Examples
Linear and Constant Functions

Based on power point presentations by Pearson Education, Inc.
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Learning Objectives

1. Define linear functions.
2. Graph linear functions by hand.
3. Define and graph constant functions by hand.

Example 1: Graph a Linear Function (1 of 4)

Graph the linear function h(x) = 2x by hand.

Let's try the Intercept Method!

x-intercept:

Given h(x) = 2x, let h(x) = y = 0 and solve for x

$$0 = 2x$$

$$0 = x \text{ (dividing both sides by 2!)}$$

The x-intercept is 0 and the point associated with the x-intercept is (0, 0), which is the origin (the point at which the two coordinate axes intersect).

Example 1: Graph a Linear Function (2 of 4)

y-intercept:

Given $h(x) = 2x$, let $x = 0$ and solve for $h(x) = y$.

$$h(0) = 2(0) = y$$

The *y*-intercept is 0 and the point associated with the y-intercept is also (0, 0).

Example 1: Graph a Linear Function (3 of 4)

Most often the *Intercept Method* produces two different points. However, this does not always happen. In our case, the *Intercept Method* only produced one point, namely (0, 0).

Since we need at least two points to graph a line, we will use the *Point-by-Point Plotting Method* to find at least one more point. How about we find two points by letting x be equal to –2 and 2? This way we have a point above the origin and below the origin.

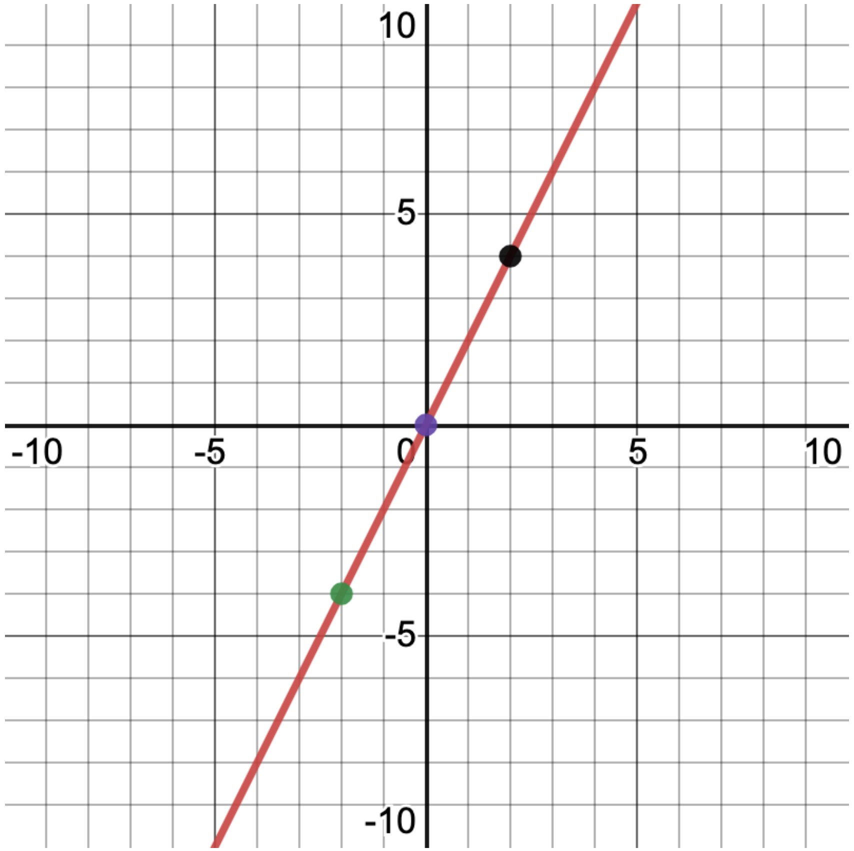
Given $h(x) = 2x$, we calculate $h(-2) = y = 2(-2) = -4$

and $h(2) = y = 2(2) = 4$

The coordinates of the additional points are (–2, – 4) and (2, 4).

Example 1: Graph a Linear Function (4 of 4)

Graph the function by drawing a line through the points found in the previous three slides.



