Examples
Uses of the Slope of a Line

Based on power point presentations by Pearson Education, Inc.
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Learning Objectives

1. Define and calculate the slope of a line.
2. Identify the slopes of increasing, decreasing, vertical, and horizontal lines.
3. Identify the slope and the y-intercept in the equation of a line.
4. Write the slope-intercept equation of a line, if possible.
5. Identify and use slopes of parallel and perpendicular lines.

Example 1: Calculate the Slope of a Line

Find the slope of the line passing through the points determined by the ordered pairs (− 1, 3) and (− 4, − 6).

We will let (− 1, 3) equal (x_1, y_1) and (− 4, − 6) equal (x_2, y_2). However, you can also let (− 4, − 6) equal (x_1, y_1) and (− 1, 3) equal (x_2, y_2). In either case, you will get the same answer.

Let's say that (− 4, − 6) equals (x_1, y_1) and (− 1, 3) equals (x_2, y_2). Be sure not to get confused!

Then $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{3-(-6)}{-1-(-4)} = \dfrac{3+6}{-1+4} = \dfrac{9}{3} = 3$

Example 2: Calculate the Slope of a Line

Find the slope of the line passing through the points (6, 3) and (6, 4).

Let (6, 3) equal (x_1, y_1) and (6, 4) equal (x_2, y_2). Be sure not to get confused!

Then $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-3}{6-6} = \frac{1}{0}$

Since there is a 0 in the denominator, this particular **slope is undefined**.

Example 3: Calculate the Slope of a Line

Find the slope of the line passing through the points determined by the ordered pairs (1, 5) and (− 9, 5).

Let (1, 5) equal (x_1, y_1) and (− 9, 5) equal (x_2, y_2). Be sure not to get confused!

Then $m = \frac{y_2 - y_1}{x_2 - y_{x1}} = \frac{5-5}{-9-1} = \frac{0}{-10} = 0$

Since there is a 0 in the numerator, this particular **slope equals 0**. Please note the difference between this example and Example 2!

Example 4: Identify the Slopes of Lines

Identify the slopes of the graphs of the following lines. State whether the lines are increasing, decreasing, horizontal, or vertical.

1. $y = 3x + 9$

   $m = 3$, the slope is positive, therefore, the line is increasing

2. $y = -5x - 2$

   $m = -2$, the slope is negative, therefore, the line is decreasing

3. $y = 6$

   horizontal line, $m = 0$

4. $x = -1$

   vertical line, m is undefined

Example 5: Identify the Slope and the *y*-Intercept

Identify the slope, the *y*-intercept and the ordered pair associated with the *y*-intercept given the equation of the line $5x + 4y = 9$.

Please note that the equation is not in any particular form. We must change it to *slope-intercept form y = mx + b* to find the slope *m* and the *y*-intercept *b*.

First, we will move the *x*-term to the right side of the equation into its proper position next to the equal sign as follows:

$4y = -5x + 9$

Next, we divide BOTH sides of the equation by 4 to get the following:

$y = -\frac{5}{4}x + \frac{9}{4}$ Please note that every term on the right side had to be divided by 4.

We find that the slope is $-\frac{5}{4}$ and the *y*-intercept is $\frac{9}{4}$.

The ordered pair associated with the *y*-intercept is $\left(0, \frac{9}{4}\right)$.

Example 6: Write the Slope–Intercept Equation of a Line

Find the equation of a line with slope $m = -5$ that passes through the point created by the ordered pair $(-1, -4)$. If possible, write the equation in slope-intercept form.

We will use $m = -5$ and $(-1, -4)$ and place them into $y = mx + b$ to find the value of b.

$-4 = -5(-1) + b$

$-4 = 5 + b$

$-9 = b$

Given $m = -5$ and $b = -9$, we can now write the slope-intercept equation of a line with slope -5 that passes through the point created by the ordered pair $(-1, -4)$:

$y = -5x - 9$

Example 7: Write the Slope–Intercept Equation of a Line

Find the equation of the line whose graph passes through the points created by the ordered pairs (4, − 2) and (− 1, 5). If possible, write the equation in slope-intercept form.

We need to find *m*. Let (4, − 2) equal (x_1, y_1) and (− 1, 5) equal (x_2, y_2).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-(-2)}{-1-4} = \frac{7}{-5} = -\frac{7}{5}$$

Now we will use $m = -\frac{7}{5}$ and one of the given ordered pairs, say (4, − 2), and place all into $y = mx + b$ to find the value of *b*.

$$-2 = -\frac{7}{5}(4) + b$$

$$-2 = -\frac{7}{5}(4) + b$$

$$-2 + \frac{28}{5} = b$$

Example 7: Write the Slope–Intercept Equation of a Line (2 of 2)

To combine fractions, we need a common denominator, namely 5. We will write – 2 as $-\frac{10}{5}$ and find *b* = $\frac{18}{5}$.

