Examples
The Reciprocal Function

Based on power point presentations by Pearson Education, Inc.

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Learning Objectives

1. Memorize the definition of the reciprocal function.
2. Memorize characteristics of the graph of the reciprocal function.
3. Apply transformations to the reciprocal function.
4. Graph the reciprocal function and its transformations by hand.

Example 1: Graph the Reciprocal Function by Hand (1 of 4)

Graph the Reciprocal Function $f(x) = \frac{1}{x}$ by hand.

1. **Equation of its Vertical Asymptote:**

   Since the function is reduced to lowest terms, we set the denominator equal to 0 and solve.

   $\boldsymbol{x = 0}$ This is the equation of the *y*-axis.

2. **Equation of its Horizontal Asymptote:**

   We notice that the degree of the numerator is 0 and that of the denominator is 1. When the degree of the numerator is smaller than that of the denominator the equation of the horizontal asymptote is $\boldsymbol{y = 0}$.

   This is the equation of the *x*-axis.

Example 1: Graph the Reciprocal Function by Hand (2 of 4)

3. Point associated with the *y*-intercept (when *x* = 0):

$f(0) = \frac{1}{0}$ The *y*-value is undefined at *x* = 0.

This means that there is no *y*–intercept.

4. Point associated with the *x*-intercept (when *y* = 0):

$0 = \frac{1}{x}$

When multiplying by sides by *x* we get 0 = 1. This is a false statement.

This means that there is no *x*-intercept.

Example 1: Graph the Reciprocal Function by Hand (3 of 4)

5. Find additional points to either side of the vertical asymptote (*y*-axis).

How about $x = -2, -1, -\frac{1}{2}, \frac{1}{2}, 1$, and 2?

Using $f(x) = \frac{1}{x}$, we get the following *y*-values:

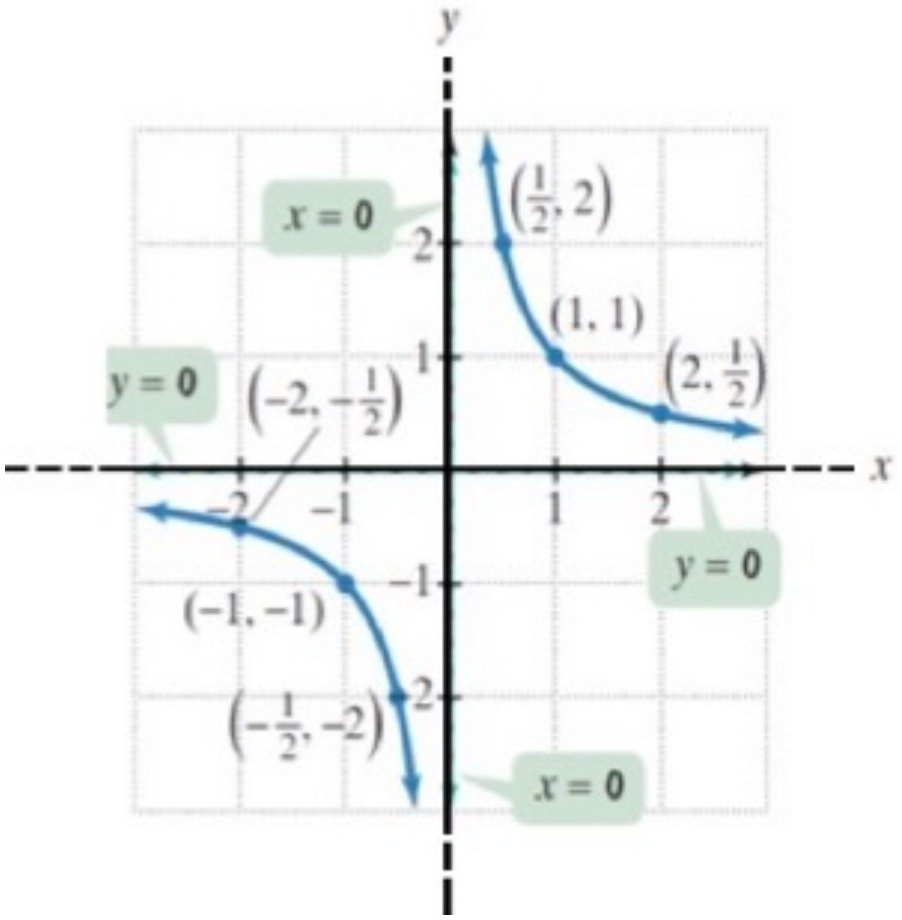
$f(-2) = \frac{1}{-2} = -\frac{1}{2}$ $f(-1) = \frac{1}{-1} = -1$ $f\left(-\frac{1}{2}\right) = \frac{1}{-\frac{1}{2}} = -2$*

