Examples
Quadratic Functions – Part 2

Based on power point presentations by Pearson Education, Inc.

Revised by Ingrid Stewart, Ph.D.

Learning Objectives

1. Define the *Standard Form* of a quadratic function.
2. Given a quadratic function in standard form, find the coordinates of the vertex and the equation of the axis of symmetry of its graph.
3. Graph quadratic functions by hand when given in standard form.

NOTE: This lesson contains some examples. You can find more examples in the “Examples” document also located in the appropriate MOM Learning Materials folder.

Example 1: Graph a Quadratic Function (1 of 6)

Graph the quadratic function $f(x) = (x + 2)^2 – 1$ by hand.

Step 1 - Determine how the parabola opens.

$a = 1$, $a > 0$; the parabola opens upward.

Step 2 - Find the coordinates of the vertex and the equation of the axis of symmetry.

The function is NOT in standard form. Therefore, we first must change to standard from as follows:

$f(x) = [x – (– 2)]^2 + (– 1)$

We now see that $h = – 2$ and $k = – 1$ and the parabola has its vertex at $(– 2, – 1)$.

The equation of the axis of symmetry is $x = – 2$.

Example 1: Graph a Quadratic Function (2 of 6)

Step 3 - Find the points associated with the *x*-intercepts by solving $y = f(x) = 0$.

Given $f(x) = (x + 2)^2 - 1$, let $f(x) = 0$ as follows:

$0 = (x + 2)^2 - 1$

Here we must solve a quadratic equation. We could do several things. We could multiply out and combine like terms, then try to factor and use the *Zero Product Principle*. We could also use the *Quadratic Formula*.

However, easiest is to use the *Square Root Property.* So, let's subtract $(x + 2)^2$ from both sides of the equation to get

$-(x + 2)^2 = -1$

and when we multiply both sides by -1 we get $(x + 2)^2 = 1$!

Example 1: Graph a Quadratic Function

Step 3 continued

We will now apply the *Square Root Property* to get

$x + 2 = \pm\sqrt{1}$

$x + 2 = \pm 1$

This means $x + 2 = 1$ and $x + 2 = -1$. It follows that $x = -1$ and $x = -3$.

The x-intercepts are at the points $(-1, 0)$ and $(-3, 0)$.

Example 1: Graph a Quadratic Function (4 of 6)

Step 4 - Find the point associated with the *y*-intercept by computing *f*(0).

$$\begin{aligned} f(0) &= (0 + 2)^2 - 1 \\ &= (2)^2 - 1 \\ &= 4 - 1 \\ &= 3 \end{aligned}$$

The *y*-intercept is at the point (0, 3).

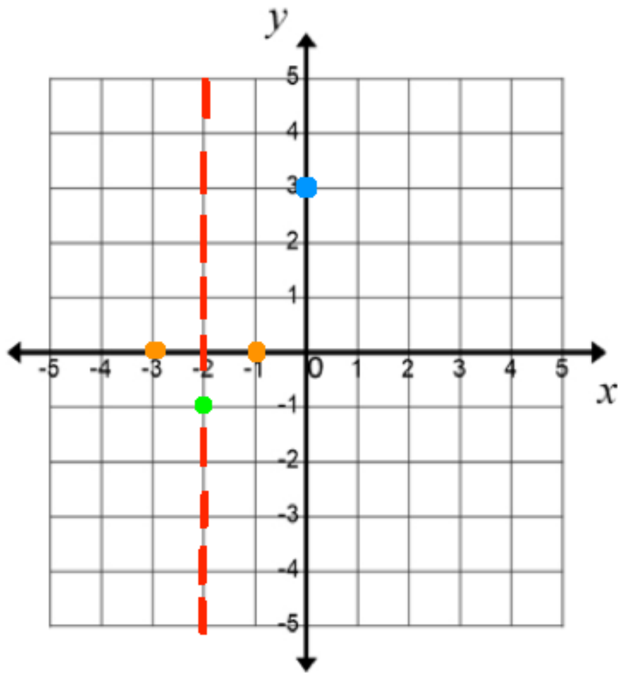
Step 5 - Graph the axis of symmetry.

The *axis of symmetr*y is the line $x = -2$. It is drawn as a dashed line to indicate that it is not a visible part of the graph.

