Examples
Exponential Functions

Based on power point presentations by Pearson Education, Inc.
Revised by Ingrid Stewart, Ph.D.

Learning Objectives

1. Memorize the definition of the common exponential function.
2. Memorize characteristics of the graphs of exponential functions.
3. Apply transformations to the common exponential function.
4. Graph the common exponential function and its transformations by hand.

Example 1: Apply Transformations to Common Exponential Functions (1 of 2)

Starting with the graph of $f(x) = 2^x$, write the equation of the graph that results from the following. Include the equation of the horizontal asymptote.

(1) a vertical shift up 3 units

$y = 2^x + 3$

The equation of the horizontal asymptote is now $y = 3$.

Remember, a vertical shift affects the horizontal asymptote!

(2) a vertical shift down 3 units

$y = 2^x – 3$

The equation of the horizontal asymptote is now $y = – 3$.

Example 1: Apply Transformations to Common Exponential Functions (2 of 2)

(3) a horizontal shift to the right 3 units

$y = 2^{x-3}$

The equation of the horizontal asymptote is still $y = 0$.

(4) a horizontal shift to the left 3 units

$y = 2^{x+3}$

The equation of the horizontal asymptote is still $y = 0$.

Example 2: Apply Transformations to Common Exponential Functions (1 of 2)

Starting with the graph of $f(x) = \boldsymbol{e}^x$, write the equation of the graph that results from the following. Include the equation of the horizontal asymptote.

(1) a reflection across the *x*-axis

$y = -\boldsymbol{e}^x$

The equation of the horizontal asymptote is still $y = 0$.

(2) a reflection across the *y*-axis

$y = \boldsymbol{e}^{-x}$

The equation of the horizontal asymptote is still $y = 0$.

Example 2: Apply Transformations to Common Exponential Functions (2 of 2)

(3) a reflection across the *x*- and *y*-axis

$y = -\boldsymbol{e}^{-x}$

The equation of the horizontal asymptote is still $y = 0$.

(4) a vertical shift up 1 unit and a horizontal shift to the right 2 units

$y = \boldsymbol{e}^{x-2} + 1$

The equation of the horizontal asymptote is now $y = 1$.

Example 3: Apply Transformations to Common Exponential Functions

Starting with the graph of $f(x) = 3^x$, write the equation of the graph that results from the following. Include the equation of the horizontal asymptote.

(1) a vertical shift up 2 units and a horizontal shift to the left 1 unit

$y = 3^{x+1} + 2$

The equation of the horizontal asymptote is now $y = 2$.

Remember, a vertical shift affects the horizontal asymptote!

(2) a vertical shift down 2 units and a horizontal shift to the right 1 unit

$y = 3^{x-1} - 2$

The equation of the horizontal asymptote is now $y = -2$.

Example 4: Graph a Common Exponential Function by Hand (1 of 3)

Graph the function $\boldsymbol{f(x) = 2^x}$ by hand.

1. **Equation of the horizontal asymptote:**

   Since the function is of the form $y = b^x$, where $b = 2$ and there are no transformations, we can conclude that the equation of the *horizontal asymptote* is $y = 0$, which is the x-axis.

2. **Point associated with the *y*-intercept (when *x* = 0):**

   $f(0) = 2^0 = 1$

   The y-intercept is at (0, 1).

Example 4: Graph a Common Exponential Function by Hand (2 of 3)

3. Find additional points to either side of the *y*-intercept:

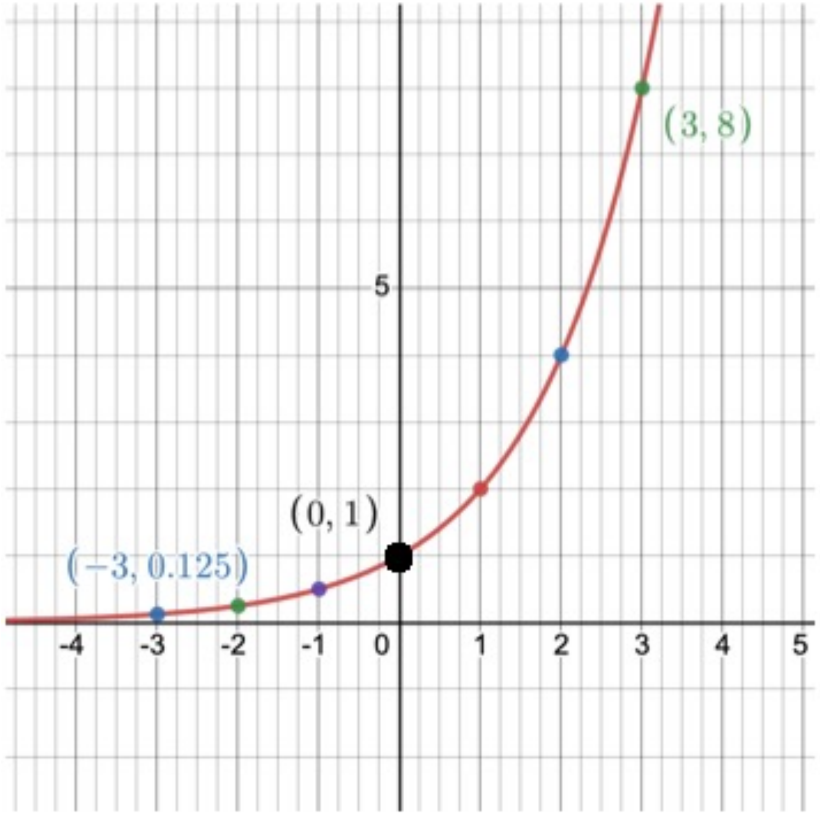
How about $x = -3, -2, -1, 1, 2,$ and 3?