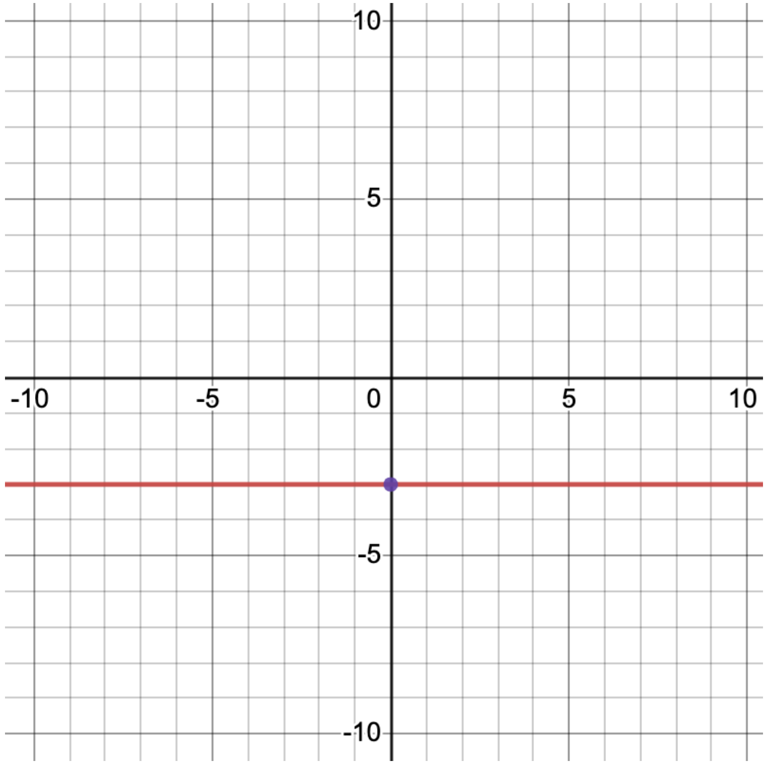
Reminder:
We found the coordinates (–2, –4), (0, 0), and (2, 4).

Example 2: Graph a Constant Function

Graph the *Constant Function* k(*x*) = – 3 by hand.

As stated already, the graph of the *constant function* is a horizontal line.

We know that the *y*-intercept is – 3, therefore the point associated with the *y*-intercept must be (0, – 3).



In the picture, we drew the point (0, – 3), which is the point associated with the *y*-intercept.

Then, we used our knowledge that the graph of the *constant function* is a horizontal line to draw a line through (0, – 3) parallel to the *x*-axis.

Example 3: Graph a Linear Function (1 of 3)

Graph the *Identity Function* $f(x) = x$ by hand.

Let's try the Intercept Method!

x-intercept:

Given $f(x) = x$, let $f(x) = y = 0$ and solve for x.

$$0 = x$$

The x-intercept is 0 and the point associated with the x-intercept is (0, 0), which is the origin (the point at which the two coordinate axes intersect).

y-intercept:

Given $f(x) = x$, let $x = 0$ and solve for $f(x) = y$.

$$f(0) = 0 = y$$

The y-intercept is 0 and the point associated with the y-intercept is also (0, 0).

Example 3: Graph a Linear Function (2 of 3)

Most often the *Intercept Method* produces two different points. However, this does not always happen. In our case, the *Intercept Method* only produced one point, namely (0, 0).

Since we need at least two points to graph a line, we will use the *Point-by-Point Plotting Method* to find at least one more point. How about we find two points by letting x be equal to –2 and 2? This way we have a point above the origin and below the origin.

