Examples
Logarithmic Equations in One Variable

Based on power point presentations by Pearson Education, Inc.

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Learning Objectives

1. Solve logarithmic equations in which not all terms are logarithms.
2. Solve logarithmic equations in which ALL terms are logarithms.

Example 1: Solve Logarithmic Equations (1 of 2)

Solve $\log_2 (x - 4) = 3$.

There already exists a single logarithm with coefficient 1. Therefore, we can change to exponential form as follows:

$x - 4 = 2^3$

$x - 4 = 8$

$x = 12$

This is the “proposed solution.

Example 1: Solve Logarithmic Equations

Check $x = 12$ in the original equation $\log_2(x - 4) = 3$ to ensure that all logarithm arguments are positive.

$\log_2(12 - 4) = 3$

$\log_2(8) = 3$ We can stop here because the logarithm argument is positive.

We verified that $x = 12$ is a true solution.

Example 2: Solve Logarithmic Equations (1 of 2)

Solve 4 ln (3*x*) = 8.

Remember ln *x* equals $\log_e x$! Here we MUST first isolate the logarithmic expression to get a coefficient of 1. We do this by dividing both sides by 4.

ln (3*x*) = 2

Now we can change to exponential form.

$$e^2 = 3x$$ We changed to exponential form.

$$x = \frac{e^2}{3}$$ This is the “proposed solution.

NOTE: In algebra, we usually do not change solutions to decimal form.

Example 2: Solve Logarithmic Equations (2 of 2)

Check $x = \frac{e^2}{3}$ in the original equation 4 ln (3*x*) = 8 to ensure that all logarithm arguments are positive.

$$4 \ln\left(3 \cdot \frac{e^2}{3}\right) = 8$$

$$4 \ln(e^2) = 8$$

We can stop here because the logarithm argument is positive.

We verified that $x = \frac{e^2}{3}$ is a true solution.

Example 3: Solve Logarithmic Equations

Solve $\log x + \log(x - 3) = 1$.

Remember that $\log x$ equals $\log_{10} x$. Here we MUST first achieve one single logarithm. We do this by using the Product Rule.

$\log(x \cdot (x - 3)) = 1$

Then we can solve as follows:

$x(x - 3) = 10^1$ We changed to exponential form.

$x^2 - 3x = 10$

$x^2 - 3x - 10 = 0$

$(x - 5)(x + 2) = 0$ We factored this quadratic equation.

Example 3: Solve Logarithmic Equations (2 of 3)

We will now use the *Zero Product Principle* to solve for x.

$x - 5 = 0$ and $x + 2 = 0$

then $x = 5$ and $x = -2$, which are ”proposed” solutions.

Check $x = 5$ in the original equation $\log x + \log (x - 3) = 1$ to ensure that all logarithm arguments are positive.

$\log (5) + \log (5 - 3) = 1$

$\log (5) + \log (2) = 1$ We can stop here because all logarithm arguments are both positive.

We verified that $x = 5$ is a true solution.

Example 3: Solve Logarithmic Equations

Check $x = -2$ in the original equation to ensure that all logarithm arguments are positive.

$\log(-2) + \log(-2-3) = 1$ Note that both logarithm arguments are NEGATIVE!

We just found that $x = -2$ is NOT a true solution!

In summary, the only true solution is $x = 5$.

Example 4: Solve Logarithmic Equations

Solve $5 \log(4x) = 10 \log(x - 3)$.

Here we must first achieve one single logarithm on the right and the left. Dividing both sides by 5 we get

$\log(4x) = 2 \log(x - 3)$

Next, we will use the Power Rule to get

$\log(4x) = \log(x - 3)^2$

Example 4: Solve Logarithmic Equations (2 of 5)

Given an equality with the logarithm bases on both sides equal and having a coefficient of 1, the arguments must be equal as well. Therefore, we can set the arguments equal to get

$4x = (x - 3)^2$

$4x = (x - 3)(x - 3)$

$4x = x^2 - 6x + 9$

$x^2 - 6x + 9 - 4x = 0$

Combining like terms, we get $x^2 - 10x + 9 = 0$

Example 4: Solve Logarithmic Equations (3 of 5)

Since we are obviously dealing with a quadratic equation, we will prepare it for a solution by factoring or possibly by the quadratic formula.

