Examples
Linear Equations in One Variable

Based on power point presentations by Pearson Education, Inc.

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Learning Objectives

1. Memorize the *Basic Principles of Equations* consisting of four axioms.
2. Memorize the definition of linear equations.
3. Solve linear equations containing integers.
4. Solve linear equations containing fractions.
5. Memorize and use the *Cross-Multiplication Principle*.

Example 1: Solve a Linear Equation

Solve the linear equation $x - 4 = 20$.

Note that this equation is not in *general form*! We use the **Addition Axiom** and add 4 to both sides of the equal sign.

$x - 4 + 4 = 20 + 4$ (if you are comfortable, you do not have to show this step)

and $x = 24$ which is the proposed solution.

Given $x = 24$, we now replace x in the original equation $x - 4 = 20$ with 24.

$24 - 4 = 20$

and $20 = 20$

This is a true statement, therefore the proposed solution $x = 24$ is an actual solution.

Example 2: Solve a Linear Equation

Solve the linear equation $x + 5 = 11$.

Note that his equation is not in *general form*! We use the **Subtraction Axiom** and subtract 5 from both sides of the equal sign.

$x + 5 - 5 = 11 - 5$ (if you are comfortable, you do not have to show this step)

and $x = 6$ which is the proposed solution.

Given $x = 6$, we now replace x in the original equation $x + 5 = 11$ with 6.

$6 + 5 = 11$

and $11 = 1$

This is a true statement, therefore the proposed solution $x = 6$ is an actual solution.

Example 3: Solve a Linear Equation (1 of 2)

Solve the linear equation $2x = 10$.

Note that this equation is not in *general form*! We can either use the **Division Axiom** or the **Multiplication Axiom**. Let's use the Division Axiom first and then the Multiplication Axiom.

Let's divide both sides of the equal sign by 2 (which is the coefficient of the variable).

$\frac{2x}{2} = \frac{10}{2}$ and $x = 5$ which is the proposed solution.

Now let's use the Multiplication Axiom. That is, let's multiply both sides of the equal sign by $\frac{1}{2}$ (which is the reciprocal of the coefficient of the variable).

$\frac{1}{2}(2x) = \frac{1}{2}(10)$ and we also find that $x = 5$.

Example 3: Solve a Linear Equation (2 of 2)

Given $x = 5$, we now replace x in the original equation $2x = 10$ with 5.

$2(5) = 10$

and $10 = 10$

This is a true statement, therefore the proposed solution $x = 5$ is an actual solution.

Example 4: Solve a Linear Equation (1 of 2)

Solve the linear equation $2(x - 4) - 5x = -5$.

Note that this equation is not in *general form*! We will first simplify the expression on the left side of the equal sign using the *Distributive Property*.

$$2x - 8 - 5x = -5$$

$$-3x - 8 = -5 \quad \text{Combined like terms on the left.}$$

Next, we collect variable terms on one side and constants on the other side.

$$-3x - 8 + 8 = -5 + 8 \quad \text{Add 8 to both sides (Addition Axiom).}$$

$$-3x = 3 \quad \text{Combined like terms (simplified).}$$

Example 4: Solve a Linear Equation (2 of 2)

Finally, we "isolate" the variable. That is, let's divide both sides of the equal sign by – 3 (which is the coefficient of the variable).

$\frac{-3x}{-3} = \frac{3}{-3}$ and $x = -1$ which is the proposed solution.

Given $x = -1$, we now replace x in the original equation $2(x - 4) - 5x = -5$ with -1.

$(-1 - 4) - 5(-1) = -5$
$2(-5) + 5 = -5$
$-10 + 5 = -5$
$-5 = -5$

This is a true statement, therefore the proposed solution $x = -1$ is an actual solution.

Example 5: Solve a Linear Equation (1 of 2)

Solve the linear equation $4(2x + 1) = 3(2x – 5) + 29$.

Note that this equation is not in *general form*! We will first simplify the expressions on each side of the equal sign using the *Distributive Property*.

$8x + 4 = 6x – 15 + 29$

$8x + 4 = 6x + 14$ Combined like terms on the right.

