Examples
Quadratic Equations in One Variable

Based on power point presentations by Pearson Education, Inc.

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Learning Objectives

1. Define quadratic equations in one variable.
2. Solve quadratic equations using the *Quadratic Formula*.
3. Solve quadratic equations using *factoring*.
4. Solve quadratic equations using the *Square Root Property*.

Example 1: Solve Quadratic Equations Using the Quadratic Formula (1 of 5)

Solve $2x^2 + 2x - 1 = 0$ using the *quadratic formula*. Find only real solutions.

We will use $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

In the given quadratic equation, $a = 2$, $b = 2$, and $c = -1$. We insert these values into the quadratic formula to get

$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(2)(-1)}}{2(2)}$$

$$x = \frac{-2 \pm \sqrt{4 + 8}}{4}$$

$$x = \frac{-2 \pm \sqrt{12}}{4}$$

Example 1: Solve Quadratic Equations Using the Quadratic Formula (2 of 5)

Mathematicians usually want us to simplify the solutions.

Given $x = \dfrac{-2 \pm \sqrt{12}}{4}$, we can write 12 as 4(3). Then $\sqrt{12} = \sqrt{4}\left(\sqrt{3}\right) = 2\sqrt{3}$.

NOTE: In mathematics, we always try to simplify radicals!

We find $x = \dfrac{-2 \pm 2\sqrt{3}}{4}$.

Now, we notice that ALL terms have the factor of 2 in common.

Example 1: Solve Quadratic Equations Using the Quadratic Formula (3 of 5)

We will factor the 2 out of the numerator as follows:

$$x = \frac{-2 \pm 2\sqrt{3}}{4} = \frac{2\left(-1 \pm \sqrt{3}\right)}{4}$$

Next, we will cancel the 2 as follows:

$$x = \frac{2\left(-1 \pm \sqrt{3}\right)}{4} = \frac{\cancelto{1}{2}\left(-1 \pm \sqrt{3}\right)}{\cancel{4}\ 2}$$

and $x = \dfrac{-1 \pm \sqrt{3}}{2}$

Example 1: Solve Quadratic Equations Using the Quadratic Formula (4 of 5)

The quadratic equation has two solutions, and they are $\frac{-1-\sqrt{3}}{2}$ and $\frac{-1+\sqrt{3}}{2}$. These are irrational solutions because $\sqrt{3}$ is an irrational number.

Please note that in algebra, we usually leave answers in this form instead of changing them to decimal form.

Example 1: Solve Quadratic Equations Using the Quadratic Formula (5 of 5)

Just for calculator practice purposes, let's change $\frac{-1-\sqrt{3}}{2}$ and $\frac{-1+\sqrt{3}}{2}$ to a decimal approximation.

We must tell the calculator that the numerator contains 2 terms. We will use the parentheses buttons to enclose the numerator. Once this is accomplished, we divide by 2.

We get the following:

$\frac{-1-\sqrt{3}}{2} \cong -1.366025404$ … (infinitely many decimal places)

$\frac{-1+\sqrt{3}}{2} \cong 0.366025404$ … (infinitely many decimal places)

Example 2: Solve Quadratic Equations Using the Quadratic Formula (1 of 2)

Solve $x^2 - 2x = -2$ using the *quadratic formula*. Find only real solutions.

Before we can continue, we must first write the equation in the general form $ax^2 + bx + c = 0$. That is, $x^2 - 2x + 2 = 0$.

We can now see that $a = 1$, $b = -2$, and $c = 2$. We insert these values into the quadratic formula to get the following:

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)} = \frac{2 \pm \sqrt{4-8}}{2}$$

and $x = \dfrac{2 \pm \sqrt{-4}}{2}$

We note that the solution contains the imaginary value $\sqrt{-4}$. Therefore, we can state that this quadratic equation has NO real solutions.

Example 2: Solve Quadratic Equations Using the Quadratic Formula (2 of 2)

Even if the solutions are imaginary, mathematicians still want to simplify them.

Given $x = \dfrac{2 \pm \sqrt{-4}}{2}$, we can write $\sqrt{-4} = i\sqrt{4} = 2i$.

We find $x = \dfrac{2 \pm \sqrt{-4}}{2} = \dfrac{2 \pm 2i}{2}$

Now, we notice that ALL terms have the factor of 2 in common. We will factor the 2 out of the numerator and cancel as follows:

$$x = \frac{2 \pm 2i}{2} = \frac{2(1 \pm i)}{2} = \frac{\overset{1}{\cancel{2}}(1 \pm i)}{\cancel{2}\,1} = -1 \pm i$$

Example 3: Solve Quadratic Equations Using Factoring

Solve $x^2 + 5x = -6$ by factoring. Find only real solutions.

Let's first change the equation to the general form $ax^2 + bx + c = 0$.

$$x^2 + 5x + 6 = 0$$

Next, we will write the quadratic expression as a product of factors.

Find all pairs of positive integers whose product is $c = 6$.

$6 = (1)(6)$ and $6 = (2)(3)$ and $6 = (-1)(-6)$ and $6 = (-2)(-3)$

Using the pairs above, find one whose sum is $b = 5$.

We notice that 2 and 3 have a sum of 5.

