Examples
Quadratic Functions – Part 1

Based on power point presentations by Pearson Education, Inc.

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Learning Objectives

1. Define the *General Form* of a quadratic function.
2. Recognize characteristics of parabolas.
3. Given a quadratic function in general form, find the coordinates of the vertex and the equation of the axis of symmetry of its graph.
4. Graph quadratic functions by hand when given in general form.

Example 1: Graph a Quadratic Function

Graph the quadratic function $f(x) = x^2 - 4x + 5$ by hand.

Step 1 - Determine how the parabola opens.

$a = 1$, $a > 0$; the parabola opens upward.

Step 2 - Find the coordinates of the vertex and the equation of the axis of symmetry.

The x-coordinate of the vertex is $-\frac{b}{2a}$.

In the given function, $a = 1$, $b = -4$, and $c = 5$. Then $x = \frac{-(-4)}{2(1)} = 2$.

The y-coordinate of the vertex is $f\left(-\frac{b}{2a}\right)$.

Then $f(2) = (2)^2 - 4(2) + 5 = 1$

The coordinates of the vertex are (2, 1).

The equation of the axis of symmetry is $x = 2$.

Example 1: Graph a Quadratic Function (2 of 5)

Step 3 - If possible, find the points associated with the *x*-intercepts by solving *f*(*x*) = 0.

Given $0 = x^2 - 4x + 5$, we are going to use the *Quadratic Formula* to solve this equation.

In the given function, $a = 1$, $b = -4$, and $c = 5$.

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{-4}}{2}$$

Reminder – Quadratic Formula!

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Note that the radicand is a negative number. Paired with a square root, we know that we are encountering imaginary numbers. Remember that $\sqrt{-1} = i$.

Example 1: Graph a Quadratic Function (3 of 5)

Step 3 continued

Since we are encountering imaginary numbers, we can conclude that there are NO *x*-intercepts.

Step 4 - Find the point associated with the *y*-intercept by computing *f*(0).

$f(0) = 0^2 - 4(0) + 5 = 5.$

The coordinates of the point associated with the *y*-intercept are (0, 5).

Step 5 - Graph the axis of symmetry.

The axis of symmetry is the line $x = 2$. It is drawn as a dashed line to indicate that it is really not a part of the graph.

Example 1: Graph a Quadratic Function (4 of 5)

Step 6 - Graph the parabola.

- The coordinates of the vertex are (2, 1).
- The coordinates of the point associated with the *y*-intercept are (0, 5).

We really do not have enough points to make an adequate graph of the function. Therefore, let's find a few more points particularly close to the vertex so that we can draw a nice bowl shape.

We are going to pick $x = -1, 1, 3, 4,$ and 5 and find the corresponding *y*-values.

$f(-1) = (-1)^2 - 4(-1) + 5 = 10$

$f(1) = (1)^2 - 4(1) + 5 = 2$

$f(3) = (3)^2 - 4(3) + 5 = 2$

$f(4) = (4)^2 - 4(4) + 5 = 5$

$f(5) = (5)^2 - 4(5) + 5 = 10$

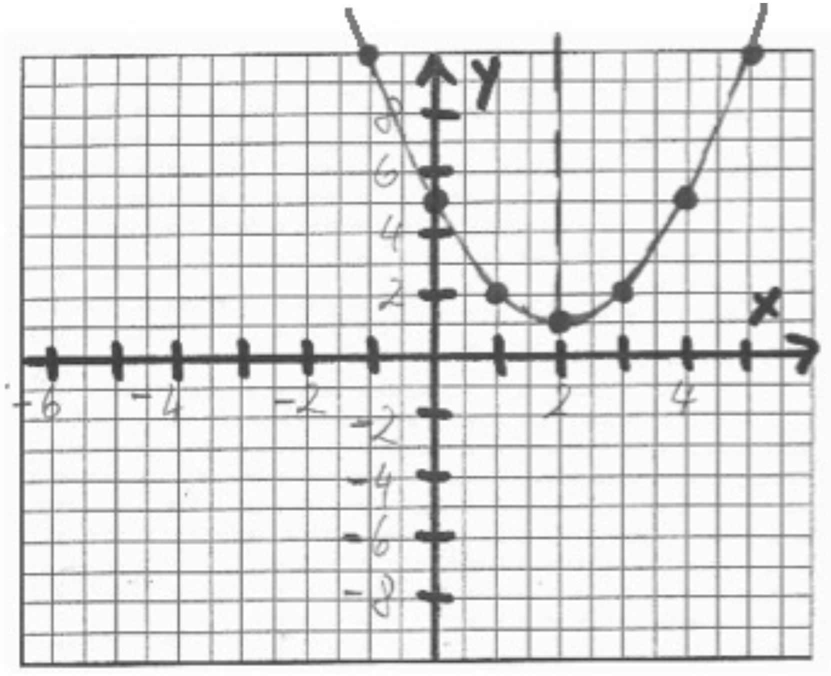
Example 1: Graph a Quadratic Function (5 of 5)

