

Examples Inverses of Functions

Based on power point presentations by Pearson Education, Inc.
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Learning Objectives

1. Define one-to-one functions.
2. Find inverse functions.
3. Verify inverses.

Example 1: Find the Inverse of a Function

The function $f(x) = 2x + 7$ is one-to-one. Find its inverse function.

Step 1 - Replace $f(x)$ with y

$$y = 2x + 7$$

Step 2 - Interchange x and y

$$x = 2y + 7$$

Step 3 - Solve for y in terms of x

$$x = 2y + 7$$

$$x - 7 = 2y$$

$$y = \frac{x - 7}{2}$$

This is a linear function, and all linear functions are one-to-one!

Step 4 - Replace y with inverse function notation.

$$f^{-1}(x) = \frac{x - 7}{2}$$

Example 2: Find the Inverse of a Function (1 of 2)

The function $f(x) = \sqrt[3]{x-5}$ is one-to-one. Find its inverse function.

Step 1 - Replace $f(x)$ with y

$$y = \sqrt[3]{x-5}$$

Step 2 - Interchange x and y

$$x = \sqrt[3]{y-5}$$

Step 3 - Solve for y in terms of x

We will raise both sides to the third power as follows:

$$x^3 = (\sqrt[3]{y-5})^3$$

$$\text{then } x^3 = y - 5$$

$$\text{and } y = x^3 + 5$$

Example 2: Find the Inverse of a Function (2 of 2)

Step 4 - Replace y with inverse function notation to get

$$f^{-1}(x) = x^3 + 5$$

Example 2: Find the Inverse of a Function (1 of 3)

The function $f(x) = \sqrt{x-2}$ is one-to-one. It has a domain of $\{x \mid x \geq 2\}$ and a range of $\{y \mid y \geq 0\}$. Find its inverse function.

Step 1 - Replace $f(x)$ with y

$$y = \sqrt{x-2}$$

Step 2 - Interchange x and y

$$x = \sqrt{y-2}$$

Example 3: Find the Inverse of a Function (2 of 3)

Step 3 - Solve for y in terms of x

We will raise both sides to the second power to get

$$x^2 = \left[\sqrt{y-2} \right]^2$$

$$x^2 = y - 2$$

$$y = x^2 + 2$$

Here we we must be careful. The function $y = x^2 + 2$ is not one-to-one. Observe that it is a quadratic function, and its graph is a parabola!

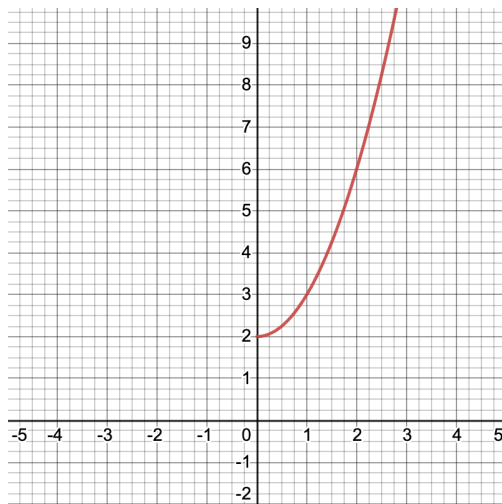
Example 3: Find the Inverse of a Function (3 of 3)

Step 4 - Replace y with inverse function notation.

Since the inverse must be one-to-one, we need to make $y = x^2 + 2$ one-to-one as follows:

We know its domain because it is the range of the original function which is $\{y \mid y \geq 0\}$. Therefore, the domain of the inverse is $\{x \mid x \geq 0\}$.

We can now state that the inverse of $f(x) = \sqrt{x-2}$ is $f^{-1}(x) = x^2 + 2$ with domain $\{x \mid x \geq 0\}$. Graphically, it is only the right half of a parabola.



Example 4: Verify Inverse Functions

Show that each function is the inverse of the other:

$$f(x) = 4x - 7 \quad \text{and} \quad g(x) = \frac{x + 7}{4}.$$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{x + 7}{4}\right) = 4\left(\frac{x + 7}{4}\right) - 7 = x + 7 - 7 = x$$

$$(g \circ f)(x) = g(f(x)) = g(4x - 7) = \frac{4x - 7 + 7}{4} = \frac{4x}{4} = x$$

Since $(f \circ g)(x) = (g \circ f)(x) = x$, we just verified that f and g are inverses of each other.