Examples
Geometric Sequences and Series

Based on power point presentations by Pearson Education, Inc.

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Learning Objectives

1. Find the common ratio of a geometric sequence.
2. Write terms of a geometric sequence.
3. Find the value of a term of a geometric sequence.
4. Evaluate the sum of a geometric sequence.
5. Evaluate the sum of an infinite geometric series.

Example 1: Find the Common Ratio of a Geometric Sequence

a. Given the geometric sequence 1, 5, 25, 125, 625, 3125, … find the common ratio r.

Dividing the second term by the first term: $r = 5 \div 1 = 5$

We check this by observing that every term after the first one is a multiple of 5 of the preceding term.

b. Given the geometric sequence – 12, 24, – 48, 96, – 192, 384, … find the common ratio r.

Dividing the second term by the first term: $r = 24 \div (-12) = -2$

We check this by observing that every term after the first one is a multiple of – 2 of the preceding term.

Example 2: Write the Terms of a Geometric Sequence

Write the first six terms of the geometric sequence with first term 12 and common ratio $\frac{1}{2}$.

$a_1 = 12$

$a_2 = 12 \cdot \frac{1}{2} = 6$

$a_3 = 6 \cdot \frac{1}{2} = 3$

$a_4 = 3 \cdot \frac{1}{2} = \frac{3}{2}$

$a_5 = \frac{3}{2} \cdot \frac{1}{2} = \frac{3}{4}$

$a_6 = \frac{3}{4} \cdot \frac{1}{2} = \frac{3}{8}$

Example 3: Find the Value of a Term of a Geometric Sequence

In Example 2, we found the first 6 terms of a geometric sequence whose first term is 12 and whose common ratio is $\frac{1}{2}$. Due to this work, we found that a_6 is equal to $\frac{3}{8}$.

Now let's find the value of this term using the formula $a_1 r^{n-1}$, where $a_1 = 12$, $n = 6$, and $r = \frac{1}{2}$.

$$a_6 = 12\left(\frac{1}{2}\right)^{6-1} = 12\left(\frac{1}{2}\right)^5$$

$$= 12\left(\frac{1}{32}\right) = \frac{12}{32}$$

$$= \frac{3}{8}$$

The sixth term is $\frac{3}{8}$.

Example 4: Find the Value of a Term of a Geometric Sequence

Find the seventh term of the geometric sequence whose first term is 5 and whose common ratio is −3.

To find the seventh term, a_7, we use the formula $a_1 r^{n-1}$, where $a_1 = 5$, n = 7, and $r = -3$.

$$a_n = a_1 r^{n-1}$$

$$a_7 = 5(-3)^{7-1} = 5(-3)^6 = 5(729) = 3645$$

The seventh term is 3645.

Example 5: Evaluate the Sum of a Geometric Series (1 of 2)

Evaluate the sum of a geometric series $\sum_{i=1}^{5} 6(2^i)$.

We can do this two different ways. We can either write out the 5 terms and then add them, or we can use the summation formula $S_n = \frac{a_1(1-r^n)}{1-r}$.

Let's find the five terms and then add them.

$$\sum_{i=1}^{5} 6(2^i) = 6(2^1) + 6(2^2) + 6(2^3) + 6(2^4) + 6(2^5)$$

$$= 12 + 24 + 48 + 96 + 192$$

$$= 372$$

Example 5: Evaluate the Sum of a Geometric Series (2 of 2)

Now, instead of writing out the terms and and then adding them, we will use the summation formula $S_n = \dfrac{a_1(1-r^n)}{1-r}$.

Let's find the values we need for this formula.

- We know that *n* must be 5.
- We find the 1st term by evaluating the series for i = 1.
  $a_1 = 6(2^1) = 12$
- We also need *r*. Writing out the first two terms of the series, then dividing the first term into the second term, we find $r = 24 \div 12 = 2$

Then $S_5 = \dfrac{12(1-2^5)}{1-2} = \dfrac{12\left(1-2^5\right)}{-1} = -12\left(1-2^5\right) = 372$

NOTE: Use your calculator and the Order of Operations to find this sum!

Example 6: Evaluate the Sum of a Geometric Series (1 of 2)

Evaluate the geometric serie $\sum_{i=1}^{20} 6(2^i)$.

This series has lots of terms. Instead of writing out and then adding them,

we will use the summation formula $S_n = \frac{a_1(1-r^n)}{1-r}$.

Let's find the values we need for this formula.

- We know that *n* must be 20.
- We find the 1st term by evaluating the series for i = 1.
  $a_1 = 6(2^1) = 12$
- We also need *r*. Writing out the first two terms of the series, then dividing the first term into the second term, we find $r = 24 \div 12 = 2$

Example 6: Evaluate the Sum of a Geometric Series (2 of 2)

Then $$S_{20} = \frac{12(1-2^{20})}{1-2} = \frac{12\left(1-2^{20}\right)}{-1} = -12\left(1-2^{20}\right) = 12582900$$

NOTE: Use your calculator and the Order of Operations to find this sum!

Example 7: Evaluate the Sum of a Geometric Series

Find the sum of the first 9 terms of the geometric series $2 + (-6) + 18 + (-54) + \ldots$.

We will use the summation formula $S_n = \frac{a_1(1-r^n)}{1-r}$. Let's find the values we need for it.

- We know that *n* must be 9.
- We know that the 1st term is 2.
- We find the common ratio *r* by dividing the first term of the series into the second term.

  $r = -6 \div 2 = -3$

Then $S_9 = \frac{2(1-(-3)^9)}{1-(-3)} = \frac{2(1+19{,}683)}{1+3} = \frac{39{,}368}{4} = 9842$

Example 8: Evaluate the Sum of a Geometric Series

Find the sum of the first 10 terms of the geometric series $1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\ldots$. Round the answer to 9 decimal places.

We will use the summation formula $S_n = \frac{a_1(1-r^n)}{1-r}$. Let's find the values we need for it.

- We know that n must be 10.
- We know that the 1st term is 1.
- We find the common ratio r by dividing the first term of the series into the second term.

  $r = \frac{1}{4}$

Then $S_{10} = \dfrac{1\left(1-\left(\frac{1}{4}\right)^{10}\right)}{1-\frac{1}{4}} = \dfrac{\left(1-\left(\frac{1}{4}\right)^{10}\right)}{\frac{3}{4}} \approx 1.33333\,2062\ldots$.

Example 9: Evaluate the Sum of an Infinite Geometric Series

Let's do Example 8 again, but this time we will find the sum of an infinite number of terms of the geometric series $1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\dots$.

Since $r=\frac{1}{4}$ which lies between – 1 and 1, we know that a sum can be found!

We will use the special sum formula $S=\dfrac{a_1}{1-r}$.

Given $\boldsymbol{a_1 = 1}$, we get $S=\dfrac{1}{1-\frac{1}{4}}=\dfrac{1}{\frac{3}{4}}=\dfrac{4}{3}\approx 1.\overline{3}\approx 1.333333333\dots$.

Notice that compared to Example 8, the sum did not change all that much.