Example 3: Solve Quadratic Equations Using Factoring

Let's create the template $(x \quad)(x \quad)$ and use 2 and 3 as the second terms.

$$(x + 2)(x + 3) = 0$$

Now, we will solve this equation using the *Zero Product Principle*:

$$x + 2 = 0 \text{ or } x + 3 = 0$$

Then $x = -2$ or $x = -3$

See see that the quadratic equation has two integer solutions, and they are $x = -2$ and $x = -3$.

NOTE: We could have used the *quadratic formula* to get the same solutions.

Example 4: Solve Quadratic Equations Using Factoring (1 of 2)

Solve $x^2 - 4x = -4$ by factoring. Find only real solutions.

Let's first change the equation to the general form $ax^2 + bx + c = 0.$

$x^2 - 4x + 4 = 0$

Next, we will write the quadratic expression as a product of factors.

Find all pairs of positive integers whose product is $c = 4.$

$4 = (1)(4)$ and $4 = (2)(2)$ and $4 = (-1)(-4)$ and $4 = (-2)(-2)$

Using the pairs above, find one whose sum is $b = -4.$

We notice that -2 and -2 have a sum of $-4.$

Example 4: Solve Quadratic Equations Using Factoring

Let's create the template $(x \quad)(x \quad)$ and use – 2 and – 2 as the second terms.

$$(x - 2)(x - 2) = 0$$

Now, we will solve this equation using the *Zero Product Principle*:

Since both factors are the same, we only need to set one equal to 0 as follows:

$x - 2 = 0$

and $x = 2$

The quadratic equation has one integer solution, and it is $x = 2$. **We could have used the *quadratic formula* to get the same solution.**

Example 5: Solve Quadratic Equations Using Factoring

Solve $2x^2 + 2x = 0$ by factoring. Find only real solutions.

In this equation, the c in $ax^2 + bx + c = 0$ missing. However, we note that both terms have a greatest common factor of $2x$. We will factor it out of each term as follows:

$2x(x + 1) = 0$

Now, we will solve this equation using the *Zero Product Principle*:

$2x = 0$ or $x + 1 = 0$

and $x = 0$ or $x = -1$

The quadratic equation has two integer solutions, and they are $x = 0$ and $x = -1$. **We could have used the *quadratic formula* to get the same solutions.**

Example 6: Solve Quadratic Equations Using the Square Root Property

Solve $x^2 = 16$ by the *Square Root Property*. Find only real solutions.

This equation is already in the form $u^2 = d$. Here, $u = x$ and $d = 16$. Therefore, we can use the *Square Root Property* to state

$$x = \pm\sqrt{16} = \pm 4$$

The quadratic equation has two integer solutions, and they are – 4 and 4.

NOTE: We could have used the *quadratic formula* with $a = 1$, $b = 0$, and $c = -16$ to get the same solutions.

Example 7: Solve Quadratic Equations Using the Square Root Property

Solve $3x^2 - 21 = 0$ by the *Square Root Property*. Find only real solutions.

Before we can continue, we must first write the equation in the form $u^2 = d$.

$3x^2 - 21 = 0$
$3x^2 = 21$
$x^2 = 7$ Note, now $u = x$ and $d = 7$.

By the *Square Root Property,* we find that $x = \pm\sqrt{7}$.

This quadratic equation has two irrational solutions. They are $x = \sqrt{7}$ and $x = -\sqrt{7}$. We do not usually change them to decimal form!

NOTE: We could have used the *quadratic formula* with $a = 3$, $b = 0$, and $c = -21$ to get the same solutions.

Example 8: Solve Quadratic Equations Using the Square Root Property

Solve $2x^2 + 18 = 0$ by the *Square Root Property*. Find only real solutions.

Before we can continue, we must first write the equation in the form $u^2 = d$.

$2x^2 + 18 = 0$
$2x^2 = -18$
$x^2 = -9$ Note, now $u = x$ and $d = -9$.

By the *Square Root Property,* we find $x = \pm\sqrt{-9} = \pm\ 3i$.

Since the values for x are imaginary, we must state that this quadratic equation has "NO real solutions."

NOTE: We could have used the *quadratic formula* with $a = 2$, $b = 0$, and $c = 18$ to get the same solutions.

Example 9: Solve Quadratic Equations (1 of 2)

Solve $3x^2 + 27x = 0$. Find only real solutions.

Certainly, we could use the *Quadratic Formula* with $a = 3$, $b = 27$, and $c = 0$. Since the *Quadratic Formula* is sort of cumbersome to work with, our first thought should always be factoring or the *Square Root Property*.

While the given equation has only two terms, the *Square Root Property* will not easily work because neither term is a constant. So how about factoring?

We do notice that both terms have the factor of $3x$ in common which we can factor out as follows:

$3x(x + 9) = 0$

Now, we will solve this equation using the *Zero Product Principle.*

Example 10: Solve Quadratic Equations (2 of 2)

We set both factors equal to 0 as follows:

$3x = 0$ and $x + 9 = 0$

So that $x = 0$ and $x = -9$

We find that the quadratic equation has two integer solution, namely, $x = 0$ and $x = -9$.

NOTE: We could have used the *quadratic formula* to get the same solutions.