Examples
Logarithmic Functions

Based on power point presentations by Pearson Education, Inc.

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Learning Objectives

1. Memorize the definition of the common logarithmic function.
2. Memorize the characteristics of the graphs of logarithmic functions.
3. Apply transformations to the common logarithmic function.
4. Graph the common logarithmic function and its transformations by hand.

Example 1: Apply Transformations to Common Logarithmic Functions (1 of 2)

Starting with the graph of $f(x) = \log_2 x$, write the equation of the graph that results from the following. Include the equation of the vertical asymptote.

(1) a vertical shift up 3 units

$y = \log_2 x + 3$

To find the equation of the vertical asymptote we will set x equal to 0 and solve for x. We find $x = 0$.

(2) a horizontal shift to the left 3 units

$y = \log_2 (x + 3)$ **Note the difference in (1) above!**

To find the equation of the vertical asymptote we will set $(x + 3)$ equal to 0 and solve for x. We find $x = -3$.

Remember, a horizontal shift affects the vertical asymptote.

Example 1: Apply Transformations to Common Logarithmic Functions (2 of 2)

(3) a vertical shift down 3 units

$y = \log_2 x - 3$

To find the equation of the vertical asymptote we will set x equal to 0 and solve for x. We find $x = 0$.

(4) a horizontal shift to the right 3 units

$y = \log_2 (x - 3)$ **Note the difference in (3) above!**

To find the equation of the vertical asymptote we will set $(x - 3)$ equal to 0 and solve for x. We find $x = 3$.

Remember, a horizontal shift affects the vertical asymptote.

Example 2: Apply Transformations to Common Logarithmic Functions (1 of 2)

Starting with the graph of $f(x) = \ln x$, write the equation of the graph that results from the following. Include the equation of the vertical asymptote.

(1) a reflection across the x-axis

$y = -\ln x$

To find the equation of the vertical asymptote we will set x equal to 0 and solve for x. We find $x = 0$.

(2) a reflection across the y-axis

$y = \ln(-x)$

To find the equation of the vertical asymptote we will set $-x$ equal to 0 and solve for x. We find $x = 0$.

Example 2: Apply Transformations to Common Logarithmic Functions (2 of 2)

(3) a reflection across the x- and y-axis

$y = -\ln(-x)$

To find the equation of the vertical asymptote we will set $-x$ equal to 0 and solve for x. We find $x = 0$.

(4) a vertical shift up 1 unit and a horizontal shift to the right 2 units

$y = \ln(x - 2) + 1$

To find the equation of the vertical asymptote we will set $(x - 2)$ equal to 0 and solve for x. We find $x = 2$.

Example 3: Apply Transformations to Common Logarithmic Functions

Starting with the graph of $f(x) = log_3\, x$, write the equation of the graph that results from the following. Include the equation of the horizontal asymptote.

(1) a vertical shift up 2 units and a horizontal shift to the left 1 unit

$y = \log_3 (x + 1) + 2$

To find the equation of the vertical asymptote we will set $(x + 1)$ equal to 0 and solve for x. We find $x = -1$.

(2) a vertical shift down 2 units and a horizontal shift to the right 1 unit

$y = \log_3 (x - 1) - 2$

To find the equation of the vertical asymptote we will set $(x - 1)$ equal to 0 and solve for x. We find $x = 1$.

Example 4: Graph a Common Logarithmic Function by Hand
(1 of 3)

Graph the function $f(x) = \log_2 x$ by hand.

1. **Equation of the vertical asymptote:**

   Set the argument equal to 0 and solve for x.

   This is simply $x = 0$

2. **Point associated with the *x*-intercept (when *y* = 0):**

   $0 = log_2\, x$

   Let's convert to exponential form to get

   $x = 2^0$ and we find that $x = 1$

   The x-intercept is at (1, 0).

Example 4: Graph a Basic Logarithmic Function by Hand

3. Find additional points to either side of the x-intercept:

How about x = 0.5, 2, 4, and 8?

Using $f(x) = \log_2 x$, we set up a table of coordinates and then plot these points.

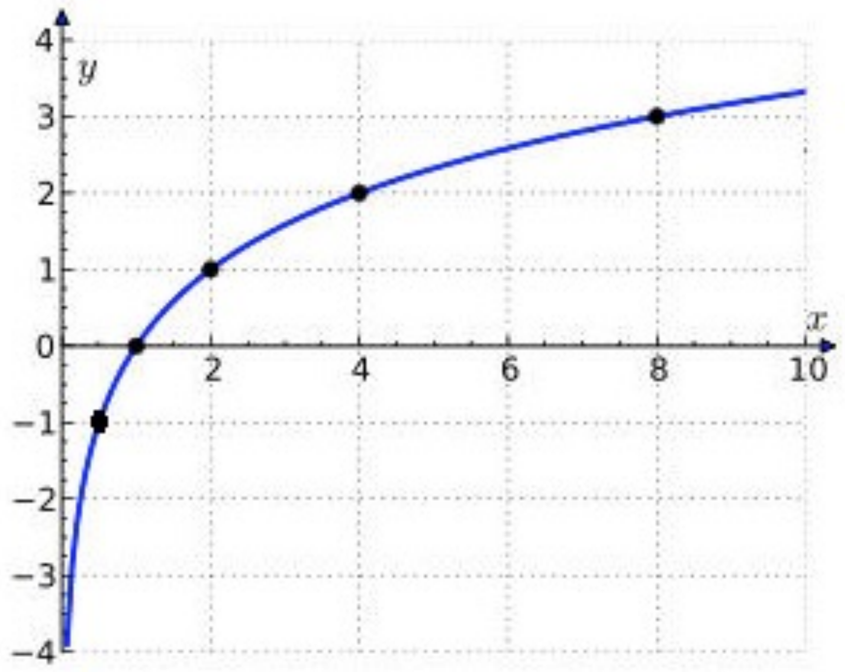
x	$f(x) = \log_2 x$
0.5	$log_2 2^{-1} = -1$
2	$log_2 2 = 1$
4	$log_2 2^2 = 2$
8	$log_2 2^3 = 3$

