Examples
Rational Equations in One Variable

Based on power point presentations by Pearson Education, Inc.

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Learning Objectives

1. Memorize the definition of a rational equation.
2. Know how to solve rational equations.

Example 1: Solve a Rational Equation (1 of 2)

Solve $\dfrac{x^2+x-6}{x^2-8x+12}=0$.

There is only one denominator. We multiply both sides of the equation by it.

$$(x^2-8x+12)\left(\frac{x^2+x-6}{x^2-8x+12}\right)=0(x^2-8x+12)$$

Distribute the denominator to ALL terms on each side and solve.

$$x^2+x-6=0$$

On the left, cancelled ($x^2 - 8x + 12$).
On the right, 0 times anything still equals 0.

$$(x-2)(x+3)=0$$

Factored the trinomial.

By the *Zero Product Principle*, $x - 2 = 0$ and $x + 3 = 0$.

We find the proposed solutions $x = 2$ and $x = -3$.

Example 1: Solve a Rational Equation (2 of 2)

Any time we solve a rational equation, we must check the proposed solutions in the original equation, rejecting any that produce 0 in any of the denominators.

Given $x = -3$ and a denominator of $x^2 - 8x + 12$, we find the following:

$(-3)^2 - 8(-3) + 12 = 45$

We find that – 3 does NOT produce 0 in the denominator which makes it a true solution.

Given $x = -2$ and a denominator of $x^2 - 8x + 12$, we find the following:

$(2)^2 - 8(2) + 12 = 0$

We find that 2 produces 0 in the denominator which leads to an undefined condition.

In summary, the rational equation has one solution, namely, $x = -3$.

Example 2: Solve a Rational Equation (1 of 2)

Solve $\frac{5}{x+6}-\frac{3}{x}=0$.

Find the smallest number divisible by the denominators in the equation.

(x + 6) is factored and is prime.
x is factored and is prime.

We will form a special product using the prime factors. That is, the smallest number divisible by the given denominators is $x(x + 6)$.

Multiply both sides of the equation by $x(x + 6)$.

$$x(x+6)\left(\frac{5}{x+6}-\frac{3}{x}\right)=x(x+6)(0)$$

Example 2: Solve a Rational Equation (2 of 2)

Distribute ***x*(*x* + 6)** to ALL terms on each side and solve.

$$5x - 3(x+6) = 0$$
$$5x - 3x - 18 = 0$$
$$2x = 18$$
$$x = 9$$

On the left, we cancelled (x + 6) in the first term and x in the second term.
On the right, 0 times anything still equals 0.

We find the proposed solution $x = 9$.

Any time we solve a rational equation, we must check the proposed solutions in the original equation, rejecting any that produce 0 in any of the denominators.

Given $x = 9$ and denominators of x and $x + 6$, **we observe that 9 does NOT produce 0 in any of the denominators which makes it a true solution.**

In summary, the rational equation has one solution, namely, $x = 9$.

Example 3: Solve a Rational Equation

Solve $\frac{5}{x-2}-\frac{17-x}{2x-4}=0$.

Find the smallest number divisible by the denominators in the equation.

(x – 2) is factored, and the factor is prime.
2x – 4 can be factored into 2(x – 2).

We will form a special product using the prime factors. However, we use all factors only once. Therefore, the smallest number divisible by the given denominators is 2(x – 2).

Multiply both sides of the equation by 2(x – 2).

$$2(x-2)\left[\frac{5}{x-2}-\frac{17-x}{2(x-2)}\right]=2(x-2)(0)$$

Example 3: Solve a Rational Equation (2 of 3)

Distribute the $2(x - 2)$ to ALL terms on each side and solve.

$$2(5) - (17 - x) = 0$$

On the left, we cancelled $(x - 2)$ in the first term and $2(x - 2)$ in the second term.
On the right, 0 times anything still equals 0.

Please note that we placed the numerator $17 - x$ into parentheses to remind us that there is a negative sign in front of the fraction.

Then $10 - 17 + x = 0$

and $x = 7$

We find the proposed solution $x = 7$.

Example 3: Solve a Rational Equation (3 of 3)

Any time we solve a rational equation, we must check the proposed solutions in the original equation, rejecting any that produce 0 in any of the denominators.

Given $x = 7$ and denominators of $x - 2$ and $2x - 4$, **we observe that 7 does NOT produce 0 in any of the denominators which makes it a true solution.**

In summary, the rational equation has one solution, namely, $x = 7$.

Example 4: Solve a Rational Equation (1 of 2)

Solve $\frac{5x}{x+1} = 3 - \frac{5}{x+1}$.

There is one denominator, namely (x + 1). The other denominator is 1 since we can write 3 as $\frac{3}{1}$. We multiply both sides of the equation by (x + 1).

$$(x+1)\left(\frac{5x}{x+1}\right) = (x+1)\left(3 - \frac{5}{x+1}\right)$$

Distribute (x + 1) to ALL terms on each side and solve.

$$5x = 3(x+1) - 5$$
$$5x = 3x + 3 - 5$$
$$5x - 3x = 3 - 5$$
$$x = -1$$

On the left, cancelled (x + 1) . On the right, cancelled (x + 1) in the second term. Note that the 3 on the right had to be multiplied by (x + 1).

