Examples
Polynomial Functions

Based on power point presentations by Pearson Education, Inc.

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Learning Objectives

1. Define polynomial functions.
2. Memorize the characteristics of graphs of polynomial functions.
3. Memorize and apply the characteristics of the *Zeros* of polynomial functions.

Example 1: Vocabulary of Polynomial Functions

a. Name the leading coefficient of the following polynomial function:

$f(x) = -x^5 + 6x^2 - 4x - 1$

The leading coefficient is – 1.

b. Name the leading coefficient of the following polynomial function:

$g(x) = 2x^3 + 10x^2$

The leading coefficient is 2.

Example 2: Vocabulary of Polynomial Functions

a. Name the degree of the following polynomial function:

$f(x) = -x^5 + 6x^2 - 4x - 1$

The degree is 5.

b. Name the degree of the following polynomial function:

$g(x) = 2x^3 + 10x^2$

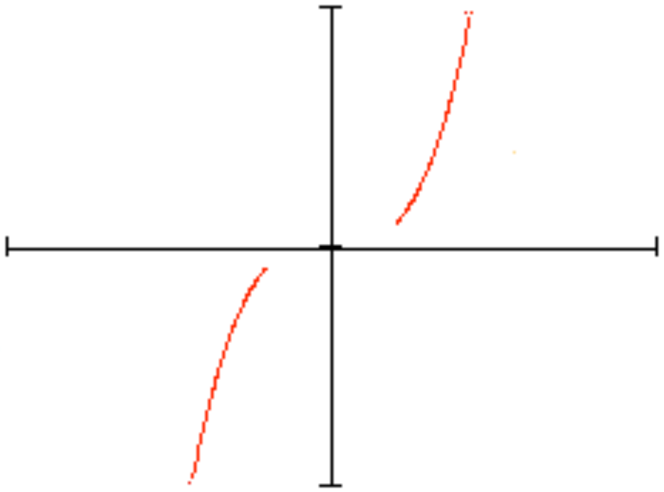
The degree is 3.

Example 3: Recognize the Characteristics of Graphs of Polynomial Functions

Determine the end behavior of the graph of $g(x) = 2x^3 + 3x^2 - 8x - 12$.

The degree of the function is 3, which is odd.

The leading coefficient 2 is positive.

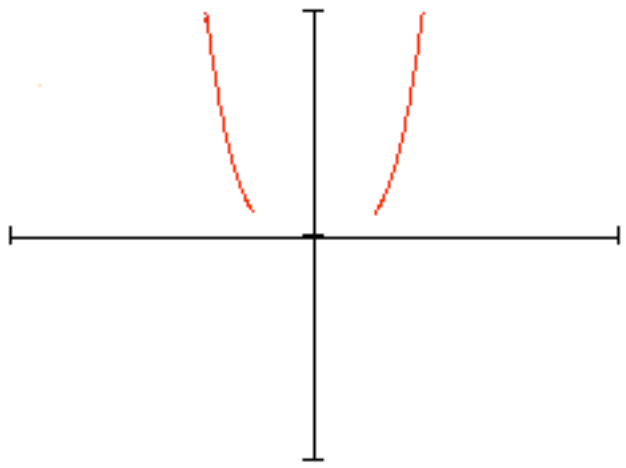


Example 4: Recognize Characteristics of Graphs of Polynomial Functions

Determine the end behavior of the graph of $h(x) = x^4 - 8x^2 - 9$.

The degree of the function is 4, which is even.

The leading coefficient, 1, is positive.

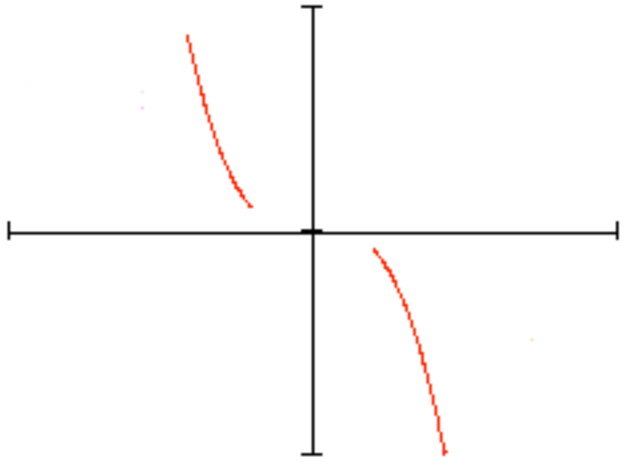


Example 5: Recognize Characteristics of Graphs of Polynomial Functions

Determine the end behavior of the graph of $g(x) = -x^3 + 2x^2 - x + 2$

The degree of the function is 3, which is odd.

The leading coefficient – 1 is negative.

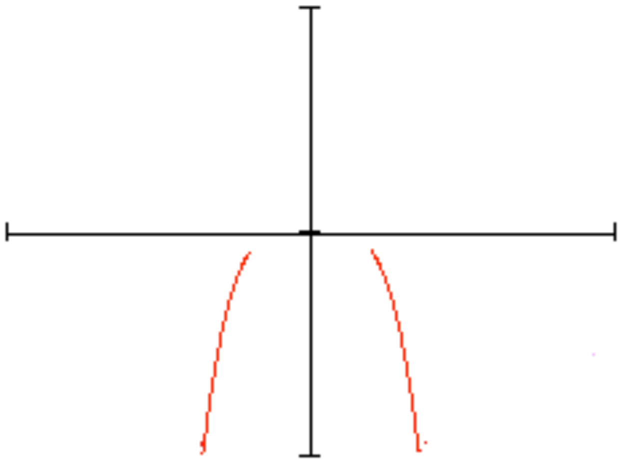


Example 6: Recognize Characteristics of Graphs of Polynomial Functions

Determine the end behavior of the graph of $f(x) = -x^4 + 16x^2$.