Please note that we specifically used x = 0.5, 2, 4, and 8 because these values allowed us to use the basic logarithmic properties to find the y-value.

We could have also used the Change-of-Base Property!

Example 4: Graph a Basic Logarithmic Function by Hand

4. Connect all points found in the previous steps keeping in mind the shape of the common logarithmic function:



Since the vertical asymptote is the *y*-axis, we will not graph it as a dashed line.

Example 5: Graph a Transformation of a Common Logarithmic Function by Hand (1 of 3)

Graph the function $g(x) = \log_2 (x - 1)$ by hand.

1. **Equation of the vertical asymptote:**

   Set the argument equal to 0 and solve for x.

   $x - 1 = 0$ and $x = 1$

2. **Point associated with the x-intercept (when $y = 0$):**

   $0 = log_2 (x - 1)$

   Let's convert to exponential form to get

   $(x - 1) = 2^0$

   then $x - 1 = 1$ and $x = 2$

   The x-intercept is at (2, 0).

Example 5: Graph a Transformation of a Common Logarithmic Function by Hand (2 of 3)

3. Find additional points to either side of the x-intercept:

How about x = 1.5, 3, and 5?

Using $f(x) = \log_2 (x - 1)$, we set up a table of coordinates and then plot these points.

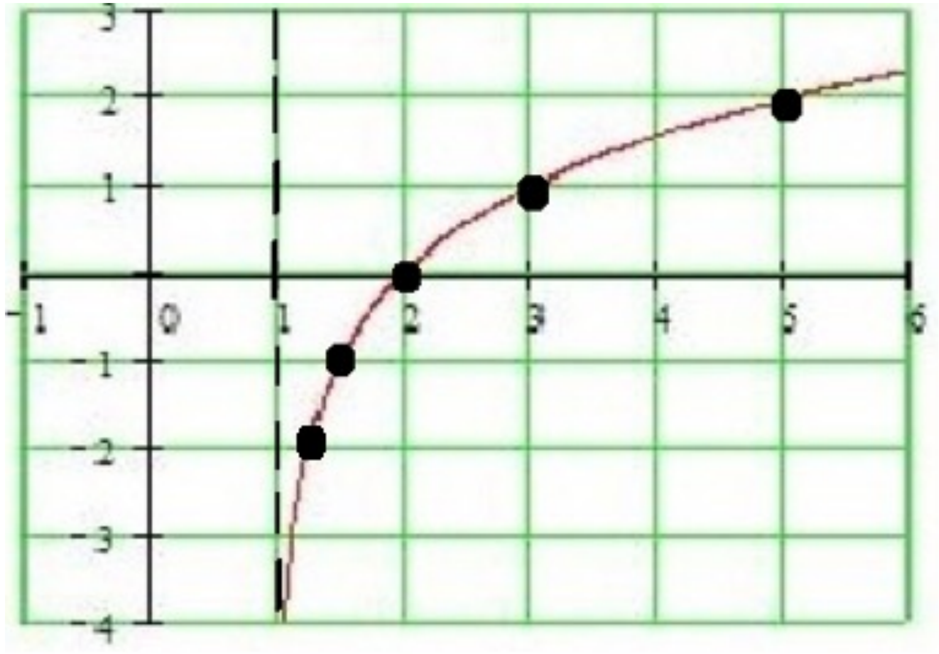
x	$g(x) = \log_2 (x - 1)$
1.5	$\log_2 (1.5 - 1)$ $= \log_2 (2^{-1}) = -1$
3	$\log_2 (3 - 1)$ $= \log_2 (2^1) = 1$
5	$\log_2 (5 - 1)$ $= \log_2 (2^2) = 2$

Please note that we specifically used x = 1.5, 3, and 5 because these values allowed us to use the basic logarithmic properties to find the *y*-value.

We could have also used the Change-of-Base Property!

Example 5: Graph a Transformation of a Common Logarithmic Function (3 of 3)

4. Connect all points found in the previous steps keeping in mind the shape of the common logarithmic function:



Here we used an additional point, namely (1.25, – 2) !
The *vertical asymptote* is drawn as a dashed line when we graph logarithmic functions by hand!