Examples
Geometric Sequences and Series

Based on power point presentations by Pearson Education, Inc.

Revised by Ingrid Stewart, Ph.D.

Learning Objectives

1. Memorize the definition of a geometric sequence and find its common ratio.
2. Find the value of a term of a geometric sequence.
3. Memorize the definition of a finite geometric series and evaluate its sum.
4. Evaluate the sum of an infinite geometric series.

Example 1: Find the Common Ratio of a Geometric Sequence

a. Given the geometric sequence – 12, 24, – 48, 96, – 192, 384, … find the common ratio *r*.

Dividing the second term by the first term: $r = 24 \div (-12) = -2$

We check this by observing that every term after the first one is a multiple of – 2 of the preceding term.

b. Given the geometric sequence 1, 5, 25, 125, 625, 3125, … find the common ratio *r*.

Dividing the second term by the first term: $r = 5 \div 1 = 5$

We check this by observing that every term after the first one is a multiple of 5 of the preceding term.

Example 2: Find the Value of a Term of a Geometric Sequence

Find the value of the 6th term of the geometric sequence whose first term a_1 is 12 and whose common ratio r is $\frac{1}{2}$.

To find the 6th term a_6, we will write out the previous 5 terms first.

$a_1 = 12$

$a_2 = 12 \cdot \frac{1}{2} = 6$

$a_3 = 6 \cdot \frac{1}{2} = 3$

$a_4 = 3 \cdot \frac{1}{2} = \frac{3}{2}$

$a_5 = \frac{3}{2} \cdot \frac{1}{2} = \frac{3}{4}$

$a_6 = \frac{3}{4} \cdot \frac{1}{2} = \frac{3}{8}$ This is the value of the 6th term.

Example 3: Find the Value of a Term of a Geometric Sequence

In Example 2, we found the value of the 6th term of a geometric sequence by writing out the 1st through the 6th term.

Now, we will use the formula $a_n = a_1 r^{n-1}$, where $a_1 = 12$, $n = 6$, and $r = \frac{1}{2}$ to find a_6.

$$a_6 = 12\left(\frac{1}{2}\right)^{6-1} = 12\left(\frac{1}{2}\right)^5$$

$$= 12\left(\frac{1}{32}\right)$$

$$= \frac{12}{32}$$

$$= \frac{3}{8}$$

We also find that the value of the 6th term is $\frac{3}{8}$.

Example 4: Find the Value of a Term of a Geometric Sequence

Find the value of the 10th term of the geometric sequence whose first term a_1 is 3 and whose common ratio r is −2.

We will use the formula $a_n = a_1 r^{n-1}$ instead of writing out the previous 9 terms. We let $a_1 = 2$, $n = 10$, and $r = -2$ to find a_{10} .

$$a_{10} = 3(-2)^{10-1}$$

$$= 3(-2)^9$$

$$= 3(-512)$$

$$= -1536$$

The value of the 10th term is – 1536.

Example 5: Evaluate the Sum of a Geometric Series (1 of 2)

Evaluate the sum of the geometric series $\sum_{k=1}^{25} 6\left(2^k\right)$.

This series has lots of terms. Instead of writing out and then adding them, we will use the summation formula $S_n = \frac{a_1(1-r^n)}{1-r}$.

Let's find the values we need for this formula.

- The number of terms *n* in the sum is 20.
- We find the first term a_1 by evaluating the series for $k = 1$.
  Specifically, $a_1 = 6(2^1) = 12$
- We also need *r*. We find the first and second terms of the geometric series, 12 and 24, then we divide 24 by 12 to find $r = 2$.

Example 5: Evaluate the Sum of a Geometric Series (2 of 2)

Finally, given a_1 = 12 and r = 2, we can find S_{25}. Specifically,

$$
\begin{aligned}
S_{25} &= \frac{12\left(1-2^{25}\right)}{1-2} \\
&= \frac{12(1-33554432)}{-1} \\
&= \frac{12(-333554431)}{-1} \\
&= 402653172
\end{aligned}
$$

Example 6: Evaluate the Sum of a Geometric Series (1 of 2)

Find the sum of the first 9 terms of the geometric series $2 + (-6) + 18 + (-54) + \ldots$.

We will use the summation formula $S_n = \dfrac{a_1(1-r^n)}{1-r}$. Let's find the values we need for it.

- The number of terms n in the sum is 9.
- We know that the first term a_1 is 2.
- We also need r. The first and second terms of the geometric series are 2 and – 6 respectively, then we divide – 6 by 2 to find $r = -3$.

Example 6: Evaluate the Sum of a Geometric Series (2 of 2)

Finally, given a_1 = 2 and r = – 3, we can find S_9. Specifically,

$$S_9 = \frac{2\left(1-(-3)^9\right)}{1-(-3)}$$

$$= \frac{2(1-(-19683))}{4}$$

$$= \frac{2(19684)}{4}$$

$$= 9842$$

Example 7: Evaluate the Sum of a Geometric Series (1 of 2)

Find the sum of the first 10 terms of the geometric series $1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\ldots$. Round the answer to 9 decimal places.

We will use the summation formula $S_n=\frac{a_1(1-r^n)}{1-r}$. Let's find the values we need for it.

- The number of terms n in the sum is 10.
- We know that the first term a_1 is 1.
- We also need r. The first and second terms of the geometric series are 1 and $\frac{1}{4}$ respectively, then we divide $\frac{1}{4}$ by 1 to find $r = \frac{1}{4}$.

Example 7: Evaluate the Sum of a Geometric Series (2 of 2)

Finally, given a_1 = 1 and $r = \frac{1}{4}$, we can find S_{10}. Specifically,

$$S_{10} = \frac{1\left(1-\left(\frac{1}{4}\right)^{10}\right)}{1-\frac{1}{4}} = \frac{\left(1-\left(\frac{1}{4}\right)^{10}\right)}{\frac{3}{4}} \approx 1.33333\,2062\ldots$$

Example 8: Evaluate the Sum of an Infinite Geometric Series
(1 of 2)

Let's use the geometric series $1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\ldots$ from Example 7 again. But this time we will try to find the sum of an infinite number of terms.

Since $r = \frac{1}{4}$ lies between – 1 and 1, we know that the sum of an infinite number of terms can indeed be be found!

Therefore, we will use the special Summation Formula $S=\frac{a_1}{1-r}$.

Example 8: Evaluate the Sum of an Infinite Geometric Series
(2 of 2)

Given $a_1 = 1$ and $r = \frac{1}{4}$, we get

$$S = \frac{1}{1 - \frac{1}{4}}$$

$$= \frac{1}{\frac{3}{4}}$$

$$= 1 \cdot \frac{4}{3}$$

$$= \frac{4}{3} \cong 1.33333\ 3333\ \ldots.$$

Notice that compared to Example 7, the sum of 10 terms and that of an infinite number of terms did not change all that much.