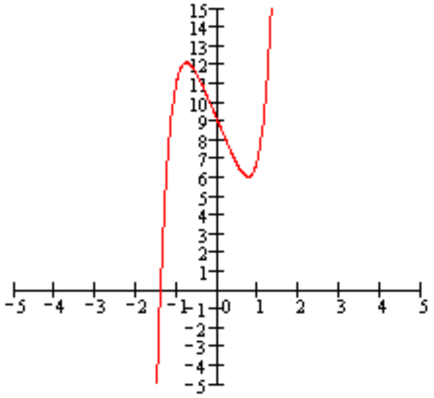
The degree of the function is 4, which is even.

The leading coefficient – 1 is negative.

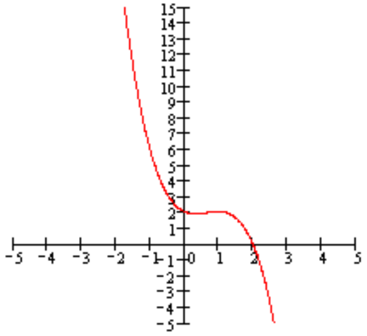


Example 7: Recognize Characteristics of Graphs of Polynomial Functions

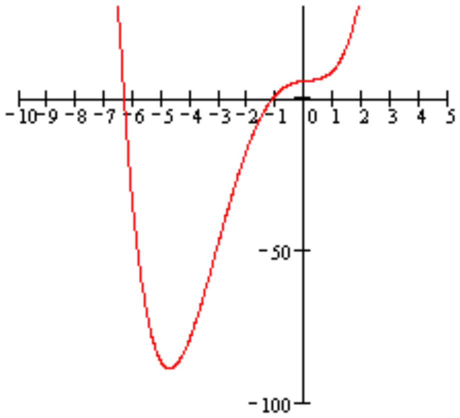
Determine the whether the degree is even or odd and whether the leading coefficient *a* is less than or greater than 0.



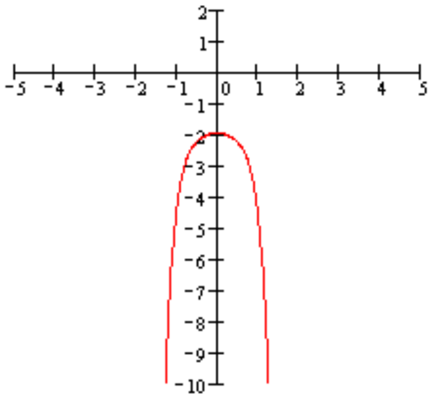
degree: odd
lead coeff: a > 0



degree: odd
lead coeff: a < 0



degree: even
lead coeff: a > 0



degree: even
lead coeff: a < 0

Example 8: Apply the Characteristics of *Zeros*

The polynomial function P is of degree 3. Its *Zeros* are 0, 1, and – 5. Its leading coefficient is 5. Find the equation of the polynomial function. Write it as a product of linear factors.

By the *Factor Theorem*, if a number ***r*** is a *Zero* of the polynomial function, then ***(x – r)*** is a *linear factor* of the function and vice versa.

Given the *Zeros* and their multiplicities, we can write the polynomial function as a product of linear factors as following:

$$P(x) = 5(x - 0)(x - 1)\,(x - (-5))$$
$$= 5x(x - 1)\,(x + 5)$$

Note: Be sure not to forget the leading coefficient 5!

Example 9: Apply the Characteristics of *Zeros*

The polynomial function P is of degree 9. Its *Zeros* are

$1 + i$ with multiplicity 3
$1 - i$ with multiplicity 3
-1 with multiplicity 2
4 with multiplicity 1

Its leading coefficient is 1. Find the equation of the polynomial function. Write it as a product of linear factors.

Given the *Zeros* and their multiplicities, we can write the polynomial function as a product of linear factors as following:

$P(x) = (x - (-1))^2\,(x - 4)\,(x - (1 + i))^3\,(x - (1 - i))^3$

$P(x) = (x + 1)^2\,(x - 4)\,(x - 1 - i)^3\,(x - 1 + i)^3$

Note: Since the leading coefficient is 1, we usually don't write it.

Example 10: Apply the Characteristics of *Zeros*

The polynomial function P is of degree 3 and has a leading coefficient of -2 . Its *Zeros* include 5 and 4i.

Find the equation of the polynomial function. Write it as a product of linear factors.

By the *Factor Theorem*, if a number ***r*** is a *Zero* of the polynomial function, then ***(x – r)*** is a *linear factor* of the function and vice versa. We also know that imaginary *Zeros* appear in conjugate pairs. Therefore, – 4i must also be a *Zero*.

Given the *Zeros* and their multiplicities, we can write the polynomial function as a product of linear factors as following:

$P(x) = -2\,(x-5)\,(x-4i)\,(x-(-4i))$

$P(x) = -2\,(x-5)\,(x-4i)\,(x+4i)$

Note: Be sure not to forget the leading coefficient – 2!