 > 36$

False (since 12 is not greater than 36)

Example 3: Graph a Linear Inequality in Two Variables (5 of 5)

Step 4 - Shade the appropriate side of the boundary line.

Since the inequality is false at (– 4, 4), we shade the side of the boundary line that does not contain (– 4, 4). This is the graph of $3y > -9x$.



Example 4: Graph a Linear Inequality in Two Variables (1 of 2)

Graph $y \geq 0$.

Step 1 - Replace the inequality symbol with an equal sign.

$y = 0$

Step 2 – Graph the boundary line.

Given $y = 0$, we see immediately that we are dealing with a horizontal line. Specifically, it is the x-axis. This is our boundary line.

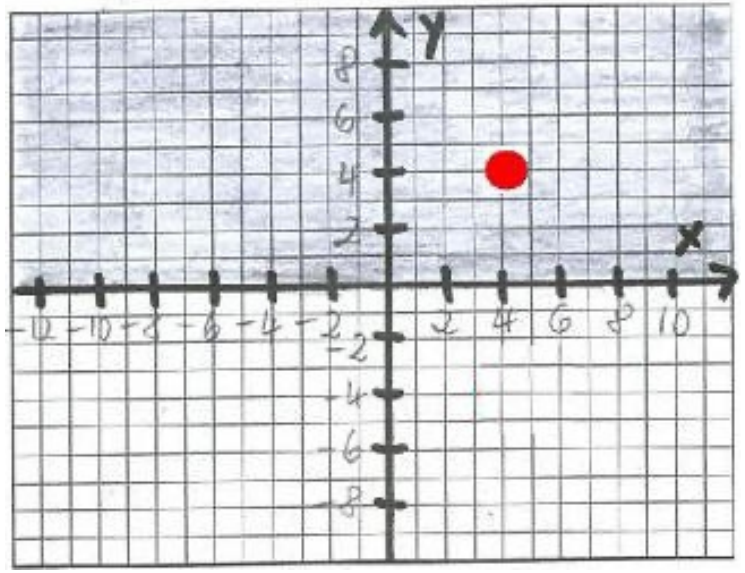
Since the inequality contains the "greater than or equal to" (≥) symbol the x-axis is included in the graph.

Example 4: Graph a Linear Inequality in Two Variables (2 of 2)

Steps 3 & 4 - Choose a test point then shade the appropriate side of the boundary line.

We use the test point (4, 4) in $y \geq 0$ to get $4 \geq 0$. **Please note while 4 is not equal to 0 it is indeed greater than 0!** Therefore, the statement is true. The symbol ≥ means one or the other or both.

We shade the side of the boundary line that contains (4, 4). See graph below.



Example 5: Graph a Linear Inequality in Two Variables (1 of 2)

Graph $x \leq 0$.

Step 1 - Replace the inequality symbol with an equal sign.

$x = 0$

Step 2 – Graph the boundary line.

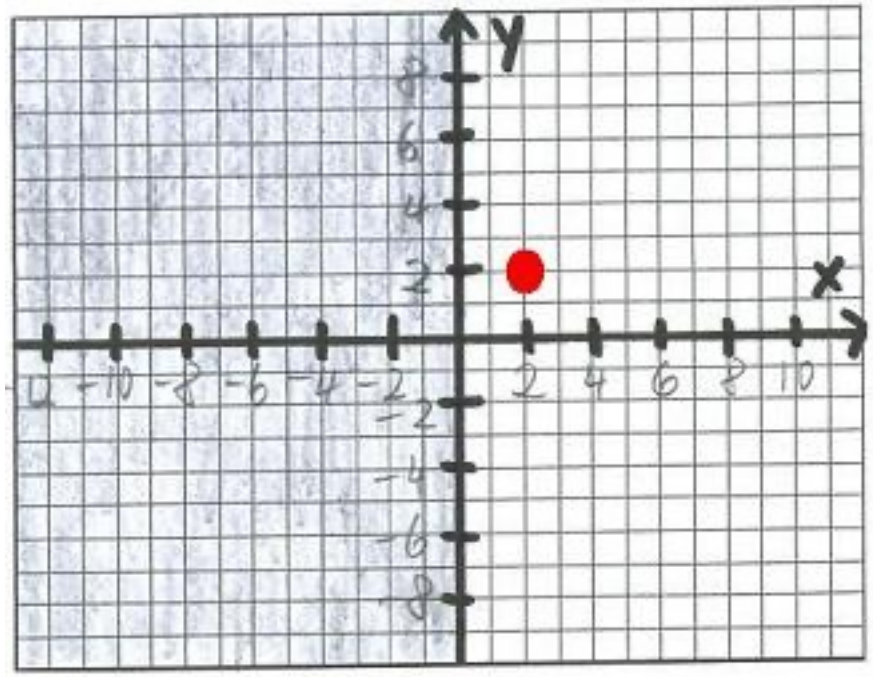
Given $x = 0$, we see immediately that we are dealing with a vertical line. Specifically, it is the *y*-axis. This is our boundary line.