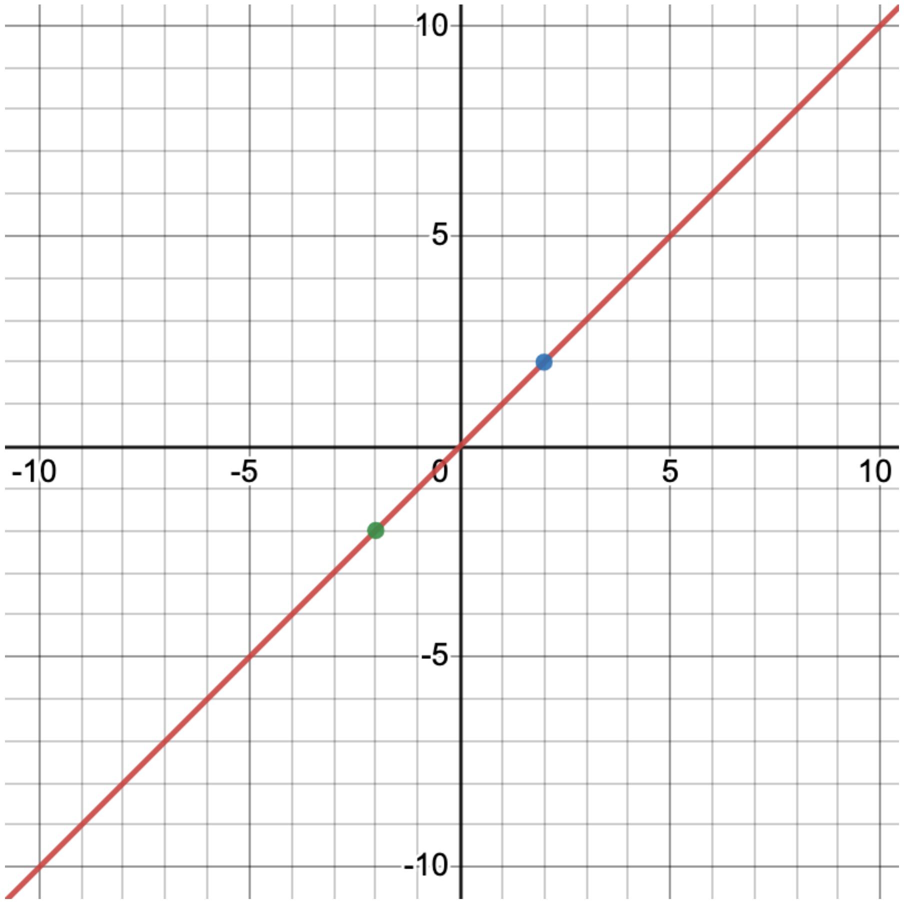
Given $f(x) = x$, we then find $f(-2) = y = -2$ and $f(2) = y = 2$.

The coordinates of the additional points are (–2, –2) and (2, 2).

NOTE: In the *Identity Function*, EVERY ordered pair has x- and y-coordinates that are equal!

Example 3: Graph a Linear Function (3 of 3)

Graph the identity function by drawing a line through the points found in the previous three slides.



Reminder:
We found the coordinates (–2, –2), (0, 0), and (2, 2).

Example 4: Graph a Linear Function (1 of 3)

Graph the linear function $g(x)=\frac{3}{2}x-3$ by hand.

Let's try the Intercept Method!

x-intercept:

Given $g(x)=\frac{3}{2}x-3$, let $g(x) = y = 0$ and solve for x.

$$0=\frac{3}{2}x-3$$

$$3=\frac{3}{2}x$$

$$3\left(\frac{2}{3}\right)=x$$

$$2 = x$$

The *x*-intercept is 2 and the point associated with it is (2, 0).

Example 4: Graph a Linear Function (2 of 3)

y-intercept:

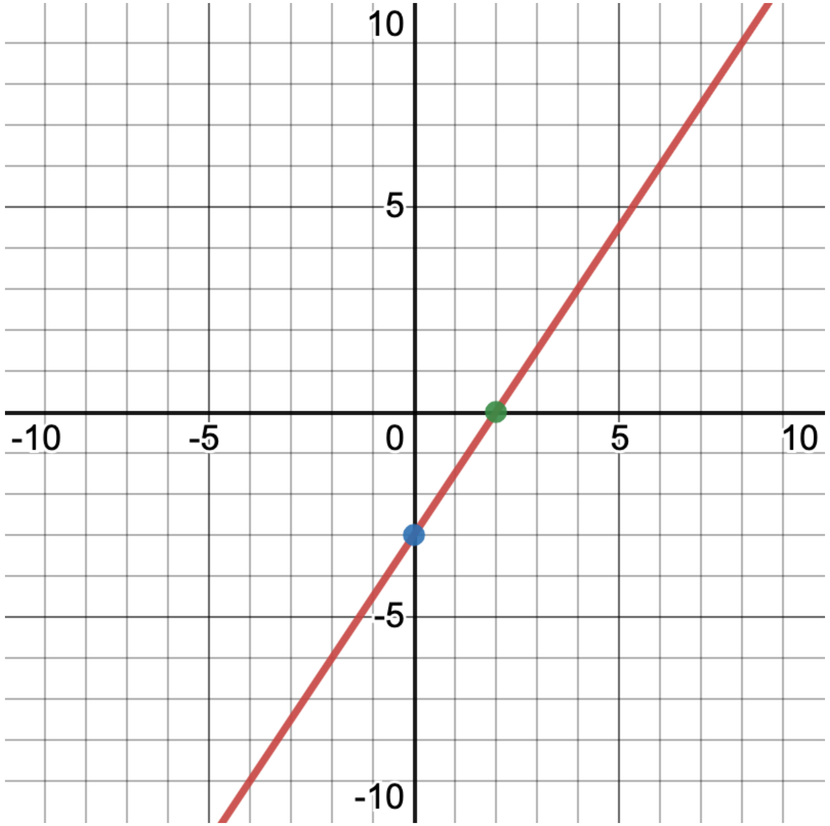
Given $g(x) = \frac{3}{2}x - 3$, let *x* = 0 and solve for *g*(*x*) = *y*.

