Examples
Logarithm Rules

Based on power point presentations by Pearson Education, Inc.

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Learning Objective

Memorize and apply the logarithm rules.

Example 1: Use Logarithm Rules

Expand $\log\left(x^4 \sqrt[3]{y}\right)$ using logarithm rules until no more can be applied.

We see that the argument is first and foremost a product. Therefore, we will use the Product Rule first.

$$\log x^4 + \log \sqrt[3]{y}$$

There are also some exponents. Please observe that $\sqrt[3]{y}$ can be written as $y^{\frac{1}{3}}$.

Therefore, we can write the following:

$$\log x^4 + \log y^{\frac{1}{3}}$$

Lastly, we will apply the Power Rule to get the following:

$$4\log x + \frac{1}{3}\log y$$

This is the expanded form of the given logarithm. No more rules can be applied.

Example 2: Use Logarithm Rules (1 of 2)

Expand $\log_5\left(\frac{\sqrt{x}}{25y^3}\right)$ using logarithm rules until no more can be applied.

We see that the argument is first and foremost a quotient. Therefore, we will use the Quotient Rule first.

$$\log_5 \sqrt{x} - \log_5 25y^3$$

There are also some exponents. Please observe that $\sqrt{x}$ can be written as $x^{\frac{1}{2}}$.

Therefore, we can write the following:

$$\log_5 x^{\frac{1}{2}} - \log_5 25y^3$$

Lastly, we will apply the Power Rule … BUT … we observe that the exponent in the second term only pertains to the *y*. Here we must use the Product Rule before we can use the Power Rule as follows:

$$\log_5 x^{\frac{1}{2}} - (\log_5 5^2 + \log_5 y^3)$$

Example 2: Use Logarithm Rules (2 of 2)

Note that the minus sign must affects the expansion of the product. Therefore, we needed the parentheses.

Finally, we can apply the Power Rule and at the same time we will open up the parentheses.

$$\frac{1}{2}\log_5 x - 2\log_5 5 - 3\log_5 y$$

Knowing that $\log_5 5 = 1$, we could state that $2 \log_5 5 = 2(1) = 2$ and write the following:

$$\frac{1}{2}\log_5 x - 2 - 3\log_5 y$$

This is the expanded form of the given logarithm. No more rules can be applied.

Example 3: Use Logarithm Rules

Expand $\log\left(\sqrt[3]{\frac{x^2}{y^3z}}\right)$ using logarithm rules until no more can be applied.

We see that the argument is first and foremost a power. Remember, we can write radicals in exponential form! Therefore, we will use the Power Rule first.

$$\log\left(\frac{x^2}{y^3z}\right)^{\frac{1}{3}} = \frac{1}{3}\log\left(\frac{x^2}{y^3z}\right)$$

Now see that the argument is first and foremost a quotient. Therefore, we can write the following:

$\frac{1}{3}\left(\log x^2 - \log y^3z\right)$ Note that the $\frac{1}{3}$ must stay in front of the parentheses ALWATYS.

Now, we will examine the inside of the parentheses. We can use the Power Rule on the argument of the first logarithm and the Product Rule on the argument in the second logarithm.

Example 3: Use Logarithm Rules (1 of 2)

$$\frac{1}{3}\left(2\log x - \left(\log y^3 + \log z\right)\right)$$

Note that the minus sign must stay in front of both terms used in the Product Rule. That's why we enclosed them in a second set of parentheses.

We can use the Power Rule again in the second set of parentheses. At the same time we will open them up.

$$\frac{1}{3}\left(2\log x - 3\log y - \log z\right)$$

This is the expanded form of the given logarithm. No more rules can be applied. If we want, we could distribute the $\frac{1}{3}$ to every time, but it is not necessary.

Example 4: Use Logarithm Rules

Condense $2 \ln x + 4 \ln(x + 5)$ to a single logarithm using logarithm rules until no more can be applied.

We notice the **+** between the two terms. We are thinking "Product Rule"! However, this rule requires coefficients of 1, which we do not currently have. But we can use the *Power Rule* to bring about the necessary changes!