Since the inequality contains the "less than or equal to" ($\leq$) symbol the *y*-axis is included in the graph.

Example 5: Graph a Linear Inequality in Two Variables (2 of 2)

Steps 3 & 4 - Choose a test point then shade the appropriate side of the boundary line.

We use the test point (2, 2) in $x \leq 0$ to get $2 \leq 0$, which is a false statement since 0 is definitely not greater than 2. Therefore, we shade the side of the boundary line that does NOT contain (2, 2). See graph below.

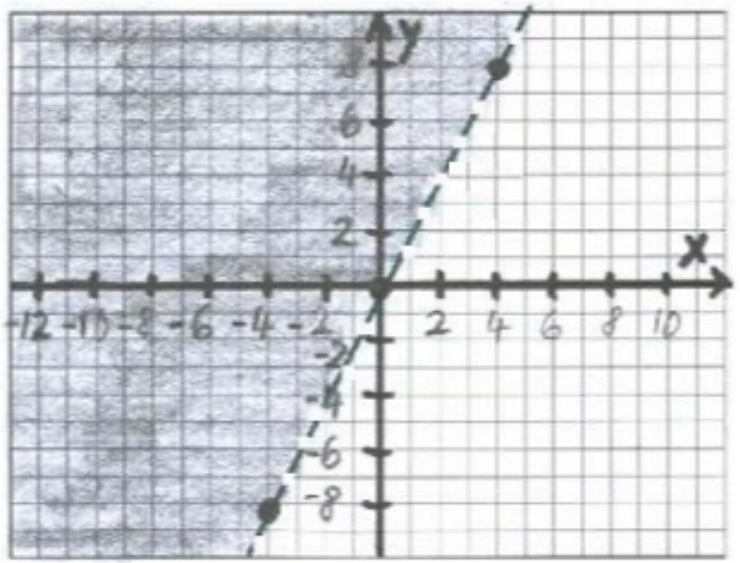


Example 6: Solve a System of Linear Inequalities (1 of 2)

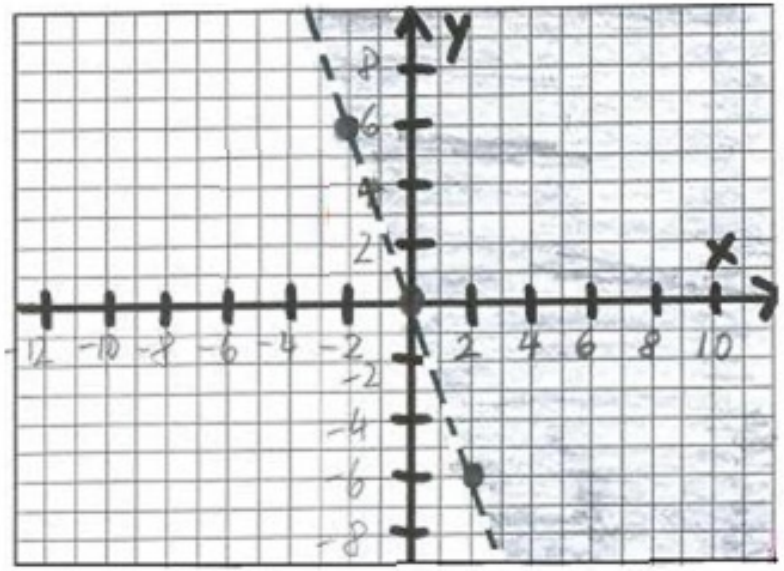
Solve $\begin{cases} y > 2x \\ 3y > -9x \end{cases}$

We can only solve this system graphically. We already graphed the two inequalities in Examples 2 and 3.

$y > 2x$ (Example 2)