$f\left(\frac{1}{2}\right) = \frac{1}{\frac{1}{2}} = 2$* $f(1) = \frac{1}{1} = 1$ $f(2) = \frac{1}{2}$

* Please note that dividing by a fraction is the same as multiplying by its reciprocal!

Example 1: Graph the Reciprocal Function by Hand (4 of 4)

6. Connect all points found in the previous steps keeping in mind the shape of the graph.



Since the horizontal asymptote is the *x*-axis and the vertical asymptote the *y*-axis, we will not graph them as a dashed line.

Example 2: Apply Transformations to the Reciprocal Function

Starting with the graph of the reciprocal function $f(x) = \frac{1}{x}$, write the equation of the graph that results from the following. Include the equations of the horizontal asymptote and the vertical asymptote.

(1) a vertical shift up 3 units

$y = \frac{1}{x} + 3$

The equation of the horizontal asymptote is now $y = 3$.

Remember, a vertical shift affects the horizontal asymptote!

The equation of the vertical asymptote is still $x = 0$.

NOTE: We could have also used one of the three theorems that allow us to find the equation of the horizontal asymptote. However, since the function is not ONE SINGLE fraction, we cannot yet apply them. We would have to use algebra to change the equation to one single fraction.

Example 2: Apply Transformations to the Reciprocal Function

(2) a vertical shift down 3 units

$y = \frac{1}{x} - 3$

The equation of the horizontal asymptote is now $y = -3$.

The equation of the vertical asymptote is still $x = 0$.

Example 3: Apply Transformations to the Reciprocal Function

(2 of 2)

Starting with the graph of the reciprocal function $f(x) = \frac{1}{x}$, write the equation of the graph that results from the following. Include the equations of the horizontal asymptote and the vertical asymptote.

(1) a horizontal shift to the right 3 units

$y = \frac{1}{x-3}$

The equation of the horizontal asymptote is still $y = 0$.

The equation of the vertical asymptote is now $x = 3$.

Remember, a horizontal shift affects the vertical asymptote!

(2) a horizontal shift to the left 3 units

$y = \frac{1}{x+3}$

The equation of the horizontal asymptote is still $y = 0$.

The equation of the vertical asymptote is now $x = -3$.

Example 4: Transformations of the Reciprocal Function (1 of 2)

Starting with the graph of the common function $f(x) = \frac{1}{x}$, write the equation of the graph that results from the following. Include the equations of the horizontal asymptote and the vertical asymptote.

(1) a vertical shift up 1 unit and a horizontal shift to the right 2 units

$y = \frac{1}{x-2} + 1$

The equation of the horizontal asymptote is now $y = 1$.

The equation of the vertical asymptote is now at $x = 2$.

Remember, a horizontal shift affects the vertical asymptote, and a vertical shift affects the horizontal asymptote!

Example 4: Transformations of the Reciprocal Function (2 of 2)

(2) a vertical shift up 3 units and a horizontal shift to the left 4 units

$y = \frac{1}{x+4} + 3$

The equation of the horizontal asymptote is now $y = 3$.

The equation of the vertical asymptote is now at $x = -4$.

(3) a vertical shift down 5 units and a horizontal shift to the right 1 unit

$y = \frac{1}{x-1} - 5$

The equation of the horizontal asymptote is now $y = -5$.

The equation of the vertical asymptote is now at $x = 1$.

(4) a vertical shift down 2 units and a horizontal shift to the left 3 unit

$y = \frac{1}{x+3} - 2$

The equation of the horizontal asymptote is now $y = -2$.

The equation of the vertical asymptote is now at $x = -3$.

