Examples
Operations on Functions

Based on power point presentations by Pearson Education, Inc.

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Learning Objectives

1. Add, subtract, multiply and divide functions.
2. Compose functions.

Example 1: Operations on Functions (1 of 2)

Let $f(x) = -5x$ and $g(x) = 2x - 1$. Find the functions $f + g$, $f - g$, fg, and $\frac{f}{g}$.

$$
\begin{aligned}
(f + g)(x) &= (-5x) + (2x - 1) \\
&= -5x + 2x - 1 \\
&= -3x - 6
\end{aligned}
$$

$$
\begin{aligned}
(f - g)(x) &= (-5x) - (2x - 1) \\
&= -5x - 2x + 1 \\
&= -7x + 1
\end{aligned}
$$

Example 1: Operations on Functions (2 of 2)

Let $f(x) = -5x$ and $g(x) = 2x - 1$. Find the functions $f + g$, $f - g$, fg, and $\frac{f}{g}$.

$$(fg)(x) = (-5x)(2x - 1)$$
$$= -10x^2 + 5x$$

(here we used the *Distributive Property* and the *power rule of exponents*!)

$$\left(\frac{f}{g}\right)(x) = \frac{-5x}{2x-1}$$

where the denominator $2x - 1$ cannot be equal to 0!

Example 2: Operations on Functions (1 of 2)

Let $h(x) = x^2 + 3$ and $k(x) = 2x - 1$. Find the functions $h + k$, $h - k$, hk, and $\frac{h}{k}$.

$$
\begin{aligned}
(h + k)(x) &= (x^2 + 3) + (2x - 1) \\
&= x^2 + 3 + 2x - 1 \\
&= x^2 + 2x + 2
\end{aligned}
$$

$$
\begin{aligned}
(h - k)(x) &= (x^2 + 3) - (2x - 1) \\
&= x^2 + 3 - 2x + 1 \\
&= x^2 - 2x + 4
\end{aligned}
$$

Please note that in mathematics terms are ALWAYS displayed in descending order of their exponent!

Example 2: Operations on Functions (2 of 2)

$(hk)(x) = (x^2 + 3)(2x - 1)$

Using FOIL, we get

$$\begin{aligned}(hk)(x) &= (x^2 + 3)(2x - 1) = \overset{F}{x^2(2x)} + \overset{O}{x^2(-1)} + \overset{I}{3(2x)} + \overset{L}{3(-1)} \\ &= 2x^3 - x^2 + 6x - 3\end{aligned}$$

$\left(\dfrac{h}{k}\right)(x) = \dfrac{x^2 + 3}{2x - 1}$ where the denominator $2x - 1$ cannot be equal to 0!

Example 3: Compose Function

Given $f(x) = x - 5$ and $g(x) = 2x - 3$, find $\mathbf{f} \circ \boldsymbol{g}$ and $\boldsymbol{g} \circ \mathbf{f}$.

We find $\mathbf{f} \circ \boldsymbol{g}$ by replacing every occurrence of x in the function f with the function g as follows:

$$(f \circ g)(x) = f(g(x)) = f(2x - 3)$$
$$= (2x - 3) - 5$$
$$= 2x - 8$$

This is the function f where x was replaced by $2x - 3$.

We find $\boldsymbol{g} \circ \mathbf{f}$ by replacing every occurrence of x in the function $\boldsymbol{g}$ with the function **f** as follows"

$$(g \circ f)(x) = g(f(x)) = g(x - 5)$$
$$= 2(x - 5) - 3$$
$$= 2x - 10 - 3$$
$$= 2x - 13$$

This is the function g where x was replaced by $x - 5$.

Please note that the results of $\mathbf{f} \circ \boldsymbol{g}$ and $\boldsymbol{g} \circ \mathbf{f}$ are not the same!

Example 4: Compose Function

Given $f(x) = x + 4$ and $g(x) = x - 1$, find $\mathbf{f} \circ \boldsymbol{g}$ and $\boldsymbol{g} \circ \mathbf{f}$.

$(f \circ g)(x) = f\,(\,g(x)\,) = f\,(x - 1)$

$= (x - 1) + 4$ This is the function f where *x* was replaced by *x – 1*.

$= x + 3$

$(g \circ f)(x) = g\,(\,f(x)\,) = g\,(x + 4)$

$= (x + 4) - 1$ This is the function *g* where *x* was replaced by *x + 4*.

$= x + 3$

In this example, the results of $\mathbf{f} \circ \boldsymbol{g}$ and $\boldsymbol{g} \circ \mathbf{f}$ are the same.

Example 5: Compose Functions

Given $f(x) = \sqrt{x}$ and $g(x) = x - 3$, find **f ∘ *g*** and ***g* ∘ f**.

We find **f ∘ *g*** by replacing every occurrence of *x* in the function *f* with the function *g* as follows:

$$(f \circ g)(x) = f(g(x)) = f(x-3)$$

$$= \sqrt{x-3}$$

This is the function f where *x* was replaced by *x – 3*.

We find ***g* ∘ f** by replacing every occurrence of *x* in the function *g* with the function *f* as follows:

$$(g \circ f)(x) = g(f(x)) = g(\sqrt{x})$$

$$= \sqrt{x} - 3$$

This is the function *g* where *x* was replaced by $\sqrt{x}$.

Please note that the results of **f ∘ *g*** and ***g* ∘ f** are not the same!