$$g(0) = \frac{3}{2}(0) - 3 = y$$

The *y*-intercept is −3 and the point associated with it is (0, −3).

Example 4: Graph a Linear Function (3 of 3)

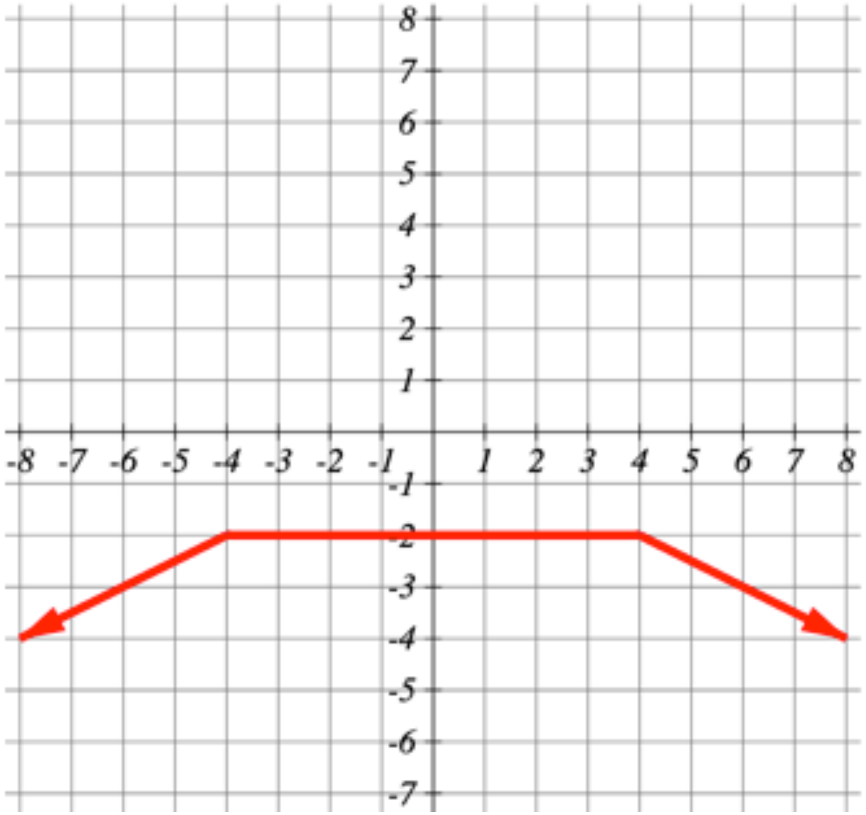
Graph the function by drawing a line through the two points associated with the intercepts.



Reminder:
We found the coordinates (2, 0) and (0, –3).

Example 5: Piecewise Defined Functions

The graph below shows what is called a *piecewise-defined function*. Specifically, this one consists of three linear branches. In the middle, we have a horizontal line (constant function). On the left end it is joined to an increasing line and on the right end it is joined to a decreasing line.



Use the graph to find the following in *Interval Notation*. Use ***open intervals***, that is, use parentheses.

The function is increasing on which open interval along the *x*-axis: $(-\infty, -4)$

The function is decreasing on which open interval along the *x*-axis: $(4, \infty)$

The function is constant on which open interval along the *x*-axis: $(-4, 4)$