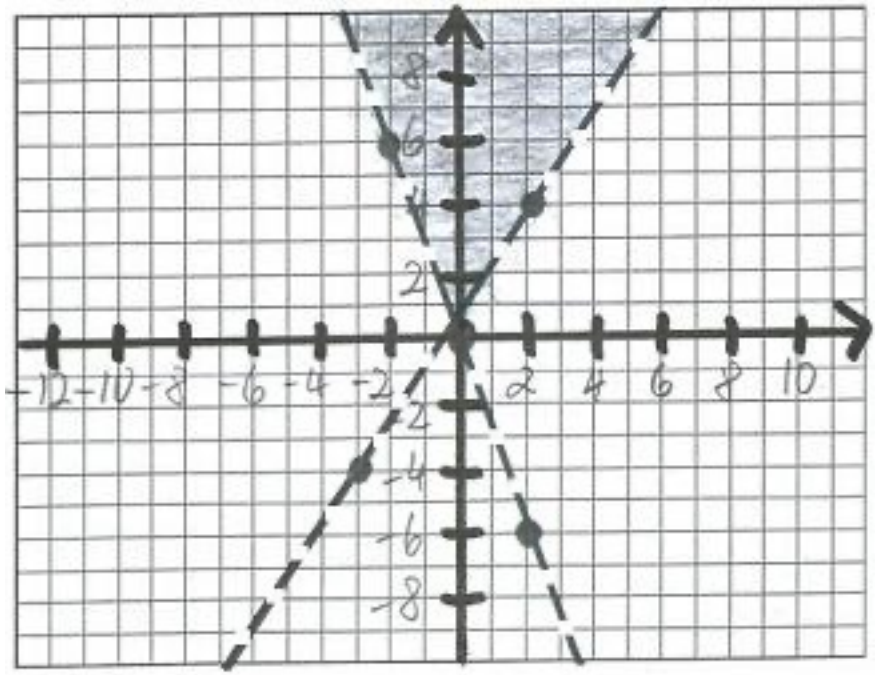


$3y > -9x$ (Example 3)



Example 6: Solve a System of Linear Inequalities (2 of 2)

Now we can determine the solution of the system of these two inequalities by observing where the shaded regions overlap. See graph below. It might take some imagination! You can always print out the two pictures and then place them on top of each other!

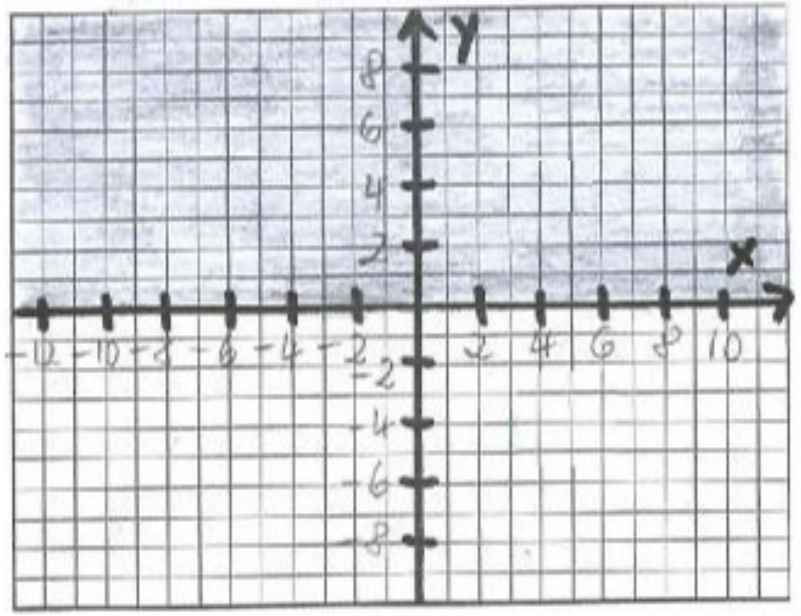


Example 7: Solve a System of Linear Inequalities (1 of 2)

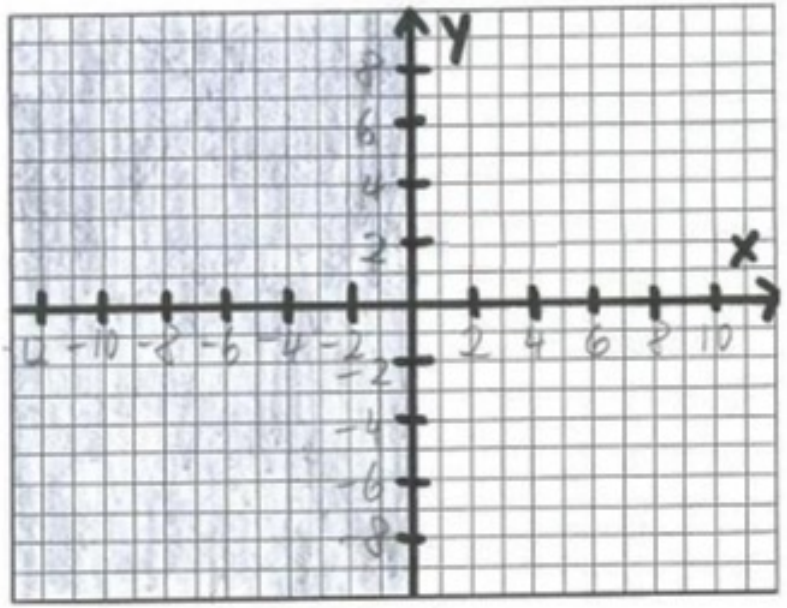
Solve $\begin{cases} y \geq 0 \\ x \leq 0 \end{cases}$

We can only solve this system graphically. We already graphed the two inequalities in Examples 4 and 5.

$y \geq 0$ (Example 4)



$x \leq 0$ (Example 5)



Example 7: Solve a System of Linear Inequalities (2 of 2)

Now we can determine the solution of the system of these two inequalities by observing where the shaded regions overlap. See graph below. It might take some imagination! You can always print out the two pictures and then place them on top of each other!