Example 1: Graph a Quadratic Function (5 of 6)

Step 6 - Graph the parabola.

- The coordinates of the vertex are (– 2, – 1).
- The coordinates associated with the *x*-intercepts are (−1, 0) and (– 3, 0).
- The coordinates of the point associated with the *y*-intercept are (0, 3).



We really do not have enough points to make an adequate graph of the function. Therefore, let's find a few more points particularly close to the vertex so that we can draw a nice bowl shape. We are going to pick $x = -4$ and find its corresponding *y*-value.

$f(2) = (-4 + 2)^2 - 1 = 3$

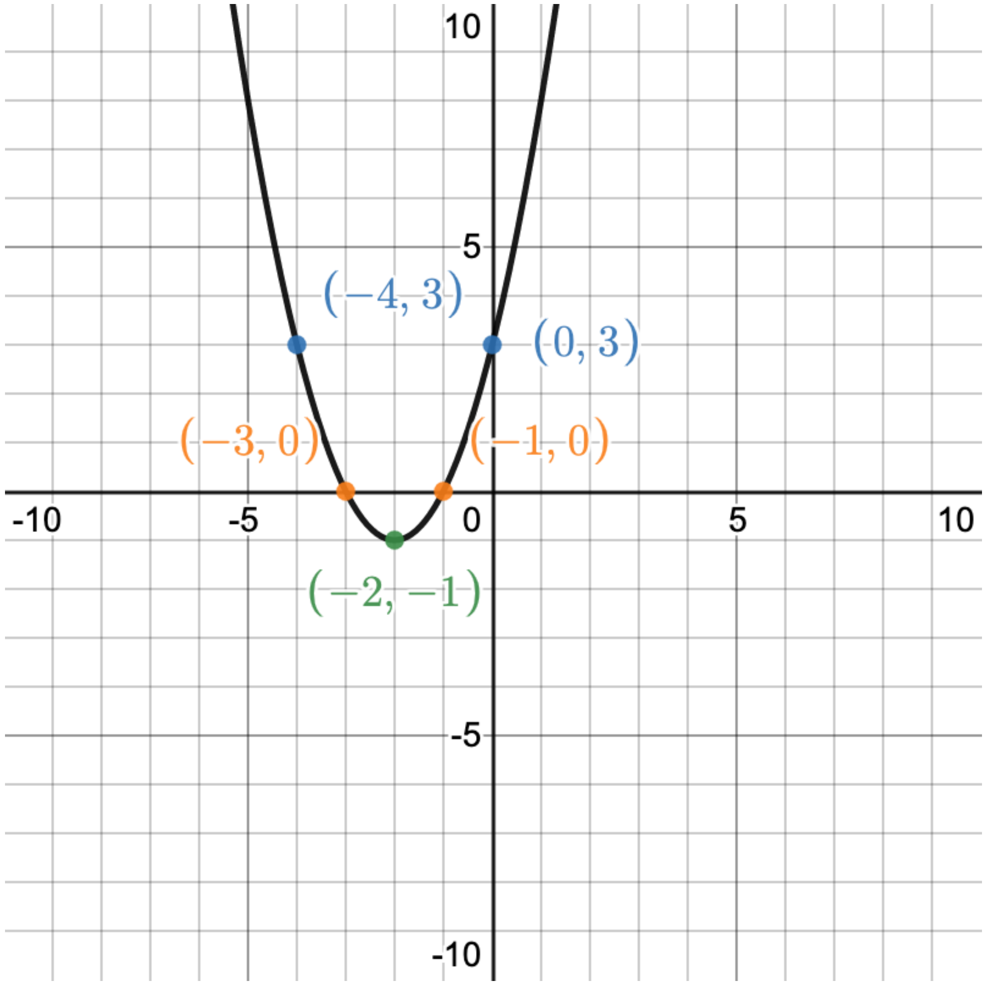
The coordinates of the additional point are (– 4 , 3).

Example 1: Graph a Quadratic Function (6 of 6)

Step 6 continued

We are now going to add the additional point (– 4, 3) to the coordinate system and then connect ALL points keeping in mind the shape of a parabola!

Please note that you NEVER end a graph at points! Be sure to extend the graph well beyond them.



Example 2: Graph a Quadratic Function (1 of 6)

Graph the quadratic function $f(x) = -(x - 1)^2 + 4$ by hand.