Step 6 continued

The coordinates of the additional points are

(– 1, 10), (1, 2), (3, 2), (4, 5), and (5, 10)

We will now add these points to the coordinate system and then connect ALL points keeping in mind the shape of a parabola!



Example 2: Graph a Quadratic Function (1 of 5)

Graph the quadratic function $f(x) = x^2 - 2x + 1$ by hand.

Step 1 - Determine how the parabola opens.

$a = 1$, $a > 0$; the parabola opens upward.

Step 2 - Find the coordinates of the vertex and the equation of the axis of symmetry.

The x-coordinate of the vertex is $-\frac{b}{2a}$.

In the given function, $a = 1$, $b = -2$, and $c = 1$. Then $x = \frac{-(-2)}{2(1)} = 1$.

The y-coordinate of the vertex is $f\left(-\frac{b}{2a}\right)$.

Then $f(1) = (1)^2 - 2(1) + 1 = 0$

The coordinates of the vertex are (1, 0).

The equation of the axis of symmetry is $x = 1$.

Example 2: Graph a Quadratic Function (2 of 5)

Step 3 - If possible, find the points associated with the *x*-intercepts by solving *f*(*x*) = 0.

Given $0 = x^2 - 2x + 1$, we are going to use factoring and the *Zero Product Principle* to solve this equation.

$(x - 1)(x - 1) = 0$

Since both factors are the same, we only need to set one equal to 0 as follows:
$x - 1 = 0$

Then $x = 1$

There is only one *x*-intercept and the coordinates of the point associated with it are (1, 0). Incidentally, this is also the coordinate of the vertex point!

Example 2: Graph a Quadratic Function (3 of 5)

Step 4 - Find the point associated with the *y*-intercept by computing *f*(0).

$f(0) = 0^2 - 2(0) + 1 = 1.$

The coordinates of the point associated with the *y*-intercept are (0, 1).

Step 5 - Graph the axis of symmetry.

The axis of symmetry is the line $x = 1$. It is drawn as a dashed line to indicate that it is really not a part of the graph.

Example 2: Graph a Quadratic Function (4 of 5)

Step 6 - Graph the parabola.

- The coordinates of the vertex are (1, 0).
- The coordinates of the point associated with the *y*-intercept are (0, 1).

We really do not have enough points to make an adequate graph of the function. Therefore, let's find a few more points particularly close to the vertex so that we can draw a nice bowl shape.

We are going to pick $x = -2, -1, 2, 3, 4$, and 5 and find the corresponding *y*-values.