Example 5: Graph a Transformation of the Reciprocal Function by Hand (1 of 5)

Graph the function $k(x) = \frac{1}{x-1} - 1$ by hand.

Find the equation of the vertical asymptote.

Since the fraction is reduced to lowest terms, we will set the denominator equal to 0 and solve.

$x - 1 = 0$

$x = 1$

The equation of the vertical asymptote is $x = 1$.

Find the equation of the horizontal asymptote.

Since the function is not ONE SINGLE fraction, we cannot yet apply one of the three theorems that allow us to find the equation of the horizontal asymptote.

We would have to use algebra to change the equation to one single fraction.

Example 5: Graph a Transformation of the Reciprocal Function by Hand (2 of 5)

Since the given function is a transformation of the reciprocal function $f(x) = \frac{1}{x}$,
we look at the horizontal shift because it affects the horizontal asymptote.
In our case then the horizontal asymptote moves to ***y* = – 1**.

Find and plot the point associated with the *y*-intercept:

This is where *x* = 0.

Then

$$k(0) = \frac{1}{0-1} - 1 = -2$$

The *y*-intercept is –2 and its associated ordered pair is **(0, –2)**.

Example 5: Graph a Transformation of the Reciprocal Function by Hand (3 of 5)

Find and plot the point associated with the *x*-intercept:

This is where $y = 0$.

Then $$0 = \frac{1}{x-1} - 1$$

$$1 = \frac{1}{x-1}$$

and multiplying both sides by $(x - 1)$ we get $\boldsymbol{x - 1 = 1}$ or $\boldsymbol{x = 2}$.

The *x*-intercept is 2 and its associated ordered pair is **(2, 0)**.

Example 5: Graph a Transformation of the Reciprocal Function by Hand (4 of 5)

Let's find some points to either side of the vertical asymptote whose equation is $x = 1$.

How about $x = -2, -1, -\frac{1}{2}, \frac{1}{2}, \frac{3}{4}, \frac{3}{2}$, and 3?
Note: We don't need to use 0 and 2 since they are the intercepts and we found them already!

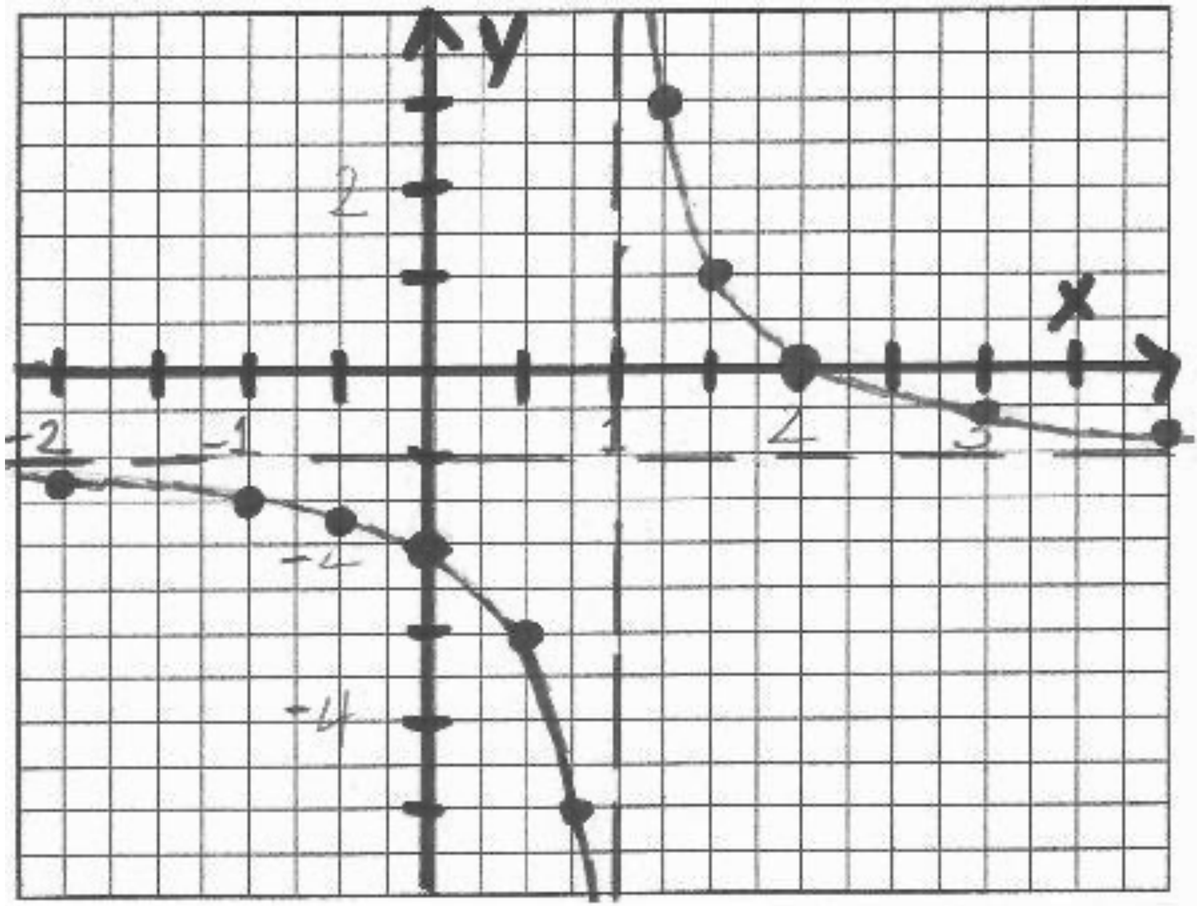
Using $k(x) = \frac{1}{x-1} - 1$, we find the following:

$p(-2) \approx -1.3$ $p(-1) = -1.5$ $p\left(-\frac{1}{2}\right) \approx -1.7$ $p\left(\frac{1}{2}\right) = -3$

$p\left(\frac{3}{4}\right) = -5$ $p\left(\frac{3}{2}\right) = 1$ $p(3) = \frac{1}{2}$

Example 5: Graph a Transformation of the Reciprocal Function by Hand (5 of 5)

Connect all points found in the previous steps keeping in mind the shape of the graph.



We graphed the vertical asymptote at $x = 1$ and the horizontal asymptote at $y = -1$ as dashed lines.

Example 6: Graph a Transformation of the Reciprocal Function by Hand (1 of 5)

Graph the function $f(x) = -\frac{1}{x} + 2$ by hand.

Find the equation of the vertical asymptote.

Since the fraction is reduced to lowest terms, we will set the denominator equal to 0 and solve.

This is pretty easy! $x = 0$

The equation of the vertical asymptote is $\boldsymbol{x = 0}$ which is the *y*-axis.

Find the equation of the horizontal asymptote.

Since the function is not ONE SINGLE fraction, we cannot yet apply one of the three theorems that allow us to find the equation of the horizontal asymptote. We would have to use algebra to change the equation to one single fraction.

Example 6: Graph a Transformation of the Reciprocal Function by Hand (2 of 5)

Since the given function is a transformation of the reciprocal function $f(x) = \frac{1}{x}$, we look at the horizontal shift because it affects the horizontal asymptote. In our case then the horizontal asymptote moves to ***y* = 2**.

Find and plot the point associated with the *y*-intercept:

This is where *x* = 0.

Then $f(0) = -\frac{1}{0} + 2$ which is **undefined**.

Therefore, there is NO *y*-intercept.

Example 6: Graph a Transformation of the Reciprocal Function by Hand (3 of 5)

Find and plot the point associated with the *x*-intercept:

This is where $y = 0$.

Then $0 = -\frac{1}{x} + 2$

$$-2 = -\frac{1}{x}$$

and multiplying both sides by – *x* we get **2*x* = 1** and $x = \frac{1}{2}$.

The *x*-intercept is $\frac{1}{2}$ and its associated ordered pair is $(\frac{1}{2}, 0)$.

Example 6: Graph a Transformation of the Reciprocal Function by Hand (4 of 5)

Let's find some points to either side of the vertical asymptote whose equation is $x = 0$.

How about $x = -3, -2, -1, -\frac{1}{2}, \frac{1}{4}, 1, 2$, and 3?
Note: We don't need to use 1 since it is an intercept which we found already!