Given $m = -\frac{7}{5}$ and $b = \frac{18}{5}$, we can now write the *slope-intercept equation* of a line that passes through the points crated by the ordered pairs (4, − 2) and (− 1, 5):

$$y = -\frac{7}{5}x + \frac{18}{5}$$

NOTE: In algebra we usually leave the equation in fraction form. We usually DO NOT change improper fractions to mixed numbers. Also, we usually do not change fractions to decimals, however, there are some exceptions to this convention.

Example 8: Write the Slope–Intercept Equation of a Line (1 of 2)

Find the equation of the line that passes through the points created by the ordered pairs (4, 2) and (− 1, 2). If possible, write the equation in slope-intercept form.

We need to find *m*. Let (4, 2) equal (x_1, y_1) and (− 1, 2) equal (x_2, y_2).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-2}{-1-4} = \frac{0}{-5} = 0$$

Now we will use $m = 0$ and one of the given ordered pairs, say (4, 2) and place into $y = mx + b$ to find the value of *b*.

$2 \quad = 0(4) + b$

$2 = 0 + b$

$2 = b$

Example 8: Write the Slope–Intercept Equation of a Line

Given $m = 0$ and $b = 2$, we can now write the *slope-intercept equation* of the line that passes through the points created by the ordered pairs (4, 2) and (− 1, 2):

$y = 0x + 2$

Lastly, we will write $y = 2$ which is the equation of a horizontal line.

Example 9: Write the Slope–Intercept Equation of a Line (1 of 2)

Find the equation of the line that passes through the points (−2, 6) and (−2, −8). If possible, write the equation in slope-intercept form.

We need to find *m*. Let (− 2, 6) equal (x_1, y_1) and (− 2, − 8) equal (x_2, y_2).

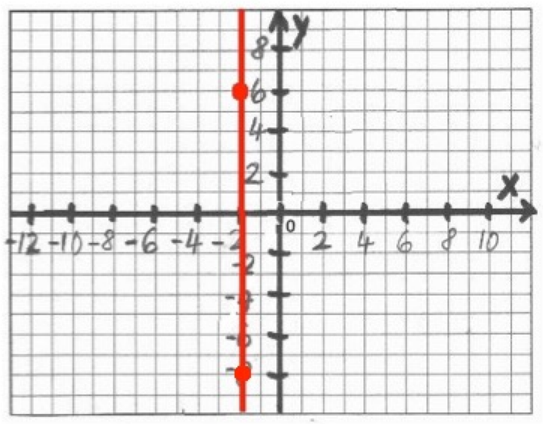
$$m = \frac{-8-6}{-2-(-2)} = \frac{-14}{-2+2} = \frac{-14}{0}$$

We note that the slope is undefined. This means, we will not be able to use $y = mx + b$ to find the equation of the line. As a matter of fact, there will not be a slope-intercept equation!

Example 9: Write the Slope–Intercept Equation of a Line (2 of 2)

We learned that only vertical lines have undefined slopes. Their equation is in the form $\boldsymbol{x = a}$ where $\boldsymbol{a}$ is the x-intercept.

To find the equation of this line, we will plot the ordered pairs $(-2, 6)$ and $(-2, -8)$ and connect them.



We find that this line intersects the x-axis at the point created by the ordered pair $(-2, 0)$. Therefore, the x-intercept must be -2.

The equation of the line is $x = -2$. It is NOT considered a *slope-intercept equation*!

Example 10: Parallel and Perpendicular Slopes (1 of 2)

Find the slopes of lines that are parallel and perpendicular to the line
$x - 3y - 12 = 0$.

First, we need to find the slope of the given line. Let's change the equation to slope-intercept form to find m.

$x - 3y - 12 = 0$

$- 3y = - x + 12$

and $y = \frac{1}{3}x - 4$

We can see that the slope of the given line is $\frac{1}{3}$.

The slope of any line parallel to the given line $x - 3y - 12 = 0$ is the same. Namely $\frac{1}{3}$.

Example 10: Parallel and Perpendicular Slopes (2 of 2)

On the other hand, the slopes of perpendicular lines are negative reciprocals.

The slope of the given line is $\frac{1}{3}$. The reciprocal of $\frac{1}{3}$ is $\frac{3}{1}$ = 3.

Therefore, the negative reciprocal of $\frac{1}{3}$ is − 3, which is the slope of any line perpendicular to the given line $x - 3y - 12 = 0$.

Example 11: Parallel and Perpendicular Slopes

Find the equation of a line passing through the point created by the ordered pair $(-2, 5)$ and parallel to the line whose equation is $y = 3x + 1$. If possible, write the equation in slope-intercept form.