$f(-2) = (-2)^2 - 2(-2) + 1 = 9$

$f(-1) = (-1)^2 - 2(-1) + 1 = 4$

$f(2) = (2)^2 - 2(2) + 1 = 1$

$f(3) = (3)^2 - 2(3) + 1 = 4$

$f(4) = (4)^2 - 2(4) + 1 = 9$

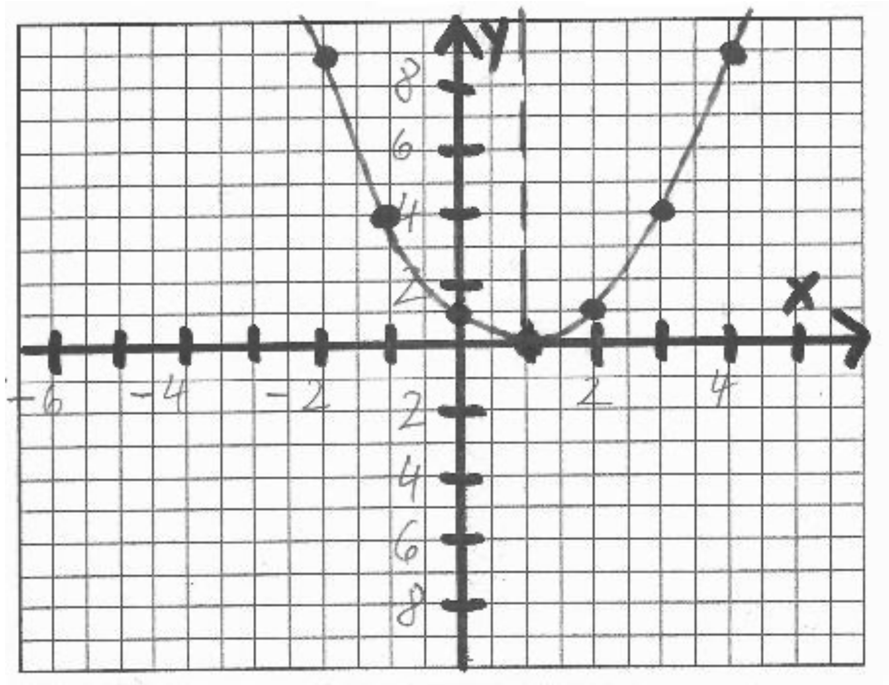
Example 2: Graph a Quadratic Function (5 of 5)

Step 6 continued

The coordinates of the additional points are

(– 2, 9), (– 1, 4), (2, 1), (3, 4), and (4, 9)

We will now add these points to the coordinate system and then connect ALL points keeping in mind the shape of a parabola!



Example 3: Graph a Quadratic Function (1 of 5)

Graph the quadratic function $f(x) = x^2 - 4x - 5$ by hand.

Step 1 - Determine how the parabola opens.

$a = 1$, $a > 0$; the parabola opens upward.

Step 2 - Find the coordinates of the vertex and the equation of the axis of symmetry.

The x-coordinate of the vertex is $-\frac{b}{2a}$.

In the given function, a = 1, b = – 4, and c = – 5 . Then $x = -\frac{-4}{2(1)} = 2$.

The y-coordinate of the vertex is $f\left(-\frac{b}{2a}\right)$.

Then $f(1) = (2)^2 - 4(2) - 5 = -9$

The coordinates of the vertex are (2, – 9).

The equation of the axis of symmetry is $x = 2$.

Example 3: Graph a Quadratic Function

Step 3 - If possible, find the points associated with the *x*-intercepts by solving $f(x) = 0$.

$0 = x^2 - 4x - 5$

We are going to use factoring and the *Zero Product Principle* to solve this equation.

$(x - 5)(x + 1) = 0$

We set both factors equal to 0 as follows:

$x - 5 = 0$ and $x + 1 = 0$

then $x = 5$ and $x = -1$

There are two *x*-intercepts and the coordinates of the points associated with them are $(5, 0)$ and $(-1, 0)$.

Example 3: Graph a Quadratic Function (3 of 5)

Step 4 - Find the point associated with the *y*-intercept by computing *f*(0).

$f(0) = 0^2 – 4(0) – 5 = – 5.$

The coordinates of the point associated with the *y*-intercept are (0, – 5).

Step 5 - Graph the axis of symmetry.

The axis of symmetry is the line $x = 2$. It is drawn as a dashed line to indicate that it is really not a part of the graph.

Example 3: Graph a Quadratic Function

Step 6 - Graph the parabola.

- The coordinates of the vertex are $(2, -9)$.
- The coordinates of the points associated with the *x*-intercepts are $(5, 0)$ and $(-1, 0)$.
- The coordinates of the point associated with the *y*-intercept are $(0, -5)$.

We really do not have enough points to make an adequate graph of the function. Therefore, let's find a few more points particularly close to the vertex so that we can draw a nice bowl shape.

We are going to pick $x = -2, 1, 3, 4$, and 6 and find the corresponding *y*-values.

$f(-2) = (-2)^2 - 4(-2) - 5 = 7$

$f(1) = (1)^2 - 4(1) - 5 = -8$

$f(3) = (3)^2 - 4(3) - 5 = -8$

$f(4) = (4)^2 - 4(4) - 5 = -5$

$f(6) = (6)^2 - 4(6) - 5 = 7$

Example 3: Graph a Quadratic Function (5 of 5)

Step 6 continued

The coordinates of the additional points are

(– 2, 7), (1, – 8), (3, – 8), (4, – 5), and (6, 7)

We will now add these points to the coordinate system and then connect ALL points keeping in mind the shape of a parabola!