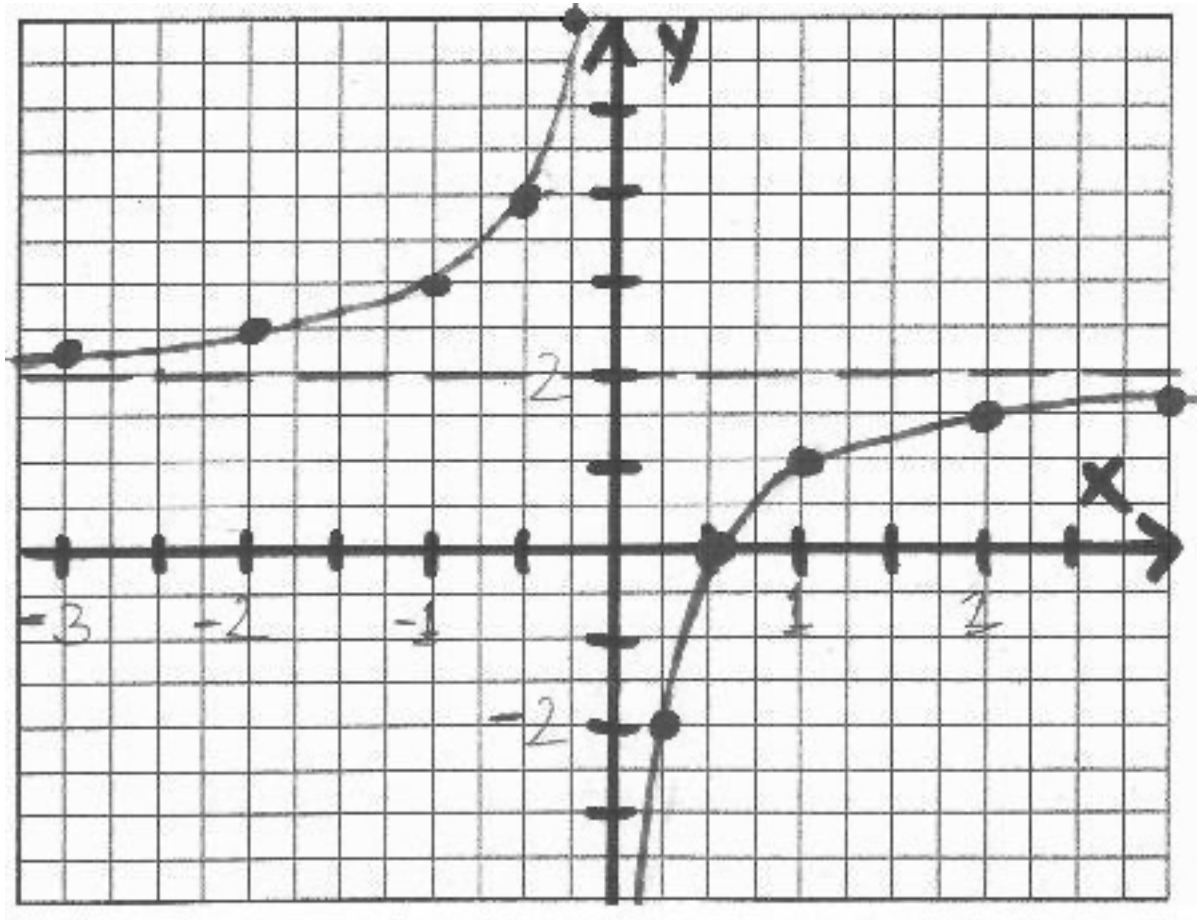
Using $f(x) = -\frac{1}{x} + 2$, we find the following:

$f(-3) \approx 2.3$ $f(-2) = 2.5$ $f(-1) = 3$ $f\left(-\frac{1}{2}\right) = 4$

$f\left(\frac{1}{4}\right) = -2$ $f(1) = -5$ $f(2) = 1.5$ $f(3) \approx 1.7$

Example 6: Graph a Transformation of the Reciprocal Function by Hand (5 of 5)

Connect all points found in the previous steps keeping in mind the shape of the graph.



We graphed the horizontal asymptote at $y = 2$ as dashed lines. The vertical asymptote is the y-axis, and we did not dash it.

Please note the reflection of the reciprocal function in the x-axis!