We find that we can factor this to get $(x - 1)(x - 9) = 0$.

We will now use the *Zero Product Principle* to solve for x.

$x - 1 = 0$ and $x - 9 = 0$

then $x = 1$ and $x = 9$, which are “proposed” solutions.

Example 4: Solve Logarithmic Equations (4 of 5)

Now we need to check BOTH solutions in the original equation to ensure that the logarithm argument is positive.

Check $x = 1$ in the original equation $5 \log (4x) = 10 \log (x - 3)$ to ensure that all logarithm arguments are positive.

$$5 \log (4 \cdot 1) = 10 \log (1 - 3)$$

$$5 \log (4) = 10 \log (-2)$$

Note that one logarithm argument is NEGATIVE!

We just found that $x = 1$ is NOT a true solution!

Example 4: Solve Logarithmic Equations

Check $x = 9$ in the original equation $5 \log (4x) = 10 \log (x - 3)$ to ensure that all logarithm arguments are positive.

$$5 \log (4 \cdot 9) = 10 \log (9 - 3)$$

$$5 \log (36) = 10 \log (6)$$

We can stop here because all logarithm arguments are positive.

We verified that $x = 9$ is a true solution.

In summary, the only true solution is $x = 9$.

Example 5: Solve a Logarithmic Equation

Solve $\ln(x - 3) = \ln(7x - 23) - \ln(x + 1)$.

Remember that $\ln x$ equals $\log_e x$. Here we must first write one single logarithmic expression on the right. We achieve this by using the *Quotient Rule*.

$$\ln(x-3) = \ln\left(\frac{7x-23}{x+1}\right)$$

Given an equality with the logarithm bases on both sides equal, the arguments must be equal as well. Therefore, we can set the arguments equal to get

$$x - 3 = \frac{7x-23}{x+1}$$

We multiply both sides by the denominator $(x + 1)$!

Example 5: Solve a Logarithmic Equation (2 of 4)

We now multiply both sides by the denominator ($x + 1$)!

$$(x+1)(x-3) = \frac{7x-23}{x+1} \cdot (x+1)$$

$$(x-3)(x+1) = 7x-23$$

Combining like terms we get $x^2 - 2x - 3 = 7x - 23$.

Since we are obviously dealing with a quadratic equation, we will prepare it for a solution by factoring or possibly by the quadratic formula.

$$x^2 - 9x + 20 = 0$$

We find that we can factor this to get $(x - 5)(x - 4) = 0$.

Example 5: Solve a Logarithmic Equation (3 of 4)

We will now use the *Zero Product Principle* to solve for x.

$x - 5 = 0$ and $x - 4 = 0$

then $x = 5$ and $x = 4$, which are ”proposed” solutions.

Check $x = 5$ in the original equation $\ln(x - 3) = \ln(7x - 23) - \ln(x + 1)$ to ensure that all logarithm arguments are positive.

$\ln(5 - 3) = \ln(7 \cdot 5 - 23) - \ln(5 + 1)$

$\ln(2) = \ln(12) - \ln(6)$

We can stop here because all logarithm arguments are positive.

We verified that $x = 5$ is a true solution.

Example 5: Solve a Logarithmic Equation (4 of 4)

Check $x = 4$ in the original equation $\ln(x - 3) = \ln(7x - 23) - \ln(x + 1)$ to ensure that all logarithm arguments are positive.

$$\ln(4 - 3) = \ln(7 \cdot 4 - 23) - \ln(4 + 1)$$

$$\ln(1) = \ln(5) - \ln(5)$$

We can stop here because all logarithm arguments are positive.

We verified that $x = 4$ is a true solution.

In summary, the equation has two true solutions, namely $x = 4$ and $x = 5$.