Examples
Inverses of Functions

Based on power point presentations by Pearson Education, Inc.

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Learning Objectives

1. Define one-to-one functions.
2. Find inverse functions.
3. Verify inverses.

Example 1: Find the Inverse of a Function (1 of 2)

The function $f(x) = 3x - 1$ is one-to-one. It has a domain and range consisting of *all real numbers*. Find its inverse function.

Note that f(x) can be replaced with y to get $y = 3x - 1$.

Now we interchange x and y to get $x = 3y - 1$.

Then, we solve for y in terms of x as follows:

$x + 1 = 3v$

$$y = \frac{x+1}{3}$$

and

This is a linear function, and all linear functions are one-to-one! Imagine horizontal lines crossing any increasing or decreasing function. They only intersect once, right?

Example 1: Find the Inverse of a Function (2 of 2)

In the final step, we replace *y* with function notation. We can use any function name or f^{-1}.

Let's write $f^{-1}(x) = \frac{x+1}{3}$.

Example 2: Find the Inverse of a Function (1 of 2)

The function $f(x) = \sqrt[3]{x-5}$ is one-to-one. It has a domain and range consisting of *all real numbers*. Find its inverse function.

Note that *f*(x) can be replaced with *y* to get $y = \sqrt[3]{x-5}$.

Now we interchange *x* and *y* to get $x = \sqrt[3]{y-5}$.

Then, we solve for *y* in terms of *x*. We are dealing with a radical of **index 3**. therefore, we will raise both sides of the equation to the **3rd power**. This eliminates the radical symbol.

$$x^3 = (\sqrt[3]{y-5})^3$$

then $x^3 = y - 5$

and $y = x^3 + 5$

Example 2: Find the Inverse of a Function (2 of 2)

The function $y = x^3 + 5$ is one-to-one. It is a transformation of $y = x^3$ whose graphs are similar to . Imagine horizontal lines crossing such graphs. They only intersect once, right?

In the final step, we replace y with function notation. We can use any function name or f^{-1}.

Let's write $f^{-1}(x) = x^3 + 5$.

Example 3: Find the Inverse of a Function (1 of 3)

The function $f(x) = \sqrt{x-2}$ is one-to-one. It has a *domain of {x |x* $\geq$ *2}* and a *range of {y |y* $\geq$ *0}.* Find its inverse function.

Note that *f* (x) can be replaced with *y* to get $y = \sqrt{x-2}$.

Now we interchange *x* and *y* to get $x = \sqrt{y-2}$.

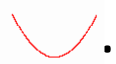
Then, we solve for *y* in terms of *x*. We are dealing with a radical of **index 2**. therefore, we will raise both sides of the equation to the **2nd power**. This eliminates the radical symbol.

$$x^2 = \left(\sqrt{y-2}\right)^2$$

then $x^2 = y - 2$

and $y = x^2 + 2$

Example 3: Find the Inverse of a Function (2 of 3)

The function $y = x^2 + 2$ is NOT one-to-one. It is a transformation of $y = x^2$ whose graphs are similar to . Imagine horizontal lines crossing such graphs. There are infinitely many that intersect more than once, right?