Example 4: Solve a Rational Equation (2 of 2)

We find the proposed solution $x = -1$.

Any time we solve a rational equation we must check the proposed solutions in the original equation, rejecting any that produce 0 in any of the denominators.

Given $x = -1$ and a denominator of $x + 1$, **we observe that – 1 produces 0 in the denominator which leads to an undefined situation.**

In summary, the rational equation has NO solutions.

Example 5: Solve a Rational Equation (1 of 3)

Solve $\frac{4}{b+7} - \frac{8}{b-3} = \frac{6b-10}{b^2+4b-21}$.

Find the smallest number divisible by the denominators in the equation.

(b + 7) is factored, and the factor is prime.
(b – 3) is factored, and the factor is prime.
(b^2 + 4b – 21) can be factored into (b + 7)(b – 3).

We will form a special product using the prime factors. However, we use the factors only once. Therefore, the smallest number divisible by the given denominators is (b + 7)(b – 3) and NOT (b + 7)(b – 3)(b + 7)(b – 3).

Multiply both sides of the equation by (b + 7)(b – 3).

Example 5: Solve a Rational Equation (2 of 3)

$$(b+7)(b-3)\left(\frac{4}{b+7}-\frac{8}{b-3}\right)=\left(\frac{6b-10}{b^2+4b-21}\right)(b+7)(b-3)$$

Distribute the $(b + 7)(b - 3)$ to ALL terms on each side of the equal sign and solve.

$4(b - 3) - 8(b + 7) = 6b - 10$

On the left, we cancelled $(b + 7)$ in the first term and $(b - 3)$ in the second term. On the right, we cancelled $(b + 7)(b - 3)$.

Then we use the *Distributive Property* to solve for b.

$4b - 12 - 8b - 56 = 6b - 10$

$-4b - 68 = 6b - 10$

$-10b = 58$

$$b = -\frac{58}{10} = -\frac{29}{5}$$

Example 5: Solve a Rational Equation (3 of 3)

We find the proposed solution $b = -\frac{29}{5}$.

Any time we solve a rational equation we must check the proposed solutions in the original equation, rejecting any that produce 0 in any of the denominators.

Given $b = -\frac{29}{5}$ and denominators of *b + 7* and *b* – *3*, **we observe that $-\frac{\mathbf{29}}{\mathbf{5}}$, which is equal to – 5.8, does NOT produce 0 in any of the denominators which makes it a true solution.**

In summary, the rational equation has one solution, namely $b = -\frac{29}{5}$.

Example 6: Solve a Rational Equation (1 of 4)

Solve $\frac{4}{x-5}+\frac{8}{x-4}=8$.

Find the smallest number divisible by the denominators in the equation.

$(x – 5)$ is factored and is prime.
$(x – 4)$ is factored and is prime.

We will form a special product using the prime factors. The smallest number divisible by the given denominators is $(x – 5)(x – 4)$.

Multiply both sides of the equation by $(x – 5)(x – 4)$.

$$(x-5)(x-4)\left(\frac{4}{x-5}+\frac{8}{x-4}\right)=8(x-5)(x-4)$$

Example 6: Solve a Rational Equation (2 of 4)

Distribute $(x – 5)(x – 4)$ to ALL terms on each side and solve.

$$4(x-4)+8(x-5)=8\left(x^2-9x+20\right)$$

On the left of the equal sign, we cancelled $(x - 5)$ in the first term and $(x - 4)$ in the second term. On the right of the equal sign, we used FOIL to multiply out $(x - 5)(x - 4)$.

$$4x-16+8x-40=8x^2-72x+160$$

Used the Distributive Property on both sides of the equal sign.

Given a $\boldsymbol{x^2}$ we notice that we are dealing with a quadratic equation. Let's write it in general form.

$12x – 56 = 8x^2 – 72x + 160$

$8x^2 – 84x + 216 = 0$

While factoring might be a possibility, let's just use the *Quadratic Formula* to solve this equation for x.

Example 6: Solve a Rational Equation (3 of 4)

Given ***a* = 8**, ***b* = – 84**, and ***c* = 216,** we will write the following:

$$x = \frac{-(-84) \pm \sqrt{(-84)^2 - 4(8)(216)}}{2(8)}$$

$$x = \frac{84 \pm \sqrt{144}}{16} = \frac{84 \pm 12}{16}$$

We find the proposed solutions $x = 6$ and $x = \frac{9}{2}$.

Example 6: Solve a Rational Equation (4 of 4)

Any time we solve a rational equation we must check the proposed solutions in the original equation, rejecting any that produce 0 in any of the denominators.

Given $x = 6$ and denominators of $x - 5$ and $x - 4$, **we observe that 6 does NOT produce 0 in any of the denominators which makes it a true solution.**

Given $x = \frac{9}{2}$ and denominators of $x - 5$ and $x - 4$, we observe that $\frac{9}{2}$, **which is equal to 4.5, does NOT produce 0 in any of the denominators which makes it a true solution.**

In summary, the rational equation has two solutions, namely $x = 6$ and $x = \frac{9}{2}$.