Using $\boldsymbol{f(x) = 2^x}$, we set up a table of coordinates and then plot these points.

x	$f(x) = 2^x$
-3	$2^{-3} = 0.125$
-2	$2^{-2} = 0.25$
-1	$2^{-1} = 0.5$
1	$2^1 = 2$
2	$2^2 = 4$
3	$2^3 = 8$

Example 4: Graph a Common Exponential Function by Hand (3 of 3)

4. Connect all points found in the previous steps keeping in mind the shape of the graph of a common exponential function:



Since the horizontal asymptote is the *x*-axis, we will not graph it as a dashed line.

Example 5: Graph a Transformation of a Common Exponential Function by Hand (1 of 3)

Graph the function $\boldsymbol{f(x) = 2^{x+1} - 3}$ by hand.

1. Equation of the horizontal asymptote:

The function is a transformation of $y = 2^x$ whose graph has a *horizontal asymptote* at $y = 0$.

We notice a horizontal shift 1 unit to the left and a vertical shift 3 units down. We learned that vertical shifts affect *horizontal asymptotes*. Here, it will move the *horizontal asymptote* to $y = -3$.

2. Point associated with the *y*-intercept (when x = 0):

$f(0) = 2^{0+1} - 3 = -1$.

The *y*-intercept is at $(0, -1)$.

Example 5: Graph a Transformation of a Common Exponential Function by Hand

3. Find additional points to either side of the *y*-intercept:

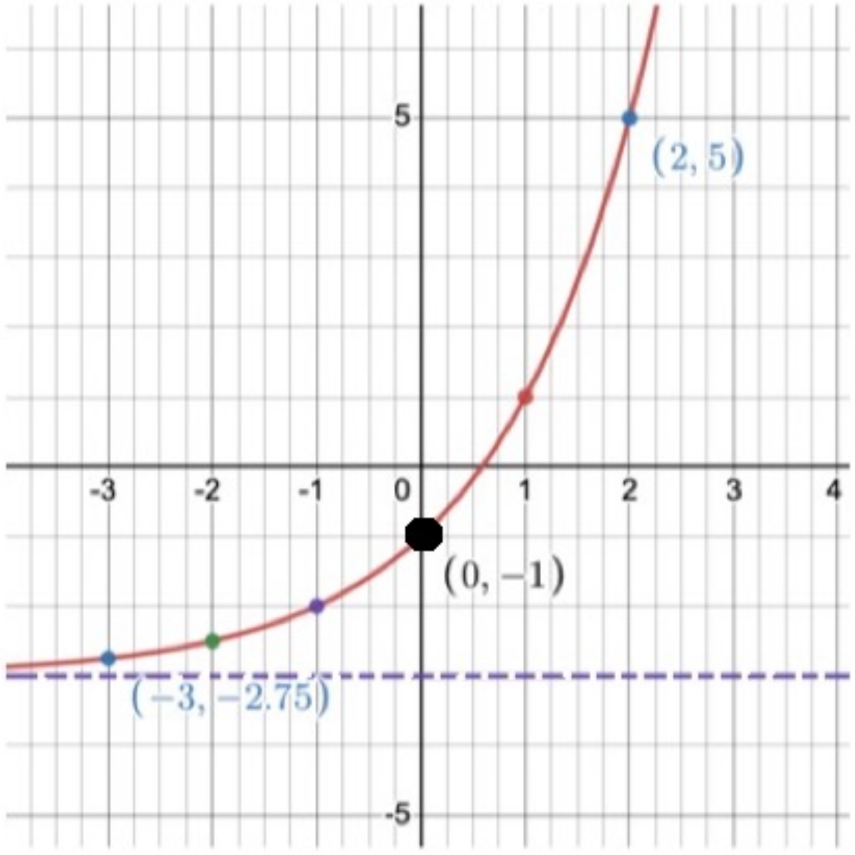
How about $x = -3, -2, -1, 1$, and 2?

Using $f(x) = 2^{x+1} - 3$, we set up a table of coordinates and then plot these points.

x	$f(x) = 2^{x+1} - 3$
-3	$2^{-3+1} - 3 = -2.75$
-2	$2^{-2+1} - 3 = -2.5$
-1	$2^{-1+1} - 3 = -2$
1	$2^{1+1} - 3 = 1$
2	$2^{2+1} - 3 = 5$

Example 5: Graph a Transformation of a Common Exponential Function by Hand (1 of 3)

4. Connect all points found in the previous steps keeping in mind the shape of the graph of a common exponential function:



Note: The *horizontal asymptote* is drawn as a dashed line when we graph exponential functions by hand!