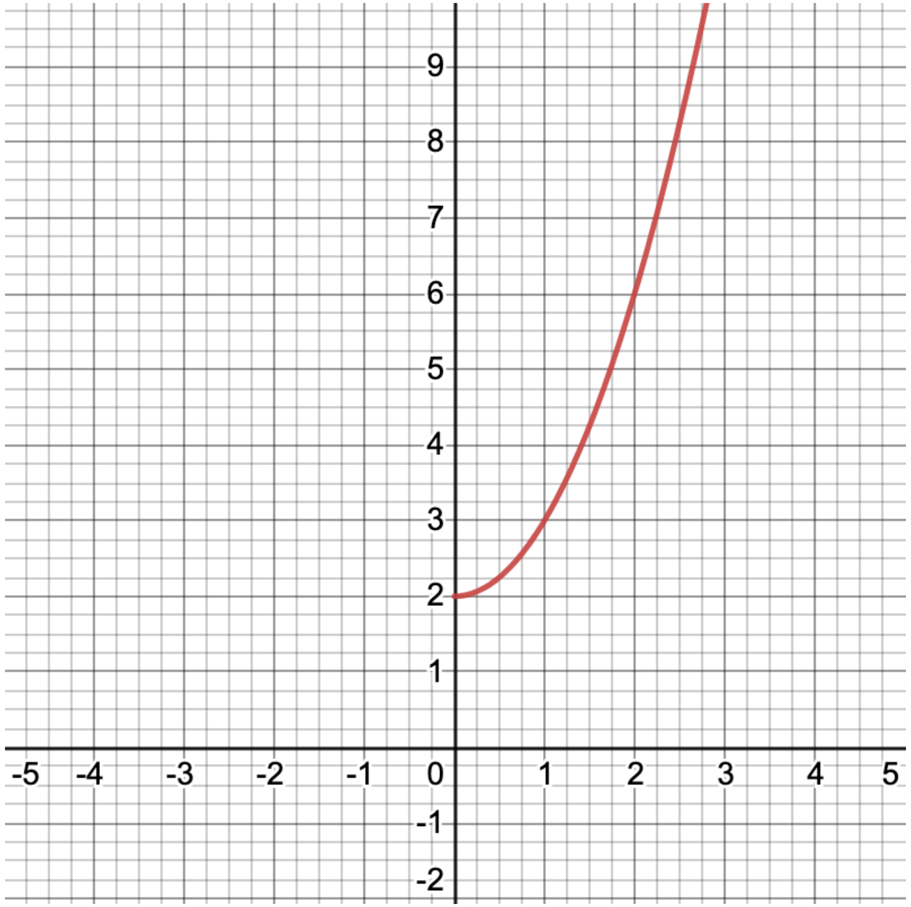
Since the inverse of a one-to-one function must also be one-to-one, we need to make $y = x^2 + 2$ one-to-one by restricting its domain.

We were told that the range of $f(x) = \sqrt{x-2}$ is $\{y \mid y \geq 0\}$. We also know that this becomes the domain of the inverse function, namely $\{x \mid x \geq 0\}$.

We can now state that the inverse of $f(x) = \sqrt{x-2}$ is $y = x^2 + 2$ with restricted domain $\{x \mid x \geq 0\}$.

Example 3: Find the Inverse of a Function (3 of 3)

Below is the graph of the inverse of $f(x) = \sqrt{x-2}$. Namely, $y = x^2 + 2$ with restricted domain $\{x \mid x \geq 0\}$.



In the final step, we replace y with function notation. Let's use the function name $\boldsymbol{g}$ to get $\boldsymbol{g}(x) = x^2 + 2$ with restricted domain $\{x \mid x \geq 0\}$.

Example 4: Verify Inverse Functions (1 of 2)

Show that the following functions are inverses:

$f(x) = \sqrt{x-9} + 7$ and $g(x) = x^2 - 14x + 58$, $\{x \mid x \geq 7\}$.

$$
\begin{aligned}
(f \circ g)(x) = f(g(x)) &= f(x^2 - 14x + 58) \\
&= \sqrt{\left(x^2 - 14x + 58\right) - 9} + 7 \\
&= \sqrt{x^2 - 14x + 49} + 7 \\
&= \sqrt{(x-7)^2} + 7 \\
&= (x-7) + 7 \\
&= x
\end{aligned}
$$

Example 4: Verify Inverse Functions (2 of 2)

$$(g \circ f)(x) = g(f(x)) = g(\sqrt{x-9}+7)$$

$$= (\sqrt{x-9}+7)^2 - 14(\sqrt{x-9}+7) + 58$$

Please note that $(\sqrt{x-9}+7)^2 = (\sqrt{x-9}+7)(\sqrt{x-9}+7)$ and we will FOIL to simplify.

$$(\sqrt{x-9})^2 + 7\sqrt{x-9} + 7\sqrt{x-9} + 7^2 = x - 9 + 14\sqrt{x-9} + 49$$

We will now continue the simplification process:

$$= x - 9 + 14\sqrt{x-9} + 49 - 14(\sqrt{x-9}+7) + 58$$

$$= x - 9 + 14\sqrt{x-9} + 49 - 14\sqrt{x-9} - 98 + 58$$

$$= x$$

Since both $(f \circ g)$ and $(g \circ f)$ equal *x*, we just verified that the functions ***f*** and ***g*** are inverses.

Example 5: Verify Inverse Functions

Show that each function is the inverse of the other:

$$f(x) = 4x - 7 \quad \text{and} \quad g(x) = \frac{x+7}{4}.$$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{x+7}{4}\right) = 4\left(\frac{x+7}{4}\right) - 7 = x + 7 - 7 = x$$

$$(g \circ f)(x) = g(f(x)) = g(4x - 7) = \frac{4x - 7 + 7}{4} = \frac{4x}{4} = x$$

Since $(f \circ g)(x) = (g \circ f)(x) = x$, we just verified that *f* and *g* are inverses of each other.