Step 1 - Determine how the parabola opens.

$a = -1$, $a < 0$; the parabola opens downward.

Step 2 - Find the coordinates of the vertex and the equation of the axis of symmetry.

The function is in standard form. Therefore, $h = 1$ and $k = 4$ and the parabola has its vertex at (1, 4).

Please note that the minus between x and 1 does not make *h* negative!

The equation of the axis of symmetry is $x = 1$.

Example 2: Graph a Quadratic Function

Step 3 - Find the points associated with the *x*-intercepts by solving $y = f(x) = 0$.

Given $f(x) = -(x - 1)^2 + 4$, let $f(x) = 0$ as follows:

$$0 = -(x - 1)^2 + 4$$

Here we must solve a quadratic equation. We could do several things. We could multiply out and combine like terms, then try to factor and use the *Zero Product Principle*. We could also use the *Quadratic Formula*.

However, easiest is to use the *Square Root Property.* So, let's add $(x - 1)^2$ to both sides of the equation to get

$$(x - 1)^2 = 4$$

Example 2: Graph a Quadratic Function (3 of 6)

Step 3 continued

We will now apply the *Square Root Property* to get

$x - 1 = \pm\sqrt{4}$

$x - 1 = \pm 2$

This means $x - 1 = 2$ and $x - 1 = -2$. It follows that $x = 3$ and $x = -1$.

The x-intercepts are at the points $(3, 0)$ and $(-1, 0)$.

Please note that parabolas can have one, two, or no *x*-intercepts!

Example 2: Graph a Quadratic Function (4 of 6)

Step 4 - Find the point associated with the *y*-intercept by computing *f*(0).

$$f(x) = -(x-1)^2 + 4$$
$$f(0) = -(0-1)^2 + 4$$
$$= -(-1)^2 + 4$$
$$= -1 + 4 = 3$$

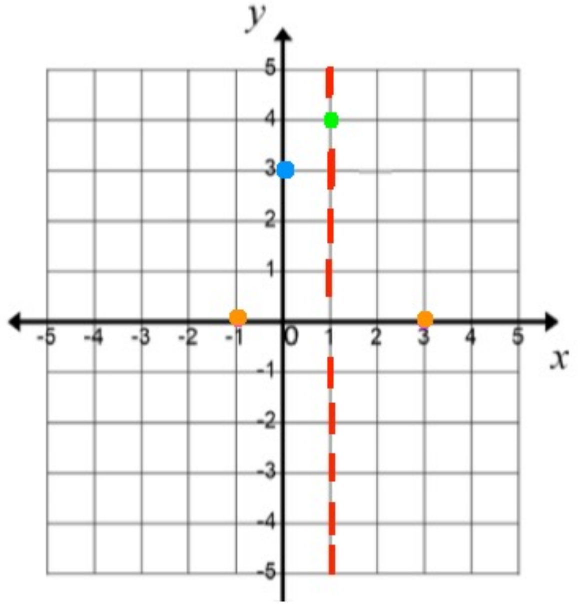
The *y*-intercept is at the point (0, 3).

Please note that parabolas always have exactly one y-intercept.

Step 5 Graph the axis of symmetry.

The axis of symmetry is the line $x = 1$. It is drawn as a dashed line to indicate that it is not a visible part of the graph.

Example 2: Graph a Quadratic Function (5 of 6)



Step 6 - Graph the parabola.

- The coordinates of the vertex are (1, 4).
- The coordinates of the points associated with the *x*-intercepts are (−1, 0) and (3, 0).
- The coordinates of the point associated with the *y*-intercept are (0, 3).

We really do not have enough points to make an adequate graph of the function. Therefore, let's find a few more points particularly close to the vertex so that we can draw a nice bowl shape. We are going to pick $x = 2$ and find its corresponding *y*-value.

$f(2) = -(2-1)^2 + 4 = 3$

The coordinates of the additional point are (2, 3).

Example 2: Graph a Quadratic Function (6 of 6)

Step 6 continued

We are now going to add the additional point (2, 3) to the coordinate system and then connect ALL points keeping in mind the shape of a parabola!

Please note that you NEVER end a graph at the *x*-intercepts! Be sure to extend the graph well beyond them.