The slope of the given line is 3. A parallel line will also have a slope of 3.

Now we will use $m = 3$ and $(-2, 5)$ and place them into $y = mx + b$ to find the value of b:

$$5 = 3(-2) + b$$

$$5 = -6 + b$$

$$11 = b$$

Example 11: Parallel and Perpendicular Slopes (2 of 2)

Given $m = 3$ and $b = 11$, we can now write the slope-intercept equation of the line parallel to the line $y = 3x + 1$ and passing through the point created by the ordered pair $(-2, 5)$:

$y = 3x + 11$

Example 12: Parallel and Perpendicular Slopes (1 of 3)

Find the equation of a line perpendicular to the line $y = 6$ and passing through the point created by the ordered pair $(4, -8)$. If possible, write the equation in slope-intercept form.

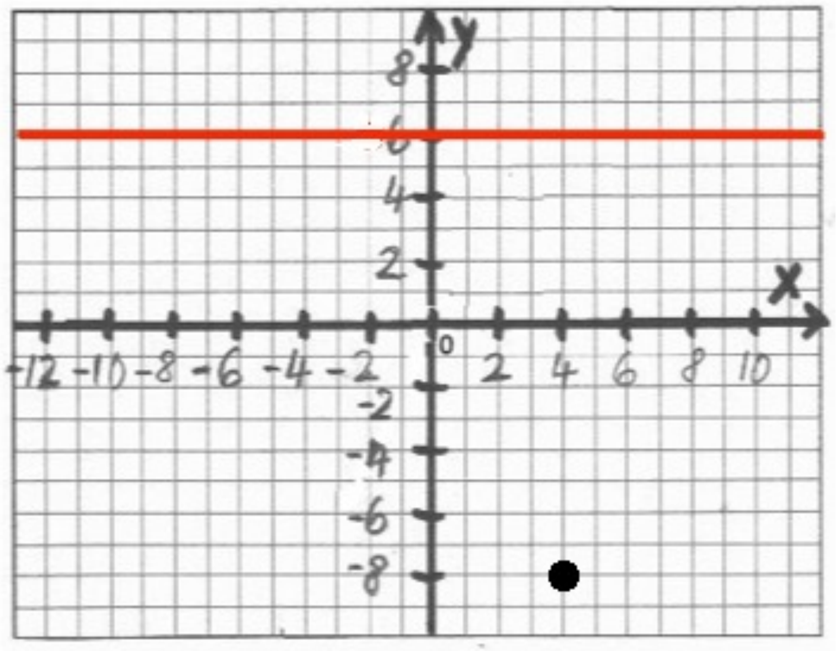
Given that there is no *x*-variable in the equation, we must recognize that we are dealing with a horizontal line. Its slope is 0.

The slope of a perpendicular line is the negative reciprocal. In our case we get $-\frac{1}{0}$ which is undefined. We know that only vertical lines have undefined slopes!

NOTE: It should be obvious that a line perpendicular to a horizontal line must be a vertical line!

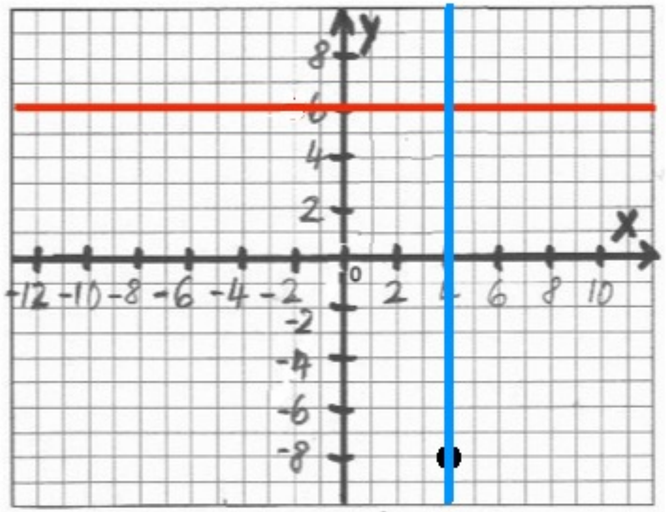
Example 12: Parallel and Perpendicular Slopes (2 of 3)

Since we are dealing with an undefined slope, we cannot use the usual route via $y = mx + b$. Instead, let's make a graph of $y = 6$ and the point created by the ordered pair $(4, -8)$.



Example 12: Parallel and Perpendicular Slopes (3 of 3)

Next, we will draw a vertical line through the point created by the ordered pair (4, − 8).



We know that a vertical line has a general equation of ***x* = *a***, where ***a*** is the *x*-intercept of the line. In the graph, we see that the *x*-intercept is 4.

Therefore, the equation of the vertical line through the point (4, – 8) and perpendicular to the line $y = 6$ must be $x = 4$.