$\ln x^2 + \ln (x + 5)^4$

Now we can apply the Product Rule to get

$\ln [x^2 (x + 5)^4]$

This is the condensed form of the given logarithms. No more rules can be applied.

Example 5: Use Logarithm Rules (1 of 2)

Condense $6 \log_5 r + 8 \log_5 s - \log_5 w$ to a single logarithm using logarithm rules until no more can be applied.

We notice a + and – between the three terms. We are thinking "Product Rule" and "Quotient Rule"! However, both rules require coefficients of 1, which we do not currently have.

So, we'll use the *Power Rule* to bring about the necessary changes!

$\log_5 r^6 + \log_5 s^8 - \log_5 w$

So now what? Which logarithm rule should we use first?

Example 5: Condense Logarithmic Expressions (2 of 2)

The correct way to approach this is to work from left to right. Therefore, we will first apply the Product Rule to the first two terms to get the following:

$\log_5 (r^6 s^8) - \log_5 w$

Finally, we will use the Quotient Rule to get

$\log_5 \left(\frac{r^6 s^8}{w}\right)$

This is the condensed form of the given logarithms. No more rules can be applied.

Example 6: Use Logarithm Rules (1 of 3)

Condense 5 ln x – 4 ln y – 3 ln z to a single logarithm using logarithm rules until no more can be applied.

We notice minuses between the three terms. We are thinking "Quotient Rule"! However, this rule requires a coefficients of 1, which we do not currently have.

So, we'll use the *Power Rule* to bring about the necessary changes!

$\ln x^5 - \ln y^4 - \ln z^3$

So now what? How do we approach this?

Example 6: Use Logarithm Rules (2 of 3)

The correct way to approach this is to work from left to right. Therefore, we will first apply the Quotient Rule to the first two terms to get the following:

$\ln\left(\frac{x^5}{y^4}\right) - \ln z^3$

Finally, we will use the Quotient Rule again to get

$\ln\frac{\frac{x^5}{y^4}}{z^3}$–

We ended up with a complex fraction in the argument. That's never something we want in mathematics. Therefore, we will change it to a proper fraction.

Example 6: Use Logarithm Rules (3 of 3)

We know the following: $\dfrac{\frac{x^5}{y^4}}{z^3} = \dfrac{x^5}{y^4} \div \dfrac{z^3}{1}$

From our work with fractions, we know that dividing by a number is the same as multiplying by its reciprocal.

Therefore, we can write $\dfrac{x^5}{y^4} \cdot \dfrac{1}{z^3} = \dfrac{x^5}{y^4 z^3}$

and we get $\ln \dfrac{x^5}{y^4 z^3}$

This is the condensed form of the given logarithms. No more rules can be applied.

Example 7: Use Logarithm Rules (1 of 2)

Condense $2 \ln (x - 3) - 3 \ln (x + 2) + 4 \ln (x + 1)$ to a single logarithm using logarithm rules until no more can be applied.

We notice a – and + between the three terms. We are thinking "Quotient Rule" and "Product Rule"! However, both rules require coefficients of 1, which we do not currently have.

So, we'll use the *Power Rule* to bring about the necessary changes!

$\ln (x - 3)^2 - \ln (x + 2)^3 + \ln (x + 1)^4$

So now what? Which logarithm rule should we use first?

Example 7: Condense Logarithmic Expressions (2 of 2)

The correct way to approach this is to work from left to right. Therefore, we will first apply the Quotient Rule to the first two terms to get the following:

$$\ln\left(\frac{(x-3)^2}{(x+2)^3}\right) + \ln\,(x+1)^4$$

Finally, we will use the Product Rule to get

$$\ln\left(\frac{(x-3)^2}{(x+2)^3}(x+1)^4\right) = \ln\left(\frac{(x-3)^2(x+1)^4}{(x+2)^3}\right)$$

This is the condensed form of the given logarithms. No more rules can be applied.