Examples
Operations on Functions

Based on power point presentations by Pearson Education, Inc.

Revised by Ingrid Stewart, Ph.D.

Learning Objectives

1. Add, subtract, multiply, and divide functions.
2. Compose functions.

Example 1: Operations on Functions (1 of 2)

Let $f(x) = -5x$ and $g(x) = 2x - 1$. Find the functions $f + g$, $f - g$, $f\,g$, and $\frac{f}{g}$ and simplify, if necessary.

$$(f + g)(x) = (-5x) + (2x - 1)$$
$$= -5x + 2x - 1$$
$$= -3x - 6$$

$$(f - g)(x) = (-5x) - (2x - 1)$$
$$= -5x - 2x + 1$$
$$= -7x + 1$$

Example 1: Operations on Functions (2 of 2)

Let $f(x) = -5x$ and $g(x) = 2x - 1$. Find the functions $f + g$, $f - g$, fg, and $\frac{f}{g}$ and simplify, if necessary.

$(fg)(x) = (-5x)(2x - 1)$

$= -10x^2 + 5x$ (here we used the *Distributive Property* and the *power rule of exponents*!)

$\left(\frac{f}{g}\right)(x) = \frac{-5x}{2x-1}$ where the denominator $2x - 1$ cannot be equal to 0!

Example 2: Operations on Functions (1 of 2)

Let $h(x) = x^2 + 3$ and $k(x) = 2x - 1$. Find the functions $h + k$, $h - k$, hk, and $\frac{h}{k}$ and simplify, if necessary.

$$\begin{aligned}(h + k)(x) &= (x^2 + 3) + (2x - 1)\\ &= x^2 + 3 + 2x - 1\\ &= x^2 + 2x + 2\end{aligned}$$

$$\begin{aligned}(h - k)(x) &= (x^2 + 3) - (2x - 1)\\ &= x^2 + 3 - 2x + 1\\ &= x^2 - 2x + 4\end{aligned}$$

Please note that the terms in a mathematical expression are ALWAYS displayed in descending order of their exponent!

Example 2: Operations on Functions (2 of 2)

$(hk)(x) = (x^2 + 3)(2x - 1)$

Using FOIL, we get

$$(hk)(x) = (x^2 + 3)(2x - 1) = \overset{F}{x^2(2x)} + \overset{O}{x^2(-1)} + \overset{I}{3(2x)} + \overset{L}{3(-1)}$$

$$= 2x^3 - x^2 + 6x - 3$$

$\left(\frac{h}{k}\right)(x) = \frac{x^2 + 3}{2x - 1}$ where the denominator $2x - 1$ cannot be equal to 0!

Example 3: Compose Function

Given $f(x) = x - 5$ and $g(x) = 2x - 3$, find $(f \circ g)(x)$ and $(g \circ f)(x)$ and simplify, if necessary.

$$(f \circ g)(x) = f(g(x)) = f(2x - 3)$$
$$= (2x - 3) - 5$$
$$= 2x - 8$$

This is the function f where x was replaced by $2x - 3$.

$$(g \circ f)(x) = g(f(x)) = g(x - 5)$$
$$= 2(x - 5) - 3$$
$$= 2x - 10 - 3$$
$$= 2x - 13$$

This is the function g where x was replaced by $x - 5$.

Please note that the results of $f \circ g$ and $g \circ f$ are NOT the same!

Example 4: Compose Function

Given $f(x) = x + 4$ and $g(x) = x - 1$, find $(f \circ g)(x)$ and $(g \circ f)(x)$ and simplify, if necessary.

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) = f(x-1) \\ &= (x-1) + 4 \\ &= x + 3\end{aligned}$$

This is the function f where x was replaced by $x - 1$.

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) = g(x+4) \\ &= (x+4) - 1 \\ &= x + 3\end{aligned}$$

This is the function g where x was replaced by $x + 4$.

In this example, the results of $f \circ g$ and $g \circ f$ are the same.

Example 5: Compose Functions

Given $f(x) = \sqrt{x}$ and *g(x) = x* – 3, find ***(f ∘ g)(x)*** and ***(g ∘ f)(x)*** and simplify, if necessary.

$$(f \circ g)(x) = f(g(x)) = f(x-3)$$

$$= \sqrt{x-3}$$

This is the function f where *x* was replaced by *x – 3*.

$$(g \circ f)(x) = g(f(x)) = g(\sqrt{x})$$

$$= \sqrt{x} - 3$$

This is the function *g* where *x* was replaced by $\sqrt{x}$.

In this example, the results of ***f*** ∘ ***g*** and ***g*** ∘ ***f*** are NOT the same.