$2x + 4 = 14$ Subtracted $6x$ from both sides (Subtraction Axiom) and combined like terms.

$2x = 10$ Subtracted 4 from both sides (Subtraction Axiom) and combined like terms.

Example 5: Solve a Linear Equation (2 of 2)

Next, we "isolate" the variable. That is, let's divide both sides of the equal sign by 2 (which is the coefficient of the variable).

$\frac{2x}{2} = \frac{10}{2}$ and $x = 5$ which is the proposed solution

Given $x = 5$, we now replace x in the original equation $4(2x + 1) = 3(2x - 5) + 29$ with 5.

$4\,(\,2(5) + 1\,) = 3\,(\,2(5) - 5\,) + 29$

$4(11) = 3(5\,) + 29$

$44 = 15 + 29$

$44 = 44$

This is a true statement, therefore the proposed solution $x = 5$ is an actual solution.

Example 6: Solve a Linear Equations

Solve the linear equation $2x + 6 = 2(x + 4)$.

Note that this equation is not in *general form*! We will first simplify the expression on the right side of the equal sign using the *Distributive Property*.

$2x + 6 = 2x + 8$

$6 = 8$ Subtracted $2x$ from both sides (Subtraction Axiom).

Note, when we tried to solve for x, we ended up with $6 = 8$, which is a FALSE statement.

In mathematics this means that the given equation has NO SOLUTION.

Example 7: Solve a Linear Equation

Solve the linear equation $4x + 6 = 6(x + 1) - 2x$.

Note that this equation is not in *general form*! We will first simplify the expression on the right side of the equal sign using the *Distributive Property*.

$4x + 6 = 6x + 6 - 2x$

$4x + 6 = 4x + 6$ Combined like terms (simplified).

$6 = 6$ Subtracted 4x from both sides (Subtraction Axiom).

Note, when we tried to solve for x, we ended up with $6 = 6$, which is a TRUE statement.

In mathematics this means that the given equation has INFINITELY MANY SOLUTIONS.

Example 8: Solve a Linear Equation (1 of 2)

Solve the linear equation $\frac{3}{4} - x = \frac{7}{8}$.

Let's find a number divisible by all denominators in the equation. Please understand that we can always find such a number by calculating the product of all denominators. In our case, that would be 4(8) = 32.

However, you might notice that 8 is also a number divisible by 4 and 8. It certainly is smaller than 32 and therefore easier to work with. Subsequently, let's use 8 and multiply both sides of the equal sign by it.

$$8\left(\frac{3}{4} - x\right) = 8\left(\frac{7}{8}\right)$$

$$8\left(\frac{3}{4}\right) - 8x = 8\left(\frac{7}{8}\right)$$

Used the Distributive Property on the left of the equal sign.

Example 8: Solve a Linear Equation (2 of 2)

After some canceling, we get $6 - 8x = 7$ and $-8x = 1$.

All that's left to do is divide both sides of the equal sign by – 8 to get

$$x = -\frac{1}{8}.$$

If we wish, we could carry out a check to prove to ourselves that this is indeed the actual solution.

Example 9: Solve a Linear Equation (1 of 2)

Solve the linear equation $\frac{x-3}{7} = \frac{7}{8}$ using *Cross-Multiplication*.

Let's make a cross over the equal sign as follows to help us remember *Cross-Multiplication.*

$$\frac{x-3}{7} = \frac{7}{8}$$

We can now state ***8(x – 3) = 7(7).***

Next, we must use the *Distributive Property* to get ***8x – 24 = 49***.

Using the **Addition Axiom**, we find ***8x = 73***.

Example 9: Solve a Linear Equation (2 of 2)

Next, we “isolate” the variable. That is, let’s divide both sides of the equal sign by 8 (which is the coefficient of the variable).

We get $x = \frac{73}{8}$.

If we wish, we could carry out a check to prove to ourselves that this is indeed the actual solution.

Now, should we change the improper fraction to a mixed number? Only if you are asked to do so. Otherwise, leave it alone.